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Competing first passage percolation on random graphs with finite variance degrees

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Abstract

We study the growth of two competing infection types on graphs generated by the configuration model with a given degree sequence. Starting from two vertices chosen uniformly at random, the infection types spread via the edges in the graph in that an uninfected vertex becomes type 1 (2) infected at rate λ_1 (λ_2) times the number of nearest neighbors of type 1 (2). Assuming (essentially) that the degree of a randomly chosen vertex has finite second moment, we show that if $\lambda_1 = \lambda_2$, then the fraction of vertices that are ultimately infected by type 1 converges to a continuous random variable $V \in (0, 1)$, as the number of vertices tends to infinity. Both infection types hence occupy a positive (random) fraction of the vertices. If $\lambda_1 \neq \lambda_2$, on the other hand, then the type with the larger intensity occupies all but a vanishing fraction of the vertices. Our results apply also to a uniformly chosen simple graph with the given degree sequence.

Keywords: Random graphs, configuration model, first passage percolation, competing growth, coexistence, continuous-time branching process.

MSC 2010 classification: 60K35, 05C80, 90B15.

1 Introduction

Fix $n \geq 1$ and let $(d_1, \ldots, d_n)$ be a sequence of positive integers that may depend on n. Consider a graph with n vertices and degrees $(d_1, \ldots, d_n)$ generated by the configuration model, that is, equip each vertex $i \in \{1, \ldots, n\}$ with d_i half-edges, and pair half-edges uniformly at random to create edges. For all half-edges to find a partner we assume that the total degree $\sum d_i$ is even. Assign independently to each edge e in the resulting graph two independent exponentially distributed passage times $X_1(e)$ and $X_2(e)$ with parameter λ_1 and λ_2, respectively. At time 0, two uniformly chosen vertices are infected with infections type 1 and type 2, respectively, and the infections then spread via nearest neighbors: When a vertex becomes type 1 (2) infected, the time that it takes for the infection to traverse an edge e emanating from the vertex is given by $X_1(e)$ ($X_2(e)$). If the other end point of the edge e is still uninfected at that time, it becomes type 1 (2) infected and remains so forever. It also becomes immune to the other infection type.

In this paper we study the above competing growth process on a random graph generated from a given degree sequence subject to the regularity conditions stated below. These conditions ensure that the graph contains a giant component occupying all but a vanishing fraction of the vertices as $n \to \infty$, and hence that almost all vertices will w.h.p. be infected when the process terminates. The question that we will be interested in is the outcome of this competition. Specifically, will both types occupy a strictly positive fraction of the vertices in the limit as $n \to \infty$? We show that the answer is yes if and only if $\lambda_1 = \lambda_2$. This question has previously been