

## A test on the location of the tangency portfolio on the set of feasible portfolios

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## Abstract

Due to the problem of parameter uncertainty, specifying the location of the tangency portfolio(TP) on the set of feasible portfolios becomes a challenging task. The set of feasible portfolios is a parabola in the mean-variance space with optimal portfolios lying on its upper part. Using statistical test theory, we want to decide if the tangency portfolio is mean-variance efficient, i.e. if it belongs to the upper limb of the efficient frontier. In the opposite case, the investor would prefer to invest into the risk-free asset or into the global minimum variance portfolio which lies in the vertex of the set of feasible portfolios. Assuming that the portfolio asset returns are independent and multivariate normally distributed, we suggest a test on the location of the tangency portfolio on the set of feasible portfolios. The distribution of the test statistic is derived under both hypotheses, which we use to assess the power of the test and construct a confidence interval. Moreover, out-of-sample performance of the test is evaluated based on real data. The robustness to the assumption of normality is investigated via an extensive simulation study where we show that the new test is robust to the violation of the normality assumption and can also be used for heavy-tailed stochastic models. Moreover, in an empirical study we apply the developed theory to real data. We find that when the sample size is relatively large and a stable period is present on the market, then the mean-variance efficiency of the tangency portfolio can be statistically justified.

*Keywords*: tangency portfolio, feasible portfolios, test theory, power function, out-of-sample performance

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