Discriminant analysis in small and large dimensions

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Abstract

We study the distributional properties of the linear discriminant function under the assumption of normality by comparing two groups with the same covariance matrix but different mean vectors. A stochastic representation for the discriminant function coefficients is derived which is then used to obtain their asymptotic distribution under the high-dimensional asymptotic regime. We investigate the performance of the classification analysis based on the discriminant function in both small and large dimensions. A stochastic representation is established which allows to compute the error rate in an efficient way. We further compare the calculated error rate with the optimal one obtained under the assumption that the covariance matrix and the two mean vectors are known. Finally, we present an analytical expression of the error rate calculated in the high-dimensional asymptotic regime. The finite-sample properties of the derived theoretical results are assessed via an extensive Monte Carlo study.

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