



Bayesian Inference of the Multi-Period Optimal Portfolio for an Exponential Utility

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Abstract

We consider the estimation of the multi-period optimal portfolio obtained by maximizing an exponential utility. Employing Jeffreys' non-informative prior and the conjugate informative prior, we derive stochastic representations for the optimal portfolio weights at each time point of portfolio reallocation. This provides a direct access not only to the posterior distribution of the portfolio weights but also to their point estimates together with uncertainties and their asymptotic distributions. Furthermore, we present the posterior predictive distribution for the investor's wealth at each time point of the investment period in terms of a stochastic representation for the future wealth realization. This in turn makes it possible to use quantile-based risk measures or to calculate the probability of default. We apply the suggested Bayesian approach to assess the uncertainty in the multi-period optimal portfolio by considering assets from the FTSE 100 in the weeks after the British referendum to leave the European Union. The behaviour of the novel portfolio estimation method in a precarious market situation is illustrated by calculating the predictive wealth, the risk associated with the holding portfolio, and the default probability in each period.

Keywords: Multi-period optimal portfolio, Bayesian estimation, Stochastic representation, Posterior predictive distribution, Default probability, Credible sets

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