Estimating the Proportion of True Null Hypotheses under Copula Dependency

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It is a well known result in multiple hypothesis testing that the proportion $\pi_0$ of true null hypotheses is not identified under general dependencies. However, it is possible to estimate $\pi_0$ if structural information about the dependency structure among the test statistics or $p$-values, respectively, is available. We demonstrate these points, and propose a marginal parametric bootstrap method. A pseudo-sample of bootstrap $p$-values is generated, which still carry information about $\pi_0$, but behave like realizations of stochastically independent random variables. Theoretical properties of resulting estimation procedures for $\pi_0$ are analyzed and their usage is illustrated on synthetic and real data.

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