Continuous-time limits of multi-period cost-of-capital valuations

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Abstract

We consider multi-period cost-of-capital valuation of a liability cash flow subject to repeated capital requirements that are partly financed by capital injections from capital providers with limited liability. Limited liability means that, in any given period, the capital provider is not liable for further payment in the event that the capital provided at the beginning of the period turns out to be insufficient to cover both the current-period payments and the updated value of the remaining cash flow. The liability cash flow is modeled as a continuous-time stochastic process on $[0, T]$. The multi-period structure is given by a partition of $[0, T]$ into subintervals, and on the corresponding finite set of times a discrete-time value process is defined. Our main objective is the analysis of existence and properties of continuous-time limits of discrete-time value processes corresponding to a sequence of partitions whose meshes tend to zero. Moreover, we provide explicit and interpretable valuation formulas for a wide class of cash flow models.

1   Introduction

1.1   Multi-period cost-of-capital valuation

The paper focuses on the multi-period cost-of-capital valuation of a cumulative liability cash flow $L = \{L_t\}_{t \in [0,T]}$ subject to repeated capital requirements at the beginning of each time period, where the time periods form a partition of $[0, T]$. Here T is a time after which no cash flow occurs. In line with current regulatory frameworks, the time periods may be one-year periods. However, we will here investigate the effects of varying the number and lengths of the periods and in particular consider a sequence of partitions of $[0, T]$ whose meshes tend to 0. That is, we will analyze continuous-time limits of discrete-time cost-of-capital valuations of the liability cash flow L. In what follows, all cash flows and financial values are discounted by a given numéraire, or equivalently, denoted in units of this numéraire. A classical bank account numéraire, a rolling one-period bond, may be a natural choice.