



# Mean-Variance Efficiency of Optimal Power and Logarithmic Utility Portfolios

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## Abstract

We derive new results related to the portfolio choice problem for a power and logarithmic utilities. Assuming that the portfolio returns follow a log-normal distribution, the closed-form expressions of the optimal portfolio weights are obtained for both utility functions. Moreover, we prove that both optimal portfolios belong to the set of mean-variance feasible portfolios and establish necessary and sufficient conditions such that they are mean-variance efficient. Furthermore, an application to the stock market is presented and the behavior of the optimal portfolio is discussed for different values of the relative risk aversion coefficient. It turns out that the assumption of log-normality does not seem to be a strong restriction.

**Keywords:** optimal portfolio selection; power utility; log-normal distribution; mean-variance analysis; logarithmic utility.

## 1 Introduction

The theory of optimal portfolio choice started with the pioneering contribution of Markowitz (1952). Markowitz used the variance as a measure of the risk of a portfolio return. He recommended to choose the portfolio weights in such a way that the portfolio variance is minimal for a given level of the expected portfolio return. All of these so-called efficient portfolios lie in the efficient frontier which is a parabola in the mean-variance space.

In the meantime many further proposals for a portfolio selection have been made. A widely made approach is based on the maximization of an investor's utility function,

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