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Stein-Haff identity for the exponential family

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Abstract

In this paper, the Stein-Haff identity is established for positive-definite and symmetric random matrices belonging to the exponential family. The identity is then applied to the matrix-variate gamma distribution, and an estimator that dominates the maximum likelihood estimator in terms of Stein's loss is obtained. Finally a simulation study is conducted in order to support the theoretical results.

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1 Introduction

The Stein-Haff identity was first derived by Stein (1977) and Haff (1979) regarding the problem of estimating the covariance matrix of multivariate normal populations. The $p \times p$ sample covariance matrix $\mathbf{W}$ of such a population follows a Wishart distribution, and is commonly estimated using the unbiased estimator $\mathbf{W}/n$, where n is the sample size. However, the eigenvalues of the estimator $\mathbf{W}/n$ tends to spread out more over the positive real line, than the equivalent eigenvalues of the population covariance matrix $\mathbf{\Sigma}$. For example, letting $\lambda_1, \ldots, \lambda_p$ be the p ordered eigenvalues of $\mathbf{\Sigma}$ and $l_1, \ldots, l_p$ be the p ordered sample eigenvalues of $\mathbf{W}/n$, l_1 is an positively biased estimator of λ_1 and l_p is a negatively biased estimator of λ_p (see e.g. Van der Vaart (1961)). As such, it can often be useful to consider estimators that aim to decrease larger sample eigenvalues and increase smaller sample eigenvalues.

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