Stein-Haff identity for the exponential family

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Abstract

In this paper, the Stein-Haff identity is established for positive-definite and symmetric random matrices belonging to the exponential family. The identity is then applied to the matrix-variate gamma distribution, and an estimator that dominates the maximum likelihood estimator in terms of Stein’s loss is obtained. Finally a simulation study is conducted in order to support the theoretical results.

Keywords and phrases: Random matrices, matrix-variate gamma distribution, decision theory.

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1 Introduction

The Stein-Haff identity was first derived by Stein (1977) and Haff (1979) regarding the problem of estimating the covariance matrix of multivariate normal populations. The $p \times p$ sample covariance matrix $W$ of such a population follows a Wishart distribution, and is commonly estimated using the unbiased estimator $W/n$, where $n$ is the sample size. However, the eigenvalues of the estimator $W/n$ tends to spread out more over the positive real line, than the equivalent eigenvalues of the population covariance matrix $\Sigma$. For example, letting $\lambda_1, \ldots, \lambda_p$ be the $p$ ordered eigenvalues of $\Sigma$ and $l_1, \ldots, l_p$ be the $p$ ordered sample eigenvalues of $W/n$, $l_1$ is a positively biased estimator of $\lambda_1$ and $l_p$ is a negatively biased estimator of $\lambda_p$ (see e.g. Van der Vaart (1961)). As such, it can often be useful to consider estimators that aim to decrease larger sample eigenvalues and increase smaller sample eigenvalues.

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