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Abstract

The general principles for determining the financial performance of a company is that revenue is earned as goods are delivered or services provided, and that expenses in the period are made up of the costs associated with this earned revenue. In the insurance industry, premium payments are typically made upfront, and can provide coverage for several years, or be paid many years before the coverage period starts. The associated costs are often not fully known until many years later. Hence, complexity arises both in determining how a premium paid should be earned over time, and in valuing the costs associated with this earned premium. IFRS 17 attempts to align the insurance industry with these general accounting principles. We bring this new accounting standard into the realm of actuarial science, through a mathematical interpretation of the regulatory texts, and by defining the algorithm for profit or loss in accordance with the new standard. Furthermore, we suggest a computationally efficient risk-based method of valuing a portfolio of insurance contracts and an allocation of this value to subportfolios. Finally, we demonstrate the practicability of these methods and the algorithm for profit or loss in a large-scale numerical example.

1 Introduction

The accrual method for calculating profit or loss has for a long time been the standard method used when large companies prepare their financial statements. Hence, profit or loss is calculated as the difference between revenue and expenses, where revenue is earned as goods are delivered or services are provided, and expenses should match revenue, i.e. expenses are recognised in the period in which the associated revenue is recognised. This is different from the more basic cash method of calculating profit or loss, where revenue is recognised when cash is received and expenses when cash is paid. Naturally, the total profit or loss over the lifetime of a company is the same irrespective of method, but the method used will determine how profit or loss is allocated to different time periods. Since the main objective of financial statements of listed companies is to provide investors with the information they need to make informed economic decisions, the accrual method is the prescribed method in both International Financial Reporting Standards (IFRS) and Generally Accepted Accounting Practices (GAAP).

For some industries, the accrual method and the cash method for calculating profit or loss would give similar results, even in the short run. This is generally not the case in the insurance industry, due to the inherent nature of insurance products. In an insurance contract, an insurer agrees to compensate a policyholder in case of an insured event occurring. In exchange, the policyholder pays a premium in advance of the provision of coverage. When the contract is issued, the claim payments are uncertain in amount or timing, or both. The premium payment, the coverage period, and the payments due to claims in this coverage period generally occur at different time points, sometimes many years apart. Hence, to recognise premium payments as revenue and claims payments as costs according to when they are paid would not accurately reflect the profit or loss in the period in question for insurance products, and would therefore be misleading.

For some types of insurance products, insurance coverage is typically purchased for one year at a time. Hence, to determine when the premium is earned is a fairly standard matter. Complexity arises when the total claims costs due to coverage provided during one reporting period only are fully determined several years later for some product lines. To faithfully represent the economics of this type of contract, the total claims cost somehow need to be measured and recognised in the period in which the premium is earned, i.e. as coverage is provided. For other types of insurance products, claims are generally paid as they are incurred, but on the other hand a premium payment often provides many years of coverage, and can be paid several years before the coverage period starts. Hence, the premium payment and any claims payments and other costs associated with the coverage that this premium provides need to be taken into account in order to derive a net liability value, i.e. an estimate of the profit or loss for the contract. This should be earned over the coverage period, since this is when services are provided. One also needs to take into account that when the coverage period is long, facts and circumstances might change in such a way that the net liability value changes compared to the estimated value at the time a contract is issued, and decide how this type of changes should affect profit or loss in the period, as well as the estimate of the remaining profit or loss for the contract.

As described in [11], there is currently considerable variation in financial reporting practices in different jurisdictions for the insurance industry. The measurement of contracts and recognition of revenue also varies greatly compared to other industries, where accrual accounting as above is the general standard. The need for a consistent framework for accounting of insurance contracts has long been recognised, now resulting in IFRS 17, which provides a systematic way of recognising unearned profit over time.

1.1 Introducing IFRS 17

In the current European solvency regulatory framework, Solvency II, the concept of risk-based valuation of liabilities is one of the cornerstones in determining the solvency positions of insurance companies. In the coming years, risk-based valuation of liabilities will also need to be incorporated in how companies measure their financial position and performance, as the new international financial standard for insurance contracts, IFRS 17, comes into force.

The current accounting standard for insurance contracts, IFRS 4, is an interim standard, which sets some minimum requirements on the accounting policies in different jurisdictions, but apart from this allows considerable variation in financial reporting practices. The main objectives of IFRS 17 is thus to make accounting practices more consistent over different jurisdictions as well as making the financial statements of insurance companies more informative. It is stressed in [10] that financial statements should "provide relevant information that faithfully represents ... [insurance] contracts" and should "reflect true underlying financial positions or performance arising from these insurance contracts", in contrast to many current accounting practices that e.g. do not use current estimates of all cash flows, require no explicit risk measurement, or disregards the time value of money in the valuation (§ BC14 in [11]).

In Solvency II, market-consistent valuation is based on a cost-of-capital approach, and the formula and parameters used for the calculation are prescribed by the regulation (see [4] and [3]). In IFRS 17, which is principles based, there is much more freedom to choose what valuation technique to use, but the general principles are largely consistent with the ones in Solvency II. It is thus up to each company to choose the appropriate technique reflecting the true economics of its insurance contracts. Furthermore, in IFRS 17 the the financial performance of a company will be linked to the valuation of its insurance liabilities, and this link is missing in Solvency II which focuses solely on solvency.

The requirements in IFRS 17 raise questions regarding how to compute the risk adjustment for non-financial risk, defined as "the compensation that the entity requires for bearing the uncertainty about the amount and timing of the cash flows that arises from non-financial risk" (§ 37 in [10]), which is part of the insurance liability value. Other considerations are how to calculate the confidence level that this risk adjustment corresponds to, as well as how to allocate the risk adjustment to a potentially large number of groups of contracts. With this new accounting standard for insurance contracts, the decisions made on these topics will influence, not only the financial position of the company at a certain date, but also the financial performance of the company for many years to come, since it directly affects the revenue streams presented by the company in its financial statements.

1.2 Our objective

Our objective is to present how to measure the financial performance of an insurer in accordance with IFRS 17 and demonstrate in terms of a realistic numerical example how the computational challenges may be handled efficiently. Throughout the paper we will disregard any costs not directly related to servicing contracts and we will only consider contracts that generate cash flows that are independent of financial asset values. In order to measure the financial performance of an insurer in accordance with IFRS 17, the following four components need to be determined:

Stochastic model. The policies written generate future payments to policyholders at random times and of random sizes. Groups of policies are therefore associated with stochastic cash flows and we must decide on a joint model for these stochastic cash flows.

Valuation method. In order to determine how much of the premium income can be considered as earned premium income resulting in profit or loss, the aggregate liability cash flow must be valued and revalued over time. We emphasise that it is the aggregate stochastic liability cash flow that is subject to capital requirements and possibly other capital costs for the insurer. Therefore we must decide on a method for valuation of aggregate stochastic cash flows.

Allocation method. Given that we assign a value to the aggregate liability cash flow, we must decide on how much of this value should be allocated to different groups of policies in order to determine profit or loss for these groups. Therefore, in addition to a risk-based valuation method we must decide on a risk-based allocation method.

Profit and loss algorithm. Given a joint stochastic model for liability cash flows, a valuation method for the aggregate liability cash flow, and an allocation method defining how the aggregate liability value should be split into contributions to this value for a partition of all policies into groups of policies, an accounting method defines profits and losses over time and across groups. We demonstrate that the IFRS 17 accounting standard defines an algorithm that, given a stochastic model, a valuation method and an allocation method, defines profits and losses of groups of policies over time.

We emphasise that the four components above can be regarded as independent. Whereas IFRS 17 essentially defines the profit and loss algorithm, the stochastic model, the valuation method, and the allocation method can be chosen independently. That is, you may replace the valuation and allocation methods advocated here by your preferred choices (given a convincing motivation for doing so)!

1.3 Organisation of the paper

This paper is organised as follows. Section 2 presents the liability cash flows and the different liability values that are key ingredients in the analysis of financial performance. Section 3 formalises the algorithm for financial performance of an insurer in accordance with IFRS 17. Section 4 presents valuation of aggregate insurance liability cash flows and allocation of these values to cash flows that are part of the aggregate cash flow. Section 4.1 presents multi-period cost-of-capital valuation, similar to what was developed in [16] and [7] and presents explicit valuation and allocation formulas that hold exactly when cash flows and covariates representing the flow of information follow a Gaussian process. Section 5 presents a life-insurance example that demonstrates how the computational challenges that result from a large-scale implementation of an IFRS 17 financial performance analysis can be handled. Moreover, based on a standard mortality model with parameters estimated on data from the Human Mortality Database we illustrate properties of the valuation and allocation method and the IFRS 17 profit and loss algorithm for a life-insurance portfolio of realistic size. The appendix provides further details on the IFRS 17 profit and loss algorithm, computational details left out in the life-insurance example, and technical arguments on the allocation of an aggregate value to groups of contracts.

2 Liability cash flows and values

Consider an integer $i_0 < 0$, the set of time points $\mathcal{T} := \{i_0 - 1, i_0, \dots\}$ and the set of time periods $\mathcal{T}_+ := \{i_0, i_0 + 1, \dots\}$. Contracts issued during time period $t \in \mathcal{T}_+$ are issued in the time interval $(t-1, t]$. Let current time be time 0. Consider development periods $1, 2, \dots, \tau$, where τ is a positive integer chosen so that all contracts issued during an issuing period i are terminated after at most τ periods after the start of the issuing period. When a contract is terminated the insurance obligation is fulfilled, e.g. due to lapses, or the end of the decumulation phase for annuities. For $t \in \mathcal{T}$, let Gr_t be the set of groups that are part of the outstanding liability at time t. Each group of contracts is associated to an issuing period but there may be several groups associated to the same issuing period. For example, contracts issued during the same period might be grouped according to their product line or profitability profile. Let $I_t^{(g)}$ denote the incremental net cash flow for group g in time period t, i.e. in the time interval $(t-1, t]$.

Let $X^{(t)}$ be the cash flow that corresponds to the outstanding liability as seen from time t (in run-off):

$$X^{(t)} := \sum_{g \in \mathrm{Gr}_t} \left(I_{t+1}^{(g)}, \dots, I_{t+\tau}^{(g)} \right).$$

Let $L^{(t)}$ denote the value of the outstanding liability cash flow as seen from time t, and let $L^{(t,g)}$ denote the liability value allocated to group g, i.e.

$$L^{(t)} = \sum_{g \in \mathrm{Gr}_t} L^{(t,g)}.$$

If $g \notin \mathrm{Gr}_t$, then $L^{(t,g)} = 0$. Note that $L^{(t,g)}$ is a part of the value $L^{(t)}$ of the sum $X^{(t)}$ that is allocated to one of its terms $X^{(t,g)} := (I_{t+1}^{(g)}, \dots, I_{t+\tau}^{(g)})$. Table 1 illustrates incremental liability cash flows and groups of contracts for the special case when $i_0 = -5$, $\tau = 5$ and there is a single group per issuing period.

The outstanding liability cash flows can be partitioned into cash flows that correspond to insured events that have not yet occurred, and cash flows that correspond to insured events that have already occurred, including occurred claims that are not yet reported. The liability for remaining coverage,

	1	2	3	4	5
-5	$I_{-5}^{(-5)}$	$I_{-4}^{(-5)}$	$I_{-3}^{(-5)}$	$I_{-2}^{(-5)}$	$I_{-1}^{(-5)}$
-4	$I_{-4}^{(-4)}$	$I_{-3}^{(-4)}$	$I_{-2}^{(-4)}$	$I_{-1}^{(-4)}$	$I_{0}^{(-4)}$
-3	$I_{-3}^{(-3)}$	$I_{-2}^{(-3)}$	$I_{-1}^{(-3)}$	$I_{0}^{(-3)}$	$I_{1}^{(-3)}$
-2	$I_{-2}^{(-2)}$	$I_{-1}^{(-2)}$	$I_{0}^{(-2)}$	$I_{1}^{(-2)}$	$I_{2}^{(-2)}$
-1	$I_{-1}^{(-1)}$	$I_{0}^{(-1)}$	$I_{1}^{(-1)}$	$I_{2}^{(-1)}$	$I_{3}^{(-1)}$
0	$I_{0}^{(0)}$	$I_{1}^{(0)}$	$I_{2}^{(0)}$	$I_{3}^{(0)}$	$I_{4}^{(0)}$
1	$I_{1}^{(1)}$	$I_{2}^{(1)}$	$I_{3}^{(1)}$	$I_{4}^{(1)}$	$I_{5}^{(1)}$
$\vdots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$

Table 1: Incremental liability cash flows illustrated for $i_0 = -5$, $\tau = 5$ and a single group per issuing period. Since there is a single group per issuing period each group can be identified with the issuing period: $\mathrm{Gr}_0 = \{-3, -2, -1, 0\}$ and $\mathrm{Gr}_1 = \{-2, -1, 0, 1\}$. The outstanding liability cash flow $X^{(0)}$ at time 0 corresponds to the cells coloured light gray.

denoted by $L_{\mathrm{RC}}^{(t,g)}$, is defined as the liability value for group g at time t allocated to the former, and the liability for incurred claims, denoted by $L_{\mathrm{IC}}^{(t,g)}$, is defined as the liability value for group g at time t allocated to the latter. Hence,

$$L^{(t,g)} = L_{\mathrm{RC}}^{(t,g)} + L_{\mathrm{IC}}^{(t,g)}.$$

Let $L_{\mathrm{SP}}^{(t,g)}$ denote the liability value for remaining coverage allocated to services provided between t and $t+1$ for group g, and $L_{\mathrm{FS}}^{(t,g)}$ the liability value allocated to future service (after $t+1$) for group g, as seen from time t, hence

$$L_{\mathrm{RC}}^{(t,g)} = L_{\mathrm{SP}}^{(t,g)} + L_{\mathrm{FS}}^{(t,g)}.$$

The liability values assigned to cash flows are computed by some method that takes the time value of money into account. It is common to handle this by considering discounting in terms of a money market account numeraire. From a modelling point of view the the parameters of the short rate model corresponding to the money market account should be such that the discount factors from the model agree with the given set of discount factors. In our context we need to consider liability values that are computed using discounting that depends on the discount factors that were realised at different times (the times of initial recognition of the various contract groups). To that end, let $L_{\mathrm{RC},t_0}^{(t,g)}$ denote the liability value for remaining coverage for group g at time t measured at the discount rates at time $t_0 \leq t$, where t_0 is the time of initial recognition of group g. This corresponds to setting the parameters of the short-rate model for the money market account numeraire to ensure consistency with discount factors at the time of initial recognition of a group of contracts. Similarly to $L_{\mathrm{RC},t_0}^{(t,g)}$, let $L_{\mathrm{FS},t_0}^{(t,g)}$ denote the liability value allocated to future service for group g as seen from time t measured at the discount rates at time t_0. Furthermore, let $d_{t_0,t}$ denote the discount factor between t_0 and t, known at time t_0. Note that $L_{\mathrm{RC},t_0}^{(t_0,g)} = L_{\mathrm{RC}}^{(t_0,g)}$, $L_{\mathrm{FS},t_0}^{(t_0,g)} = L_{\mathrm{FS}}^{(t_0,g)}$ and $d_{t_0,t_0} = 1$.

Remark 1 (Initial recognition)**.** *Initial recognition of a group of contracts is defined as the earliest of the following three dates: the beginning of the coverage period of the group; the date when the first payment from a policyholder in the group becomes due; and, for an onerous group, when the group becomes onerous (§ 25 in [10]). A group of contracts is onerous at initial recognition if the value of the outstanding liability cash flow for the group exceeds any cash flows for the group at that date (e.g. premium income received), resulting in a net outflow for the group (§ 47 in [10]). Hence, if the insurer has reason to believe that a contract group is onerous (due to e.g. pricing), this needs to be determined as the contracts are issued, by calculating the value of the outstanding liability cash flow at that date.*

Let $P^{(t,g)}$ denote the premium income during period t, i.e. between time $t-1$ and t, for contracts belonging to group g at time t, *issued during this period.* To avoid double counting, $I_t^{(g)}$ does not include premium income from contracts belonging to group g that are issued in period t, but can include premium income for contracts belonging to group g that have been issued in previous periods, if this premium income is within the contract boundary. If the premium income is not within the contract boundary it is viewed as a new contract, and hence is included in $P^{(t,g)}$ if this new contract belongs to group g.

Remark 2 (Contract boundary)**.** *In IFRS 17 premiums are within the contract boundary if the insurer can force the policyholder to pay them, or if the insurer has to accept future premium payments from the policyholder, without being able to reprice the contract or change the benefit level to fully reflect the risks of the policyholder or the risks of the portfolio that the contract in question belongs to (§ 34 in [10]).*

3 Formalising financial performance in IFRS 17

The key principles when measuring the financial performance of a company in accordance with IFRS 17 is that insurance contracts need to be divided into groups which should be valued as the sum of the fulfilment cash flows and the contractual service margin, the former being "a risk-adjusted present value of the future cash flows ... consistent with observable market information" and the latter "an amount representing the unearned profit in the group." (§ IN6 in [10]). Hence, in contrast to Solvency II, IFRS 17 does not allow the company to recognise unearned future profit as a gain when the group of contracts is initially recognised. Instead the profit from the group will be recognised over the period that insurance coverage is provided, and as it is released from risk. However, the contractual service margin cannot be negative, i.e. if a group is or becomes loss-making, this loss will be recognised immediately.

The procedure for determining the financial position of a company can be summarised as follows:

1. Divide contracts into groups according to product line, profitability, and date issued (contracts in the same group have to be issued within one year of each other).

2. Valuation of the outstanding insurance liability as "estimates of future cash flows, ... an adjustment to reflect the time value of money and the financial risks related to the future cash flows, ... and a risk adjustment for non-financial risk" (§ 32 in [10]).

3. Allocate the aggregate insurance liability value to each group of contracts.

4. Determine the contractual service margin, if any, as the amount that ensures that the initial recognition of a group of contracts does not result in any gain. Hence the sum of the insurance liability value allocated to the group, the contractual service margin, and any cash flows arising from the contracts at that date should be equal to zero (§ 38 in [10]). The contractual service margin cannot be negative, hence if the sum of the insurance liability value allocated to the group and any cash flows arising from the contracts at initial recognition is positive, this amount will instead be the value of the loss component at initial recognition (§ 47 in [10]).

5. In subsequent measurement, the contractual service margin at the end of the period, for each group, consists of the contractual service margin at the beginning of the period, adjusted for new contracts added to the group, if any; interest accreted on the contractual service margin in the period; the change in the liability value due to future service allocated to the group; and allocation of part of the contractual service margin to profit or loss in the period (§ 44 in [10]). If the contractual service margin at the beginning of the period is zero, i.e. there is a loss component, the development in the period will only give rise to a contractual service margin at the end of the period if the loss component is first fully reversed.

The profit or loss of a company consists of:

(i) The part of the contractual service margin allocated to profit or loss in step 5 above, if any.

(ii) Experience adjustments, i.e. the difference between "the estimate at the beginning of the period of the amounts expected to be incurred in the period and the actual amounts incurred in the period." (Appendix A in [10])

(iii) The release from risk in the period, i.e. the change in the risk adjustment for non-financial risk due to services provided in the period.

(iv) The loss component at initial recognition for onerous groups of contracts, or any increase or reversal of the loss component at subsequent measurement.

(v) The change in the outstanding liability value and contractual service margin due to the effect of the time value of money and changes in the time value of money.

Note that step (ii), (iii), and (v) are only valid in the case of a contractual service margin at the beginning of the period. If there instead is a loss component at the beginning of the period a proportion of the estimate at the beginning of the period in step (ii), of the release from risk in step (iii), and of the change due to the time value of money in step (v) will instead adjust the loss component. Hence, in order to avoid double counting, only the remaining part of these amounts will be included in step (ii), (iii), and (v). Furthermore, note that the estimate at the beginning of the period in step (ii) together with the release from risk in step (iii) is the same as the liability value for remaining coverage allocated to services provided, $L_{\mathrm{SP}}^{(t)}$.

Let $\mathrm{CSM}^{(t,g)}$ denote the contractual service margin and $\mathrm{LC}^{(t,g)}$ the loss component at time t for group g. The definition of the contractual service margin and the loss component as mathematical objects is as follows. We use the notation $x^+ := \max(x, 0)$ and $x^- := \max(-x, 0) = -\min(x, 0)$. The definition involves weights $W_t^{(g)}$ that, for a given time t and group g, $W_t^{(g)}$ represents the proportion of the unearned profit at time t for group g that is allocated to profit or loss in periods $> t$. Therefore, $1 - W_t^{(g)}$ represents the proportion of the unearned profit at time t for group g that is allocated to profit or loss in period t. $W_t^{(g)}$ is a $[0, 1]$-valued random variable known at time t.

Definition 1 (Algorithm for calculating CSM and LC)**.** *Consider a sequence of groups* $(\mathrm{Gr}_t)_{t\in\mathcal{T}}$ *and a sequence* $(W_t^{(g)})_{t\in\mathcal{T}}$*, where* $W_t^{(g)}$ *is* $[0, 1]$*-valued and* $W_t^{(g)} = 0$ *for* $g \notin \mathrm{Gr}_t$*. Fix* $(t, g) \in \mathcal{T}_+ \times \cup_{s\in\mathcal{T}} \mathrm{Gr}_s$*. The time of initial recognition of the group, denoted by* t_0*, is defined by* g*: if* $g \notin \mathrm{Gr}_{t-1}$ *and* $g \in \mathrm{Gr}_t$*, then* $t_0 = t$*.*

If $g \notin \mathrm{Gr}_{t-1}$*, then set* $\mathrm{CSM}^{(t-1,g)} := 0$ *and* $\mathrm{LC}^{(t-1,g)} := 0$*.*

If $g \in \mathrm{Gr}_{t-1} \cup \mathrm{Gr}_t$*,* $\mathrm{CSM}^{(t-1,g)} \geq 0$ *and* $\mathrm{LC}^{(t-1,g)} = 0$*, then set*

$$\Delta_1 := \frac{d_{t_0,t-1}}{d_{t_0,t}} \mathrm{CSM}^{(t-1,g)} + \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)} + P^{(t,g)} \tag{1}$$

and set $\mathrm{CSM}^{(t,g)} := W_t^{(g)} \Delta_1^+$ *and* $\mathrm{LC}^{(t,g)} := \Delta_1^-$*.*

If $g \in \mathrm{Gr}_{t-1} \cup \mathrm{Gr}_t$*,* $\mathrm{CSM}^{(t-1,g)} = 0$ *and* $\mathrm{LC}^{(t-1,g)} > 0$*, then set*

$$\Delta_2 := -\mathrm{LC}^{(t-1,g)} \frac{L_{\mathrm{RC}}^{(t,g)}}{L_{\mathrm{RC}}^{(t-1,g)}} - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}} \Big(\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)} \Big), \tag{2}$$

$$\Delta_3 := -\Delta_2^- + \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)} + P^{(t,g)} \tag{3}$$

and set $\mathrm{CSM}^{(t,g)} := \Delta_3^+$ *and* $\mathrm{LC}^{(t,g)} := \Delta_3^-$*.*

The underlying principles for the development of the contractual service margin is that changes in the liability value only adjust the contractual service margin to the extent that these changes relate to future service, and that the contractual service margin is measured at discount rates locked in at initial recognition.

Remark 3 (Change in the liability value relating to future service)**.** *The term* $\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)}$ *appearing in* (1)*,* (2) *and* (3) *in Definition 1 is the change in the liability value relating to future service in the period, measured at the discount rates determined at initial recognition of the contracts, and* *excluding any effect of the time value of money, since this effect is not seen as relating to future service in IFRS 17 (see § B97 in [10]). Note that* $L^{(t-1,g)}_{\mathrm{FS},t_0}$ *is the liability value at* $t-1$ *(measured at the* t_0*-discount rates) allocated to cash flows after time* t*, and unwinding the discount one period is captured by multiplying with* $\frac{d_{t_0,t-1}}{d_{t_0,t}}$*. Hence* $\frac{d_{t_0,t-1}}{d_{t_0,t}} L^{(t-1,g)}_{\mathrm{FS},t_0}$ *is the liability value at time* t *for cash flows after* t*, measured at the* t_0*-discount rates, but where all non-financial assumptions are based on the information at time* $t-1$*, while* $L^{(t,g)}_{\mathrm{RC},t_0}$ *is the liability value at time* t *for cash flows after* t*, measured at the* t_0*-discount rates, where all non-financial assumptions have been updated to include the information in period* t*.*

Remark 4. *In Definition 1, the weight* $\mathrm{LC}^{(t-1,g)} / L^{(t-1,g)}_{\mathrm{RC}}$ *appears in the definition of* Δ_2*. This is a specific choice of allocation of the change in the liability value due to services provided and of the change in the liability value due to the effect of the time value of money between the loss component and the liability value excluding the loss component. A more general setting and a motivation of our choice of weight can be found in Section A.3.*

In order for the definition of the contractual service margin $\mathrm{CSM}^{(t,g)}$ to make economic sense, the weights $W^{(g)}_t$ in Definition 1 must be suitably chosen. These weights correspond to quantities that are derived from so-called coverage units. Precise details are found in Section 3.1 below. For further details on the expressions in (1), (2), and (3), see Section A.1.

Remark 5 (Termination of CSM and LC)**.** *Definition 1 ensures that the contractual service margin and the loss component equal zero at the end of the coverage period, i.e. that* $\mathrm{CSM}^{(t,g)} = 0$ *and* $\mathrm{LC}^{(t,g)} = 0$ *when* $g \in \mathrm{Gr}_{t-1}$ *and* $g \notin \mathrm{Gr}_t$*. Note that* $g \in \mathrm{Gr}_{t-1}$ *and* $g \notin \mathrm{Gr}_t$ *together imply that* $L^{(t-1,g)}_{\mathrm{RC}} \neq 0$, $L^{(t,g)}_{\mathrm{RC}} = L^{(t-1,g)}_{\mathrm{FS},t_0} = L^{(t,g)}_{\mathrm{RC},t_0} = 0$*, and* $P^{(t,g)} = 0$*. Hence, if* $\mathrm{CSM}^{(t-1,g)} \geq 0$ *and* $\mathrm{LC}^{(t-1,g)} = 0$*, then* Δ_1 *in* (1) *is* $\Delta_1 = \frac{d_{t_0,t-1}}{d_{t_0,t}} \mathrm{CSM}^{(t-1,g)} \geq 0$*, hence* $\mathrm{LC}^{(t,g)} = 0$*, and* $\mathrm{CSM}^{(t,g)} = 0$ *since* $W^{(g)}_t = 0$*. If instead* $\mathrm{CSM}^{(t-1,g)} = 0$ *and* $\mathrm{LC}^{(t-1,g)} > 0$*, then* Δ_2 *in* (2) *is equal to zero, from which it follows that* Δ_3 *in* (3) *is zero, thus* $\mathrm{LC}^{(t,g)} = \mathrm{CSM}^{(t,g)} = 0$*.*

Remark 6 (Initial recognition)**.** *Note that when reporting period* t *is the period during which the group of contracts is initially recognised, we have* $L^{(t-1,g)}_{\mathrm{RC}} = \mathrm{CSM}^{(t-1,g)} = \mathrm{LC}^{(t-1,g)} = 0$*, and* $L^{(t,g)}_{\mathrm{RC},t_0} = L^{(t,g)}_{\mathrm{RC}}$*. Furthermore, since no unearned profit should be allocated to profit or loss at initial recognition of the group it follows that* $1 - W^{(g)}_{t_0} = 0$*, hence* $W^{(g)}_t = W^{(g)}_{t_0} = 1$*. Thus* $\mathrm{CSM}^{(t,g)}$ *and* $\mathrm{LC}^{(t,g)}$ *correspond to the contractual service margin and loss* *component at initial recognition of the group:*

$$\mathrm{CSM}^{(t,g)} = \left(P^{(t,g)} - L_{\mathrm{RC}}^{(t,g)}\right)^{+},$$
$$\mathrm{LC}^{(t,g)} = \left(L_{\mathrm{RC}}^{(t,g)} - P^{(t,g)}\right)^{+}.$$

Profit or loss in reporting period t for group g, $\mathrm{P\&L}_t^{(g)}$, is defined as follows in terms of the contractual service margin and loss component in Definition 1.

Definition 2 (Profit or loss)**.** *Profit or loss determined at time* t *for reporting period* t *and group* g *of contracts is given by*

$$\mathrm{P\&L}_t^{(g)} := L^{(t-1,g)} + \mathrm{CSM}^{(t-1,g)} + P^{(t,g)} - (L^{(t,g)} + \mathrm{CSM}^{(t,g)}) - I_t^{(g)}. \quad (4)$$

Remark 7 (Long time P&L)**.** *For a group of contracts initially recognised at time* t_0*, where all claims are paid by time* $t_0 + \tau$*, the total profit or loss for the group over its lifetime is given by*

$$\sum_{t=t_0}^{t_0+\tau} \mathrm{P\&L}_t^{(g)} = \sum_{t=t_0}^{t_0+\tau} P^{(t,g)} - \sum_{t=t_0}^{t_0+\tau} I_t^{(g)}, \quad (5)$$

i.e. the premium income minus the sum of the net liability cash flows for the group. This follows immediately from the definitions of the involved quantities, see Definition 1 and Remark 5. Any other proposed definition of $\mathrm{P\&L}_t^{(g)}$ *for which* (5) *does not hold would be seriously flawed.*

Remark 8. *Note that the loss component* $\mathrm{LC}^{(t,g)}$ *does not appear explicitly in Definition 2. Instead, the loss component is part of the liability value, i.e. any changes in the loss component will be included in the difference* $L^{(t-1,g)} - L^{(t,g)}$*. This means that changes in the loss component in the period will directly affect profit or loss in the period, whether positive (decreasing loss component) or negative (increasing loss component). Since the loss component is already taken into account in both the financial position and performance of the company through the computation of the liability value, one might wonder why it is necessary to track the loss component through the algorithm in Definition 1. The reason is that the loss component is needed in order to determine if a currently onerous group gives rise to a contractual service margin at the end of the period. Here we get to one of the key principles in IFRS 17 regarding how unearned profit versus unearned losses should be treated. The definition of profit or loss in Definition 2 ensures that losses for groups of contracts are recognised immediately in the reporting period in which a previously profitable group becomes onerous, i.e. when* $\mathrm{CSM}^{(t-1,g)} > 0$ *and* $\mathrm{LC}^{(t,g)} > 0$*, or* *an onerous group becomes more onerous, i.e. when* $\mathrm{LC}^{(t,g)} > \mathrm{LC}^{(t-1,g)}$*. At the same time, the appearance of the change in the contractual service margin in Definition 2 together with its development in Definition 1 ensure that for a profitable group of contracts a proportion of the unearned profit is recognised later, due to appearance of the weight factor* $W_t^{(g)}$*.*

Remark 9. *Note that it is the change in the total liability value,* $L^{(t-1,g)} - L^{(t,g)}$ *that appears in Definition 2. At the same time, the algorithm for calculating the contractual service margin in Definition 1 only depends on the liability for remaining coverage. Hence, the liability for remaining coverage affects profit or loss in period* t *both directly through the change in the liability value,* $L_{\mathrm{RC}}^{(t-1,g)} - L_{\mathrm{RC}}^{(t,g)}$*, and indirectly through the change in the contractual service margin,* $\mathrm{CSM}^{(t-1,g)} - \mathrm{CSM}^{(t,g)}$*, while the liability for incurred claims only affects profit or loss through the change in the liability value,* $L_{\mathrm{IC}}^{(t-1,g)} - L_{\mathrm{IC}}^{(t,g)}$*. This is a natural consequence of that the liability for incurred claims is the liability value allocated to cash flows corresponding to insured events that have already occurred, hence it relates to past or current service, and the contractual service margin should only be adjusted for changes in the liability value that relate to future service.*

Remark 10 (Alternative definition of P&L)**.** *In [10] profit or loss is defined in a way that is not obviously equivalent to Definition 2. We prove the equivalence in Proposition 1 in Section A.2 and choose the more intuitive expression for* $\mathrm{P\&L}_t^{(g)}$ *in Definition 2.*

3.1 Coverage units

The weights $W_t^{(g)}$ appearing in the definition of the contractual service margin are essential quantities since profit or loss is defined in terms of the contractual service margin. The weights are derived from so-called coverage units. The estimation of coverage units for the group should ensure that the release of the contractual service margin into profit or loss is in accordance with how services are provided. The number of coverage units in a group correspond to "the quantity of coverage provided by the contracts in the group, determined by considering for each contract the quantity of the benefits provided under a contract and its expected coverage duration." (§ B119 in [10]). No further details are provided in [10], hence it is largely up to the judgement of the insurer. Minimal guidance is provided in [11], apart from that IASB rejected allocating the contractual service margin based on the pattern of expected cash flows (§ BC279 in [11]). Our interpretation of coverage units is as follows:

Let $B_s^{(g,k)}$ be the quantity of the benefits provided under the kth contract in group g in period s, and let $T_t^{(g,k)}$ be the time until the coverage period ends for the contract as seen from time t, taking non-negative integer values. We define the coverage units $\mathrm{CU}_s^{(t,g,k)}$ in period s, as seen from time t, as

$$\mathrm{CU}_s^{(t,g,k)} := B_s^{(g,k)} \mathbb{1}_{\{T_t^{(g,k)} > s-t-1\}}.$$

$\mathrm{CU}_t^{(t,g,k)}$ is known at time t. The coverage unit $\mathrm{CU}_s^{(t,g)}$ is the sum of the coverage units $\mathrm{CU}_s^{(t,g,k)}$ over the index k.

$W_t^{(g)}$ is defined as the ratio of the expected remaining number of coverage units at time t over the lifetime of the group to the total number of coverage units for the group. The latter is the sum of the number of coverage units provided in reporting period t and the expected remaining number of coverage units at time t. We write

$$W_t^{(g)} := \frac{\mathbb{E}_t[\sum_{s=t+1}^{t+\tau_g} \mathrm{CU}_s^{(t,g)}]}{\mathrm{CU}_t^{(t,g)} + \mathbb{E}_t[\sum_{s=t+1}^{t+\tau_g} \mathrm{CU}_s^{(t,g)}]},$$

where $t+\tau_g := \max\{s : g \in \mathrm{Gr}_s\}$ is the time at which all contracts in group g are terminated, and $\mathbb{E}_t[\cdot]$ denotes the expectation conditional on information up to time t. For convenience we let $\mathrm{ERCU}^{(t,g)} := \mathbb{E}_t[\sum_{s=t+1}^{t+\tau_g} \mathrm{CU}_s^{(t,g)}]$ and write

$$W_t^{(g)} := \frac{\mathrm{ERCU}^{(t,g)}}{\mathrm{CU}_t^{(t,g)} + \mathrm{ERCU}^{(t,g)}}.$$

For one contract, omitting the indices g and k, note that if B_s is known at time t for $s = t+1, \dots, t+\tau$, then

$$\mathbb{E}_t\Big[\sum_{s=t+1}^{t+\tau} \mathrm{CU}_s^{(t)}\Big] = \sum_{s=t+1}^{t+\tau} B_s \mathbb{E}_t[\mathbb{1}_{\{T_t > s-t-1\}}] = \sum_{s=t+1}^{t+\tau} B_s \mathbb{P}_t(T_t > s-t-1).$$

Furthermore, if $B_s := B$ for $s = t+u+1, \dots, t+\tau$ and zero otherwise, where B is known at time t and $0 \le u \le \tau - 1$, then

$$\mathbb{E}_t\Big[\sum_{s=t+1}^{t+\tau} \mathrm{CU}_s^{(t)}\Big] = B \sum_{s=u+1}^{\tau} \mathbb{P}_t(T_t > s-1).$$

Note that if $u = 0$, then the above expectation equals $B\mathbb{E}_t[T_t]$, i.e. the expected remaining coverage units at time t for the contract is the constant quantity of benefits times the expected coverage duration.

We still need a more precise definition of the quantity of benefits B_t and the the time until the coverage period ends T_t. To that end, we use the opinions of the IFRS 17 Transition Resource Group (TRG) [12] as guidance when making the following definitions. For the case when the coverage period starts at time t, or has started before time t, we want $\mathbb{E}_t[T_t]$ to be the expected remaining coverage period for the contract. Hence T_t should take into account lapses and cancellations, as well as deaths and other insured events to the extent that they affect the remaining coverage period (e.g. a contract that is terminated after the insured event occurs). This means that, depending on the type of contract under consideration and the contract terms, T_t can be fixed as seen from time t, or it can be a random variable.

To define the quantity of the benefits B_t is more complex, since there are many different product types and contract terms. The definition needs to be consistent with § B119 in [10] which states that the allocation of the contractual service margin to profit or loss in each period should reflect the services provided under the group in that period. The services provided to a contract in one period is the coverage in the form of economic compensation if the insured event occurs in that period. Hence, as noted by the TRG in [12], the quantity of the benefits should be based on the claims payment in case of a valid claim, which is not the same as the amount that the insurer expects to pay out per contract. Some possible methods for determining B_t mentioned in [12] are using the maximum contractual cover for the contract in each time period, or using the benefit amount that the policyholder is expected to receive if the insured event occurs in each time period. The former method might be appropriate for some life insurance contracts, where a fixed benefit payment is specified in the contract terms and paid out if the insured person is still alive at some future time point. This is how we have defined B_t in the examples provided below and in Section 5. The latter method might be appropriate when no maximum contractual cover is specified. As an example, consider a contract which provides income when the policyholder is unable to work due to disability, and the benefit the policyholder receives depends on policyholder's salary before the insured event occurs. For this type of contract the quantity of the benefits for future reporting periods can be determined as the current salary adjusted for some measure of inflation. Another way to determine the quantity of the benefits for a group of contracts with no maximum contractual cover specified could be to use some simple model for the claim severity, i.e. the average payment per claim, based on historical data for similar contracts, and incorporating a trend component or inflation as needed.

There are also cases where a maximum contractual cover is specified, but using it to determine B_t might not be appropriate. E.g. consider a group of contracts consisting of some contracts with a very high maximum contractual cover, and others with a much lower maximum contractual cover, but where the historical claim severity is similar for both contract types. In this case, it might be more suitable to define the quantity of the benefits based on a model for the claim severity rather than the contractual maximum cover, since the very high maximum cover for some of the contracts has essentially no economic effect on the performance of the group. It is up to each insurer to judge how B_t should be defined in each specific case, to ensure that the number of coverage units in each period reflects the services provided.

When a contract is terminated ahead of time (due to e.g. lapse of the contract or death of the policyholder), the number of expected remaining coverage units that would have remained had the contract not been terminated needs to be derecognised from the group (§ 76 in [10]). Our interpretation is that the coverage units provided in the period should be adjusted in a similar manner to ensure that part of the contractual service margin due to this contract is recognised in profit or loss for the period. Hence, if $\mathbb{E}_t[\sum_{s=t+1}^{t+\tau_{g,k}} B_s^{(g,k)} \mathbb{1}_{\{T_t^{(g,k)}>s-t-1\}}]$ are the expected remaining coverage units for contract k in group g at time t had the contract not been terminated in period t, then we let $\mathrm{CU}_t^{(t,g,k)} = \mathbb{E}_t[\sum_{s=t+1}^{t+\tau_{g,k}} B_s^{(g,k)} \mathbb{1}_{\{T_t^{(g,k)}>s-t-1\}}]$ and $\mathrm{ERCU}^{(t,g,k)} = 0$ after the termination of the contract.

The number of coverage units provided in period t and the expected remaining number of coverage units at time t for some life insurance contracts are defined below. We consider a group g of $N_{t-1}^{(g)}$ contracts at time $t-1$. For the kth contract of the group, let $\mathrm{CU}_t^{(g,k)}$ and $\mathrm{ERCU}^{(t,g,k)}$ denote the corresponding coverage units and expected remaining coverage units and set

$$\mathrm{CU}_t^{(t,g)} := \sum_{k=1}^{N_{t-1}^{(g)}} \mathrm{CU}_t^{(t,g,k)}, \quad \mathrm{ERCU}^{(t,g)} := \sum_{k=1}^{N_t^{(g)}} \mathrm{ERCU}^{(t,g,k)}.$$

In the three examples below, consider the kth insured person in group g, x periods old at time t, whose remaining lifetime is $T_{x,t}$.

Example 1 (Survival benefit)**.** *Consider an insurance contract with survival benefit B, paid if the insured person is still alive at time $\tau + t$. If the insured person is alive at time t, then*

$$\mathrm{CU}_t^{(t,g,k)} := 0, \quad \mathrm{ERCU}^{(t,g,k)} := B\mathbb{P}_t(T_{x,t} > \tau)$$

If the insured person dies in period t, then

$$\mathrm{CU}_t^{(t,g,k)} := B\mathbb{P}_t(T_{x,t} > \tau), \quad \mathrm{ERCU}^{(t,g,k)} := 0$$

Example 2 (Life annuity I)**.** *Consider a life annuity during the decumulation phase, with benefit B paid out at the beginning of each period if the insured* *person is alive. If the insured person is alive at time t, then*

$$\mathrm{CU}_t^{(t,g,k)} := B, \quad \mathrm{ERCU}^{(t,g,k)} := B \sum_{s=1}^{\infty} \mathbb{P}_t(T_{x,t} > s-1)$$

If the insured person dies in period t, then

$$\mathrm{CU}_t^{(t,g,k)} := B \sum_{s=0}^{\infty} \mathbb{P}_t(T_{x,t} > s-1), \quad \mathrm{ERCU}^{(t,g,k)} := 0$$

Example 3 (Life annuity II)**.** *Consider a life annuity during the accumulation phase, with benefit B paid out at the beginning of each period after time $\tau + t$ if the insured person is still alive. For this type of contract the total number of coverage units is $B\sum_{s=1}^{\infty} \mathbb{P}(T_{x,t} > \tau + s - 1)$: If the insured person is alive at time t, then*

$$\mathrm{CU}_t^{(g,k)} := 0, \quad \mathrm{ERCU}^{(t,g,k)} := B \sum_{s=1}^{\infty} \mathbb{P}_t(T_{x,t} > \tau + s - 1)$$

If the insured person dies in period t, then

$$\mathrm{CU}_t^{(g,k)} := B \sum_{s=1}^{\infty} \mathbb{P}_t(T_{x,t} > \tau + s - 1), \quad \mathrm{ERCU}^{(t,g,k)} := 0$$

4 Valuation of liability cash flows and allocation to subgroups

In IFRS 17 the valuation of insurance liabilities should consist of the fulfilment cash flows and the contractual service margin. The fulfilment cash flows consist of estimates of future cash flows, and a risk adjustment for non-financial risk. The estimate of future cash flows is defined as "the expected value (ie the probability-weighted mean) of the full range of possible outcomes", and it should be adjusted "to reflect the time value of money and the financial risks related to the future cash flows" (§§ 32-33 in [10]). This is similar to the definition of the technical provisions in Solvency II, which consists of a best estimate and a risk margin (Article 77, §§ 1-2 in [4]). However, there is a difference between the valuation in the two regulations. In Solvency II, the risk margin "shall be such as to ensure that the value of the technical provisions is equivalent to the amount that insurance and reinsurance undertakings would be expected to require in order to take over and meet the insurance and reinsurance obligations" (Article 77, § 3 in [4]).

In IFRS 17, the risk adjustment for non-financial risk is defined as an adjustment "to reflect the compensation that the entity requires for bearing the uncertainty about the amount and timing of the cash flows that arises from non-financial risk" (§ 37 in [10]), and it should reflect the fact that insurers "generally fulfil insurance contracts directly over time by providing services to policyholders, rather than by transferring the contracts to a third party" (§ BC17 in [11]). Hence, in IFRS 17 it is the company's own perspective that should be taken into account, rather than that of a reference undertaking. In most other aspects, the definition of the risk adjustment is consistent with the principles underlying the calculation of the risk margin. However, in Solvency II the method for calculating the risk margin is prescribed by the regulation, which states that it should be "calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof" (Article 77, § 5 in [4]), and furthermore it specifies a formula for the calculation as well as certain parameter values. This is in contrast to IFRS 17 where there is no specification of what technique should be used as long as the risk adjustment has some general characteristics (see further § B91 in [10]).

The risk margin in Solvency II has been criticised for lacking a proper definition and theoretical foundation, which is addressed in e.g. [16]. Through the framework developed in [7], building on the setup in [16], a multi-period cost-of-capital margin is computed, and given certain assumptions explicit computations are possible. This cost-of-capital margin is consistent with the requirements on the risk adjustment in IFRS 17, if taking the company's own perspective on the level of the cost-of-capital rate and risk aversion, and not that of a reference undertaking.

4.1 Multi-period cost-of-capital valuation

We value an aggregate insurance liability by considering a hypothetical transfer of the liability to a subsidiary, whose sole purpose is to manage the run-off of the liability. Hence, the subsidiary is a financially separate entity, but is owned by the original entity, and thus has the same view on risk and the compensation required for bearing that risk as the parent entity. Note that this construction is in line with the hypothetical transfer to a reference undertaking in Solvency II. We use the term subsidiary to emphasise that the view on risk and compensation should be the company's own, not that of a third party.

We consider a money market account numeraire, and express values and cash flows via this numeraire. The evolution of the money market account is determined by a short-rate model that is assumed consistent with a given set of discount factors.

The liabilities considered throughout this paper are entirely non-replicable by financial instruments and therefore we do not need to consider any replicating portfolio that may be transferred along with the liability except for a numeraire position: an amount of cash invested fully in the money market account. The liability can be identified with a stochastic cash flow $X = (X_t)_{t=1}^{\tau}$, where τ corresponds to the time τ periods after the transfer has occurred when the run-off is complete. Time $t = 0$ here stands for the actual valuation time when the hypothetical transfer of the liabilities is considered. By considering a discrete-time liability cash flow we implicitly make the simplifying assumption that cash flows occur only at the preset finite set of times.

Along with the transfer of the liability, a cash amount V_0 is transferred to the subsidiary. V_0 is the amount that the parent entity would require as compensation for bearing the non-financial risks in the subsidiary. Consequently, V_0 is defined as the value of the liability cash flow X. In order to compute the compensation for bearing the risks in the subsidiary, and thus V_0, capital costs throughout the run-off must be considered. Capital costs originate from the need for the subsidiary to hold buffer capital. This buffer capital is not necessarily the same as the regulatory required capital, instead it is the amount of capital that the risk averse owner requires the subsidiary to hold to ensure that it can fulfil the obligations to policyholders in most situations. What is meant by most situations, and thus the amount of buffer capital required, is based on the degree of risk aversion of the parent entity. On the other hand the owner of the subsidiary does not need to fulfil the obligations to the policyholders in all situations. More precisely, the owner of the subsidiary has limited liability which here means that if the required buffer capital turns out to be insufficient to meet the obligations to policyholders, then the subsidiary may be terminated at no further cost upon transferring the buffer capital to the policyholders.

In order to make the arguments leading to the valuation of the liability cash flow in run-off precise, we consider the following mathematical setup. Let $(\mathcal{H}_t)_{t=0}^{\tau}$, with $\mathcal{H}_0 := \{\emptyset, \Omega\}$, be the run-off filtration representing the flow of information throughout the run-off of the liability. The cash flow X is assumed adapted to $(\mathcal{H}_t)_{t=0}^{\tau}$ but this filtration may be larger than the filtration generated by X. For any $t = 1, \dots, \tau$, let V_t denote the value of the remaining liability cash flow at that time. Obviously, $V_\tau := 0$ since the run-off is complete at time τ. Let RC_t denote the required capital at time t.

At the time of the transfer of the liability from the original insurer to the subsidiary, the subsidiary receives V_0 but is required to hold capital $\mathrm{RC}_0 > V_0$. Hence, the owner injects $\mathrm{RC}_0 - V_0$ in order to operate the subsidiary. At time

1 the policyholders demand X_1 and, given the information available at that time, the value of the remaining liability cash flow beyond time 1 is V_1. Taking limited liability into account the payoff at time 1 for the owner of the subsidiary on the initial investment $\mathrm{RC}_0 - V_0$ is therefore $(\mathrm{RC}_0 - X_1 - V_1)^+$. However, the capital injection $\mathrm{RC}_0 - V_0$ at time 0 would only be made available if the expected rate of return on this investment was sufficiently attractive:

$$\frac{\mathbb{E}[(\mathrm{RC}_0 - X_1 - V_1)^+] - (\mathrm{RC}_0 - V_0)}{\mathrm{RC}_0 - V_0} = \eta_0 \tag{6}$$

for some cost-of-capital rate η_0, which is determined by the parent entity, based on its required rate of return on capital invested in the subsidiary. The same reasoning holds at any time during the run-off, conditional on that the subsidiary has not been terminated at an earlier time. Therefore, the acceptability criterion at time t corresponding to (6) reads

$$V_t = \mathrm{RC}_t - \frac{1}{1+\eta_t}\mathbb{E}[(\mathrm{RC}_t - X_{t+1} - V_{t+1})^+ \mid \mathcal{H}_t]. \tag{7}$$

If we were to assume, in line with the Solvency II regulatory framework, that at any time t the capital requirement is defined in terms of the conditional 99.5%-quantile of the $X_{t+1} + V_{t+1}$, conditional on $\mathcal{H}_t$, written

$$\mathrm{RC}_t := \mathrm{VaR}_{99.5\%}(X_{t+1} + V_{t+1} \mid \mathcal{H}_t), \tag{8}$$

then, upon replacing RC_t by the expression in (8) and noting that $V_\tau := 0$, we have arrived at a non-linear backward recursion for determining the value V_0 of the original liability. Notice that IFRS 17 does not prescribe the use of this specific risk measure.

Solving the backward recursion for $(V_t)_{t=0}^{\tau}$ numerically is in general a challenging task. However, under the strong assumption that the liability cash flow together with other variables generating the filtration can be described by a Gaussian process, and that the cost-of-capital rates $(\eta_t)_{t=0}^{\tau-1}$ are non-random, an explicit formula can be derived for V_0:

$$V_0 = \mathbb{E}[R] + \sum_{t=0}^{\tau-1} c_t \Big(\mathrm{Var}\big(R \mid \mathcal{H}_t\big) - \mathrm{Var}\big(R \mid \mathcal{H}_{t+1}\big)\Big)^{1/2}, \tag{9}$$

where $R := \sum_{t=1}^{\tau} X_t$ and in case the risk measure is chosen as in (8):

$$c_t = \Phi^{-1}(0.995) - \frac{1}{1+\eta_t}\Big(0.995\Phi^{-1}(0.995) + \varphi(\Phi^{-1}(0.995))\Big),$$

where Φ and φ denote the distribution and density function, respectively, of a standard normal random variable. If instead of $\mathrm{VaR}_{99.5\%}(\cdot \mid \mathcal{H}_t)$ the risk measure Expected Shortfall at the level 99% is used, $\mathrm{ES}_{99\%}(\cdot \mid \mathcal{H}_t)$,

$$c_t = \frac{\varphi(\Phi^{-1}(0.99))}{0.01} - \frac{1}{1+\eta_t}\Big(\frac{\varphi(\Phi^{-1}(0.99))}{0.01}\Phi\Big(\frac{\varphi(\Phi^{-1}(0.99))}{0.01}\Big) + \varphi\Big(\frac{\varphi(\Phi^{-1}(0.99))}{0.01}\Big)\Big).$$

Notice again that IFRS 17 does not prescribe the use of this specific risk measure.

Since V_0 is non-random the expression (9) may look strange. However, due to the special property of conditional (co)variances for the multivariate normal distribution, the conditional variances in (9) are indeed non-random.

Remark 11 (Run-off filtration). *We could take the filtration $(\mathcal{H}_t)_{t=0}^{\tau}$ to be the filtration generated by the aggregate cash flow $(X_t)_{t=1}^{\tau}$. However, when considering allocation of values to n groups it is more appropriate to consider the larger filtration generated by all the cash flows of the n groups, i.e. the filtration generated by $(X_t^{(k)})_{t=1}^{\tau}$, $k = 1, \dots, n$. In many cases, there may be relevant additional information available that can be expressed as the outcomes of a stochastic process. In any case we take $(\mathcal{H}_t)_{t=0}^{\tau}$ to be the filtration generated by a d-dimensional, where $d \geq n$, Gaussian process $(G_t)_{t=1}^{\tau}$ having $(X_t^{(k)})_{t=1}^{\tau}$, $k = 1, \dots, n$, as (a subset of its) marginal processes.*

Remark 12 (Alternative valuation approaches). *There are easily applied alternatives to the multi-period cost-of-capital valuation applied to the discounted outstanding liability R for which all the analysis in this paper holds with minor modification (and interpretation) of constants. The expression for V_0 in* (9) *corresponds to, under the aforementioned Gaussian model assumption, the solution to the backward recursion $V_t = \varphi_t(X_{t+1} + V_{t+1})$, $V_\tau := 0$, with*

$$\varphi_t(Y) = \rho_t(Y) - \frac{1}{1+\eta_t}\mathbb{E}[(\rho_t(Y) - Y)^+ \mid \mathcal{H}_t],$$

where ρ_t is a conditional version of VaR *or* ES *given the information corresponding to $\mathcal{H}_t$ (notice that here ρ_t is defined as operating on a loss variable rather than future net worth of a position, otherwise $\rho_t(Y)$ needs to be replaced by $\rho_t(-Y)$). For the expression for V_0 in* (9) *to hold, the only relevant mathematical properties of the map $\varphi_t : L^1(\mathcal{H}_{t+1}) \to L^1(\mathcal{H}_t)$ are the following:*

$$\begin{aligned}
&\text{if } \lambda \in L^1(\mathcal{H}_t), \text{ then } \varphi_t(Y+\lambda) = \varphi_t(Y) + \lambda,\\
&\text{if } \lambda \in [0,\infty), \text{ then } \varphi_t(\lambda Y) = \lambda\varphi_t(Y),\\
&\text{if } \mathbb{P}(Y \in A \mid \mathcal{H}_t) = \mathbb{P}(\widetilde{Y} \in A \mid \mathcal{H}_t), \text{ then } \varphi_t(Y) = \varphi_t(\widetilde{Y}).
\end{aligned}$$

Therefore, an application of the well-established standard deviation premium principle is straightforward since this corresponds to choosing

$$\varphi_t(Y) = \mathbb{E}[Y \mid \mathcal{H}_t] + \gamma_t \operatorname{Var}(Y \mid \mathcal{H}_t)^{1/2},$$

where $(\gamma_t)_{t=0}^{\tau-1}$ *is some sequence of non-negative constants. This choice of* φ_t *satisfies all requirements and gives the expression for* V_0 *in (9) with* $c_t = \gamma_t$*. There are many papers on multi-period valuation of insurance liabilities, many of which study a situation where the liability can be partly hedged by trading in financial assets. For further details, we refer to [1, 5, 6, 8, 16, 17, 18] and the references therein.*

Though IFRS 17 does not prescribe any specific technique for computing the risk adjustment, there is a requirement to disclose the confidence level that the result of the computation corresponds to (§ 119 in [10]). This requirement was included in order to supply users of financial statements with some means of comparing the risk adjustment between different companies, despite the fact of them potentially using different techniques for the calculation. This is not to say that IASB favours the confidence level technique over other techniques, but is instead a reflection of that the confidence level technique is the simplest to implement and that they did not want to burden companies with having to use more complex techniques for this benchmark in case the the confidence level technique is deemed appropriate by the individual company (§§ BC215-BC217 in [11]).

From the Gaussian assumption follows immediately from (9) that

$$\mathbb{P}(R \leq V_0) = \Phi\left(\frac{\sum_{t=0}^{\tau-1} c_t \Big(\operatorname{Var}\big(R \mid \mathcal{H}_t\big) - \operatorname{Var}\big(R \mid \mathcal{H}_{t+1}\big)\Big)^{1/2}}{\operatorname{Var}(R)^{1/2}}\right)$$

and if $\eta_t = \eta_0$ for all t then the expression simplifies further to

$$\mathbb{P}(R \leq V_0) = \Phi\left(c_0 \sum_{t=0}^{\tau-1} \left(\frac{\operatorname{Var}\big(R \mid \mathcal{H}_t\big) - \operatorname{Var}\big(R \mid \mathcal{H}_{t+1}\big)}{\operatorname{Var}(R)}\right)^{1/2}\right).$$

It is seen that the confidence level $\mathbb{P}(R \leq V_0)$ is an elementary function of η_0 and the variance profile, i.e. how the conditional variance of the outstanding liability cash flow decays to zero as the run-off progresses.

4.1.1 Allocations to contract groups

Since the risk adjustment in IFRS 17 should be based on the insurer's own view of risk, it can reflect diversification between different contract groups

(§ B88 in [10]). Thus, many insurers would likely want to calculate the cost-of-capital margin for the company as a whole. However, the total value still needs to be allocated to a potentially large number of groups of contracts, since the measurement requirements in IFRS 17 need to be applied at this level.

A group of contracts in IFRS 17 is defined as follows: first the portfolio level of contracts has to be identified. A portfolio is a group of contracts "subject to similar risks and managed together". The next step is to subdivide each portfolio into a minimum of three groups depending on the estimated profitability of each contract at initial recognition. The first group consists of contracts that are onerous at initial recognition, the second group are contracts without any significant possibility of ever becoming onerous, and the third group consists of all other contracts. Finally, a group of contracts is only allowed to include contracts that are issued within one year of each other, hence for every future year newly issued contracts cannot be merged with previous groups of contracts, instead new groups will need to be formed. The group that an individual contract belongs to is decided when the contract is initially recognised and does not change subsequently - hence a group of onerous contracts can later on become profitable, and vice versa. (See further §§ 14-24 in [10]).

A contract is onerous at initial recognition if the net of the fulfilment cash flows and any other cash flows arising from the contract is negative. For a group of contracts that is onerous, an immediate loss of this net value will have to be recognised in profit or loss. The allocation of the risk adjustment to each group of contracts is thus important, since it can both influence if a group is onerous at initial recognition as well as if a group later on becomes onerous. For groups of contracts that are not onerous, the allocation of the risk adjustment will instead influence the size of the contractual service margin (estimated future profits) for the group, which will be released into profit or loss over the lifetime of the group. Hence the allocation of the risk adjustment to each group of contracts will directly affect the revenue stream for the company. This is a new regulatory concept compared to Solvency II, where the risk margin is allocated to lines of business (likely similar to the portfolio level of contracts in IFRS 17), but no further subdivision is needed, and no connection to any financial performance of the company is required.

IFRS 17 thus introduces a regulatory requirement on the allocation of risk being economically sound in order to ensure that the financial performance of the company reflects the true economic substance of the contracts. There is no other specific requirement in IFRS 17 on how the allocation should be made, only that the company "is able to include the appropriate fulfilment cash flows in the measurement of the group ... by allocating such estimates to the groups of contracts." (§ 24 in [10]). For individual companies it is likely important to have an allocation scheme that does not introduce unnecessary instability in the future revenue streams.

Consider n groups of contracts and write $R = \sum_{k=1}^{n} R_k$, where $R_k = \sum_{t=1}^{\tau} X_t^{(k)}$ is the sum of the (discounted) cash flow of group k throughout the run-off of the aggregate liability. The so-called Euler or gradient allocation of the value V_0 of R attributed to R_k is given by

$$\Lambda(R_k, R) := \mathbb{E}[R_k] + \sum_{t=0}^{\tau-1} c_t \frac{\operatorname{Cov}\Big(R_k, R \mid \mathcal{H}_t\Big) - \operatorname{Cov}\Big(R_k, R \mid \mathcal{H}_{t+1}\Big)}{\Big(\operatorname{Var}\Big(R \mid \mathcal{H}_t\Big) - \operatorname{Var}\Big(R \mid \mathcal{H}_{t+1}\Big)\Big)^{1/2}}. \quad (10)$$

The Euler allocation of the aggregate value to contract groups is sound in the sense that the aggregate value $V_0 = \Lambda(R, R)$ is fully allocated to the contract groups and, for each group, the value $\Lambda(R_k, R)$ allocated to the group does not exceed the corresponding stand-alone value $\Lambda(R_k, R_k)$ of the group's liability cash flow (obtained by replacing R by R_k in (9)):

$$\sum_{k=1}^{n} \Lambda(R_k, R) = \Lambda(R, R), \quad \Lambda(R_k, R) \leq \Lambda(R_k, R_k).$$

A proof of this statement is found in Appendix C.

5 Numerical insurance valuation and allocation to groups

We consider an insurance company issuing only contracts where claims payment is contingent on the policyholder being alive at certain future time points. We assume that the liability cash flow together with other variables generating the run-off filtration can be described by a Gaussian process, and that the cost-of-capital rate $\eta_t = \eta_0$ for $t = 0, 1, \ldots, \tau$. Hence, the cost-of-capital margin V_0 is given by (9) with $c_t = c_0$ for $t = 0, 1, \ldots, \tau$. Furthermore, the allocation of the value V_0 of R attributed to R_k (i.e. allocated to group k of contracts) is given by (10). We use a discount rate of zero across all reporting periods.

In order to calculate the cost-of-capital margin V_0 and allocate this value to the contract groups, we need estimates of the conditional variances and covariances in (9) and (10). Since the claims payment for all contracts are contingent on the policyholder still being alive, we need a model for the stochastic mortality rate. To this end, we estimate parameters of the Poisson log-bilinear model for mortality rates as proposed in [2], where the Lee-Carter

[15] model for mortality rates is embedded in a Poisson regression model. For the estimation we use the R package StMoMo [19], which defines the family of generalised age-period-cohort (GAPC) stochastic mortality models, of which the Poisson log-bilinear model in [2] is a special case. The model assumes that the number of deaths $D_{x,t}$ in a population aged x years during period t satisfy

$$\mathcal{L}(D_{x,t} \mid E_{x,t}, \mu_{x,t}) = \text{Pois}(E_{x,t}\mu_{x,t}),$$

where $E_{x,t}$ is the so-called exposure to risk, and $\mu_{x,t}$ is the mortality rate at age x during period t. The mortality rate is modelled as

$$\log(\mu_{x,t}) = \alpha_x + \beta_x \kappa_t, \quad \kappa_t = \delta + \kappa_{t-1} + \xi_t,$$

where α_x measures the time-independent age effect, κ_t is the mortality trend, and β_x gives the sensitivity of the predictor at age x to variations in κ_t. The mortality trend κ_t is modeled as a Gaussian random walk with drift: δ is the drift parameter and (ξ_t) is an iid sequence with $\xi_t \sim N(0, \sigma_\kappa^2)$. The parameter constraints $\sum_t \kappa_t = 0$ and $\sum_x \beta_x = 1$, proposed in [15], are imposed to ensure model identification. The model parameters are estimated from data for Swedish males for year 1985 to 2018, for ages 0-90, from the Human Mortality Database [9] resulting in estimates $\widehat{\delta} \approx -2.00$, $\widehat{\sigma}_\kappa \approx 1.60$, and $(\widehat{\alpha}_x)_{x=0}^{90}$, $(\widehat{\beta}_x)_{x=0}^{90}$ and $(\widehat{\kappa}_t)_{t=1985}^{2018}$ illustrated in Figure 1. We have chosen to exclude data for ages above 90 years in the estimation, due to data being sparse for these ages. Instead, we will use the estimated values $\widehat{\alpha}_{90}$ and $\widehat{\beta}_{90}$ for ages above 90 in our calculations. All estimated parameter values will be kept fixed for the future whole period modelled, i.e. no parameter risk is included.

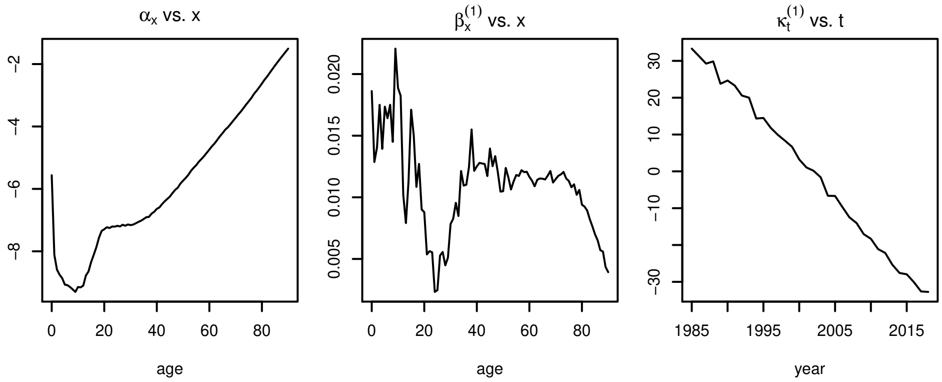

Figure 1: Parameter estimates for mortality model.

5.1 Portfolio of life annuities

We consider an insurance company issuing only one type of contracts, a life annuity. If the insured person is still alive at age 65 years, a benefit of $B = 1$ is paid out in each reporting period until the period when the person dies, or the period when the insured person reaches age 100 years, whichever comes first. The annuity can be issued to individuals between the ages of 30 and 64 years. Denote the set of ages for which the benefit B is paid out by $\mathrm{PA} := \{65, \dots, 100\}$.

Starting at time $t = -20$, new customers arrive according to a marked Poisson process, where the Poisson process has intensity $\lambda = 3000$, and the marks determine the age of the arriving customers according to a multinomial distribution. The parameters of the multinomial distribution are determined according to the proportion of different ages (between 30 and 64) of the general population of Sweden in year 2018 (age 30-40: 32.5%, age 41-50: 29.1%, age 51-64: 38.4%). Assuming that all contracts are profitable at the time of issue, we have one group of contracts per issuing year.

Let $N^{(g)}_{x,t}$ be the number of active contracts of x years old individuals belonging to group g at time t. We assume that deaths at time t of individuals conditional on the mortality rates at time $t-1$ are independent events. Hence, the number of active contracts at each time point t for groups of contracts issued before this time can be described as a nested binomial process as follows:

$$\mathcal{L}(N^{(g)}_{x,t} \mid N^{(g)}_{x-1,t-1}, \mu_{x-1,t-1}) = \mathrm{Bin}(N^{(g)}_{x-1,t-1}, p_{x-1,t-1}),$$

with $p_{x-1,t-1} := \exp\{-\mu_{x-1,t-1}\}$.

We simulate one trajectory for the customer arrival process and one trajectory for the mortality trend $(\kappa_t)^0_{t=-19}$ over 20 years. At time $t = 0$ the outstanding liability consists of 20 groups of contracts, $\mathrm{Gr}_0 = \{-19, \dots, 0\}$, and the liability cash flow $X := X^{(0)}$ takes the form

$$X = B \sum_{g \in \mathrm{Gr}_0} \sum_{x \in \mathrm{PA}} \left(N^{(g)}_{x,1}, \dots, N^{(g)}_{x,70} \right),$$

where $N^{(g)}_{x,t}$ denotes the number of x years old individuals at time t belonging to group g under the assumption that no new contracts are issued after time 0. At time 0, the number $N^{(g)}_{x,0}$ of x years old individuals belonging to group g are considered as (non-random) model parameters $n^{(g)}_{x,0}$. Similarly, at time 0, the value κ_0 of the mortality trend is considered as a (non-random) model parameter. We denote by

$$X^{(g)} = B \sum_{x \in \mathrm{PA}} \left(N^{(g)}_{x,1}, \dots, N^{(g)}_{x,70} \right), \quad g \in \mathrm{Gr}_0,$$

the liability cash flows of the individual groups, and by

$$N_t^{(g)} = \sum_{x=30}^{100} N_{x,t}^{(g)}, \quad t \in \{1, \dots, 70\},$$

the total number of active contracts at time t in each group. We calculate the value of the outstanding liability according to (9) and allocate this value to the groups of contracts according to (10). The run-off filtration $(\mathcal{H}_t)_{t=0}^{\tau}$ is here taken to be the filtration generated by the cash flows of the 20 groups, and the total number of active contracts in each group:

$$\mathcal{H}_0 := \{\emptyset, \Omega\}, \quad \mathcal{H}_t := \sigma\Big(X_t^{(g)}, N_t^{(g)}; g \in \mathrm{Gr}_0\Big) \vee \mathcal{H}_{t-1}, \quad t = 1, \dots, 70.$$

In order to calculate the value of outstanding liability cash flows according to (9) we need to calculate $\mathbb{E}[R]$ and $\mathrm{Var}(R \mid \mathcal{H}_t)$, where $R := \sum_{s=1}^{\tau} X_s$. Furthermore, to calculate the values allocated to contract groups according to (10) we need to calculate $\mathrm{Cov}(R^{(g)}, R \mid \mathcal{H}_t)$, where $R^{(g)} := \sum_{s=1}^{\tau} X_s^{(g)}$. We calculate means and covariances based on the nested binomial process for the evolutions of active contracts over time, using the Lee-Carter model for the evolution of mortality rates. Given these means and covariances, the use of (9) and (10) means that we are assuming that

$$\Big((X_t^{(g)}, N_t^{(g)}); g \in \mathrm{Gr}_0, t \in \{1, \dots, 70\}\Big)$$

is a random vector with a normal distribution. Although this is inconsistent with how the model for the cash flows and number of active contracts is defined, it is a reasonable approximation which enables efficient computation of all involved quantities.

1) The expected value

$$\mathbb{E}[R] = B \sum_{s=1}^{\tau} \sum_{x \in \mathrm{PA}} \mathbb{E}[N_{x,s}], \quad N_{x,t} := \sum_{g \in \mathrm{Gr}_0} N_{x,t}^{(g)},$$

of the outstanding liability cash flows is calculated as follows.

$$\mathbb{E}[N_{x,t} \mid \mu] = N_{x-t,0} \prod_{i=0}^{t-1} p_{x-t+i,i} = n_{x-t,0} \exp\Big\{ -\sum_{i=0}^{t-1} \mu_{x-t+i,i} \Big\},$$

where $n_{x-t,0} = \sum_{g \in \mathrm{Gr}_0} n_{x-t,0}^{(g)}$. Taylor approximation around the mean yields the approximation

$$\mathbb{E}[N_{x,t} \mid \mu] \approx n_{x-t,0} \exp\Big\{ -\sum_{i=0}^{t-1} \mathbb{E}[\mu_{x-t+i,i}] \Big\} \Big(1 - \sum_{i=0}^{t-1} \big(\mu_{x-t+i,i} - \mathbb{E}[\mu_{x-t+i,i}]\big)\Big).$$

Hence,

$$\mathbb{E}[N_{x,s}] = \mathbb{E}[\mathbb{E}[N_{x,s} \mid \mu]] \approx n_{x-s,0} \exp\left\{ -\sum_{i=0}^{s-1} \mathbb{E}[\mu_{x-s+i,i}] \right\},$$
$$\mathbb{E}[\mu_{x,t}] = \exp\left\{ \alpha_x + \beta_x(t\delta + \kappa_0) + \frac{1}{2}\beta_x^2 t \sigma_\kappa^2 \right\}.$$

2) In order to calculate the variances $\mathrm{Var}(R \mid \mathcal{H}_t)$ and covariances $\mathrm{Cov}(R^{(g)}, R \mid \mathcal{H}_t)$ we first need to calculate the following covariances for the model at time 0:

$$\mathrm{Cov}(X_t, X_s) = B^2 \sum_{x,y\in \mathrm{PA}} \mathrm{Cov}(N_{x,t}, N_{y,s}),$$
$$\mathrm{Cov}(X_t^{(g)}, X_s^{(h)}) = B^2 \sum_{x,y\in \mathrm{PA}} \mathrm{Cov}(N_{x,t}^{(g)}, N_{y,s}^{(h)}),$$
$$\mathrm{Cov}(X_t, X_s^{(g)}) = B^2 \sum_{x,y\in \mathrm{PA}} \sum_{h\in \mathrm{Gr}_0} \mathrm{Cov}(N_{x,t}^{(h)}, N_{y,s}^{(g)}).$$

The calculation of the covariances on the right-hand side above are found in Section B.1.

3) Given the covariances in 2) we form the high-dimensional covariance matrix $\Sigma^{(t)}$, for the model at time 0, of the random vector

$$\left(\sum_{s=t+1}^{\tau} X_s, \left(\sum_{s=t+1}^{\tau} X_s^{(g)} \right)_{g\in \mathrm{Gr}_0}, \left((X_s^{(g)})_{s=1}^t, (N_s^{(g)})_{s=1}^t \right)_{g\in \mathrm{Gr}_0} \right) \tag{11}$$

and notice that

$$\mathrm{Var}(R \mid \mathcal{H}_t) = \mathrm{Var}\left(\sum_{s=t+1}^{\tau} X_s \mid \left((X_s^{(g)})_{s=1}^t, (N_s^{(g)})_{s=1}^t \right)_{g\in \mathrm{Gr}_0} \right),$$
$$\mathrm{Cov}(R^{(g)}, R \mid \mathcal{H}_t) = \mathrm{Cov}\left(\sum_{s=t+1}^{\tau} X_s^{(g)}, \sum_{s=t+1}^{\tau} X_s \mid \left((X_s^{(g)})_{s=1}^t, (N_s^{(g)})_{s=1}^t \right)_{g\in \mathrm{Gr}_0} \right).$$

We emphasise at this point that due to the assumed multivariate normality of the random vector in (11), these conditional variances and covariances are indeed non-random as seen from time 0. To calculate the conditional variances and covariances needed, we use properties of the multivariate normal distribution. For a multivariate normal vector $Z \sim N_n(\mu, \Sigma)$, write

$$\mu = \begin{pmatrix} \mu_{1:m} \\ \mu_{m+1:n} \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_{1:m,1:m} & \Sigma_{1:m,m+1:n} \\ \Sigma_{m+1:n,1:m} & \Sigma_{m+1:n,m+1:n}, \end{pmatrix}$$

for $m \in \{1, \dots, n-1\}$. Then the conditional distribution of $(Z_1, \dots, Z_m)$ given $(Z_{m+1}, \dots, Z_n)$ is multivariate normal with mean vector

$$\mu_{1:m|m+1:n} = \mu_{1:m} + \Sigma_{1:m,m+1:n}\Sigma^{-1}_{m+1:n,m+1:n}(Z_{m+1:n} - \mu_{m+1:n}),$$

assuming that the inverse $\Sigma^{-1}_{m+1:n,m+1:n}$ exists, and covariance matrix

$$\Sigma_{1:m|m+1:n} = \Sigma_{1:m,1:m} - \Sigma_{1:m,m+1:n}\Sigma^{-1}_{m+1:n,m+1:n}\Sigma_{m+1:n,1:m}.$$

Hence, the conditional distribution of $(\sum_{s=t+1}^{\tau} X_s, (\sum_{s=t+1}^{\tau} X_s^{(g)})_{g\in \mathrm{Gr}_0})$ given $((X_s^{(g)})_{s=1}^t, (N_s^{(g)})_{s=1}^t, g \in \mathrm{Gr}_0)$ is multivariate normal with covariance matrix

$$\Sigma^{(t)}_{1:m|m+1:n} = \Sigma^{(t)}_{1:m,1:m} - \Sigma^{(t)}_{1:m,m+1:n}(\Sigma^{(t)}_{m+1:n,m+1:n})^{-1}\Sigma^{(t)}_{m+1:n,1:m},$$

where $\Sigma^{(t)}$ is the covariance matrix for the random vector in (11).

The covariance structure of the liability cash flows $(X_t)_{t=1}^{70}$ seen from time $t = 0$ is shown in Figure 2. The left figure shows the terms $t \mapsto (\mathrm{Var}(R \mid \mathcal{H}_t) - \mathrm{Var}(R \mid \mathcal{H}_{t+1}))^{1/2}$, $t = 0, \dots, 69$, in the formula for the outstanding liability cash flow at time $t = 0$. The right figure shows the correlation matrix for the liability cash flows $(X_t)_{t=1}^{70}$ seen from time $t = 0$.

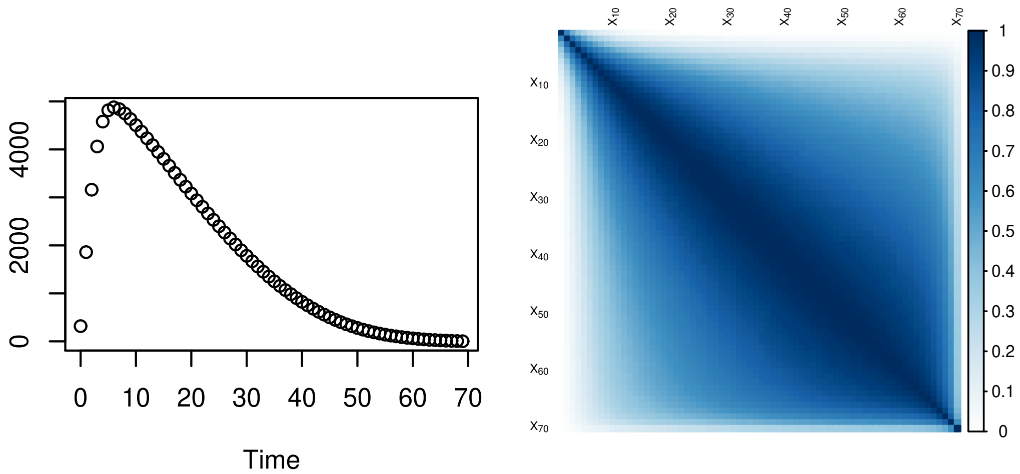

Figure 2: The left figure shows the terms $t \mapsto (\mathrm{Var}(R \mid \mathcal{H}_t) - \mathrm{Var}(R \mid \mathcal{H}_{t+1}))^{1/2}$, $t = 0, \dots, 69$, in the formula for the outstanding liability cash flow at time $t = 0$. The right figure shows the correlation matrix for the outstanding liability cash flow at time $t = 0$.

The allocation to contract groups is demonstrated in Figure 3. The upper-left figure corresponds to a 10% cost-of-capital rate, with parameters for the mortality model estimated from data for Swedish males, as described previously. Hence in this example, we have used $\sigma_\kappa = 1.60$ as the volatility for the mortality trend. From this figure it is clear that the liability value is just marginally larger than the expected value of the liability cash flows. However, we would expect a larger difference in a real world setting for a number of reasons. Firstly, in this example the only risk modelled is longevity risk, since we assume that the only expenses for the insurer are payments to policyholders. In reality, the insurer would also have administrative expenses, and thus be exposed to the risk that these expenses increase more than expected over the lifetime of the portfolio. Furthermore, we have only modelled contracts with a single premium paid upfront, and no right for the policyholder to terminate the contract ahead of time. With periodic premium payments or a surrender value being paid out if the contract is terminated by the policyholder, there would be uncertainty associated with future fees drawn by the insurer, giving rise to lapse risk. Both expense risk and lapse risk should be included in the valuation, as per the definition of non-financial risk in [10].

Secondly, in this example we have estimated the parameters for the mortality model based on data for the whole population of Swedish males. When valuing the outstanding liability of an insurer, the mortality model would need to be adjusted to capture the mortality experience of the insurer, i.e. to a much smaller population. How to go about adjusting the mortality model based on data for a large population to the subset consisting of only insured persons, and further, to the even smaller population of an individual insurance company, is a question in its own right. It is however reasonable to assume that the variability in data for an insurer's portfolio would be higher than in data for the population as a whole.

To conclude, the risk that an individual insurer is exposed to would generally be higher than what is captured by our example. In order to illustrate this effect in a simple manner without complicating our model, we have adjusted the volatility parameter σ_κ in the mortality trend, the results of which can be seen in Figure 3, where we also show the effect of changing the cost-of-capital rate from 10% to 20%. We emphasise that this is simply a way to illustrate the effect of overall higher volatility in the cash flow model than what can be captured in our simple setting, and should not be seen as a suitable way of adjusting the mortality model for an insurer. In fact, one can argue that the model for the mortality trend estimated for the whole population should be valid for smaller subsets of that population, at least long-term. Furthermore, even if the insurer has reason to believe that its particular portfolio consists of a certain subset of the population that would not see the same longevity improvements as the population as a whole, this is likely difficult to ascertain statistically for a portfolio of this size. One way of including the unsystematic mortality risk due to the size of the individual insurer's portfolio is given in [13]. In order to derive an alternative to the longevity risk calculations in the standard formula in Solvency II, they assume that the mortality rates of each insurer is proportional to the mortality rates for the whole insurance industry, with a mortality trend estimated based on data for the Danish population. However, the proportionality constant is unknown and has to be determined based on the insurer's mortality experience over a certain time period. The unsystematic risk is due to the fact that this proportionality constant needs to be reestimated every year.

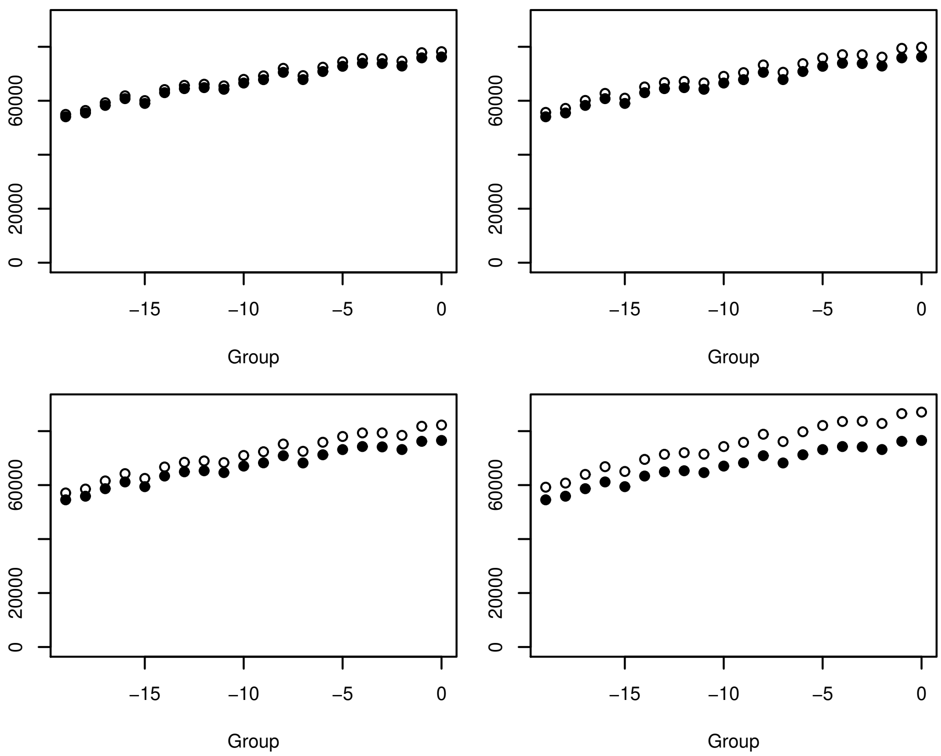

Figure 3: The figures show the allocation $\Lambda(R^{(g)}, R)$, $g = -19, \ldots, 0$, of the liability value (empty circles) and the expected value of the outstanding liability cash flows (filled circles) at time $t = 0$ to the 20 groups of contracts with the following parameters: 10% cost-of-capital rate (left), 20% cost-of-capital rate (right), $\sigma_\kappa = 1.60$ (top), and $\sigma_\kappa = 4.81$ (bottom).

5.2 Profit or loss for a portfolio of survival benefits

To illustrate the development of profit or loss and the contractual service margin in accordance with IFRS 17, we consider an insurance company that at time $t = 0$ issues 1000 insurance contracts to policyholders aged 50 years. The insurer make a single benefit payment $B = 1$ to each policyholder who survives until age 70. All policyholders pay the same single premium P at time $t = 0$, and no new contracts are issued after this time, hence the liability consists of a single group of contracts. We simulate the financial performance of the company until all contracts are terminated, which happens either when the policyholder dies, or when the benefit is paid at time $\tau = 20$, whichever comes first.

The financial performance is calculated through the algorithm for calculating the contractual service margin and loss component (Definition 1) and profit or loss as in Definition 2. Since we only have one group of contracts, we drop the superscript (g) in the following. For this simple example, we note that $L^{(t)}_{\mathrm{SP}} = 0$, $t = 0, \ldots, \tau - 2$, since service is only provided at time τ (i.e. in period τ). Furthermore, since payments are made immediately as claims are incurred (when an insured person survives until age 70), $L^{(t)}_{\mathrm{IC}} = 0$. Hence, $L^{(t)} = L^{(t)}_{\mathrm{RC}} = L^{(t)}_{\mathrm{FS}}$, $t = 0, \ldots, \tau - 2$, and $L^{(\tau-1)} = L^{(\tau-1)}_{\mathrm{RC}} = L^{(\tau-1)}_{\mathrm{SP}}$. We also note that $L^{(-1)} = L^{(\tau)} = 0$.

We simulate five independent trajectories $(\kappa^{(i)}_t)_{t=1}^{\tau}$, $i = 1, \ldots, 5$, for the mortality trend, and, as in the previous example, we let the number of active contracts remaining at each time point t be described as a nested binomial process. For the i-th mortality trend trajectory and at each time point t, the value $L^{(i,t)}$ of the outstanding liability is calculated according to (9) using the run-off filtration $(\mathcal{H}^{(i,t)}_s)_{s=0}^{\tau}$ that applies to a hypothetical run-off starting at time t and finishing at time $t + \tau$. Seen from time t, $\kappa^{(i)}_t$ and $N^{(i)}_t$ are (non-random) model parameters. Since here we do not consider any new contracts, and the only cash flows to the policyholders are at $\tau = 20$, the run-off filtration $(\mathcal{H}^{(i,t)}_s)_{s=0}^{\tau}$ only depends on the development of the number $N^{(i)}_t$ of contracts that are active at time t:

$$\mathcal{H}^{(i,t)}_0 = \{\emptyset, \Omega\}, \quad \mathcal{H}^{(i,t)}_s = \sigma(N^{(i)}_{t+s}) \vee \mathcal{H}^{(i,t)}_{s-1}, \quad s = 1 \ldots, \tau - t,$$

where $N^{(i)}_{t+s}$ denotes the number of active contracts at time $t + s$ for a run-off starting at time t given a development of the mortality trend up to time t according to $(\kappa^{(i)}_s)_{s=1}^{t}$.

In order to use the algorithm for CSM and LC in Definition 1, we also need to calculate the weights $(W_t)_{t=1}^{\tau}$. These are determined based on the coverage units provided in each period, and the expected remaining coverage units (see Section 3.1). Since all contracts considered in this example are survival benefits, the coverage units provided and remaining for each time point can be determined as in Example 1 in Section 3.1, hence if the kth insured person is alive at time t, then

$$\mathrm{CU}_t^{(t,k)} := 0, \quad \mathrm{ERCU}^{(t,k)} := B\mathbb{P}_t(T_{x,t} > 70 - x)$$

and if the kth insured person dies in period t, then

$$\mathrm{CU}_t^{(t,k)} := B\mathbb{P}_t(T_{x,t} > 70 - x), \quad \mathrm{ERCU}^{(t,k)} := 0,$$

where $T_{x,t}$ is the remaining lifetime of a randomly chosen individual who is x periods old at time t, and

$$\mathrm{CU}_t^{(t)} := \sum_{k=1}^{N_{t-1}} \mathrm{CU}_t^{(t,k)}, \quad \mathrm{ERCU}^{(t)} := \sum_{k=1}^{N_t} \mathrm{ERCU}^{(t,k)}$$

Assuming that the ith trajectory for the mortality trend corresponds to the observable information, the subscript t in $\mathbb{P}_t$ means that the probability is calculated with respect to the model with parameters (α_x), (β_x), δ, σ_κ and $\kappa_0^{(i,t)} := \kappa_t^{(i)}$. To calculate the probability at time t of an individual aged x at time t surviving until time $t + 70 - x$, $\mathbb{P}_t(T_{x,t} > 70 - x)$, we use the same approximations as when calculating the liability value.

$$\begin{aligned}
\mathbb{P}_t(T_{x,t} > 70 - x) &= \mathbb{E}_t\Big[\mathbb{P}_t\Big(T_{x,t} > 70 - x \mid \mu^{(i,t)}\Big)\Big] = \mathbb{E}_t\Big[\exp\Big\{-\sum_{s=0}^{69-x} \mu_{x+s,s}^{(i,t)}\Big\}\Big] \\
&\approx \exp\Big\{-\sum_{s=0}^{69-x} \mathbb{E}_t[\mu_{x+s,s}^{(i,t)}]\Big\} \\
&= \exp\Big\{-\sum_{s=0}^{69-x} \exp\Big\{\alpha_{x+s} + \beta_{x+s}(s\delta + \kappa_0^{(i,t)}) + \frac{1}{2}\beta_{x+s}^2 s\sigma_\kappa^2\Big\}\Big\}.
\end{aligned}$$

The trajectories for the mortality trend $\kappa_t^{(i)}$, $t = 1 \ldots, \tau$ are demonstrated in Figure 4, together with the trajectories for the accumulated number of deaths. The profit or loss, the contractual service margin and the loss component is shown in Figure 5. Note the different patterns of profit or loss depending on the size of the initial premium P compared to the initial liability value $L^{(0)}$. The initial premium is determined according to $nP = (1+m)L^{(0)}$, where m is the margin added to the initial liability value. The figures on the left demonstrate profit or loss, contractual service margin and loss component when $m = 0$, i.e. no margin is added to the initial liability value when determining the premium. Note that the development of the contractual service margin and loss component is closely related to the trajectory for the mortality trend. Furthermore, the profit or loss for trajectories where a CSM is built up is either zero or slightly above zero, until time $\tau = 20$ when

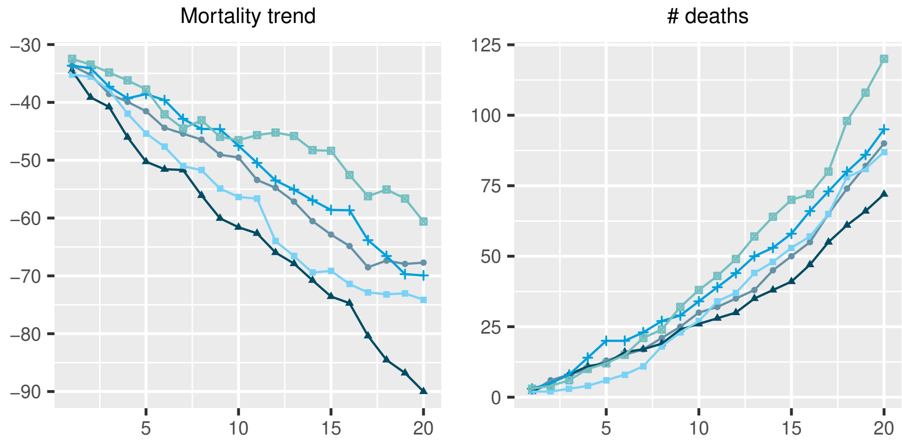

Figure 4: The left figure shows the trajectories for the mortality trend $\kappa_t^{(i)}$, $t = 1, \ldots, \tau$, $i = 1 \ldots, 5$. The right figure shows the trajectories for the accumulated number of deaths in the portfolio consisting of 1000 contracts.

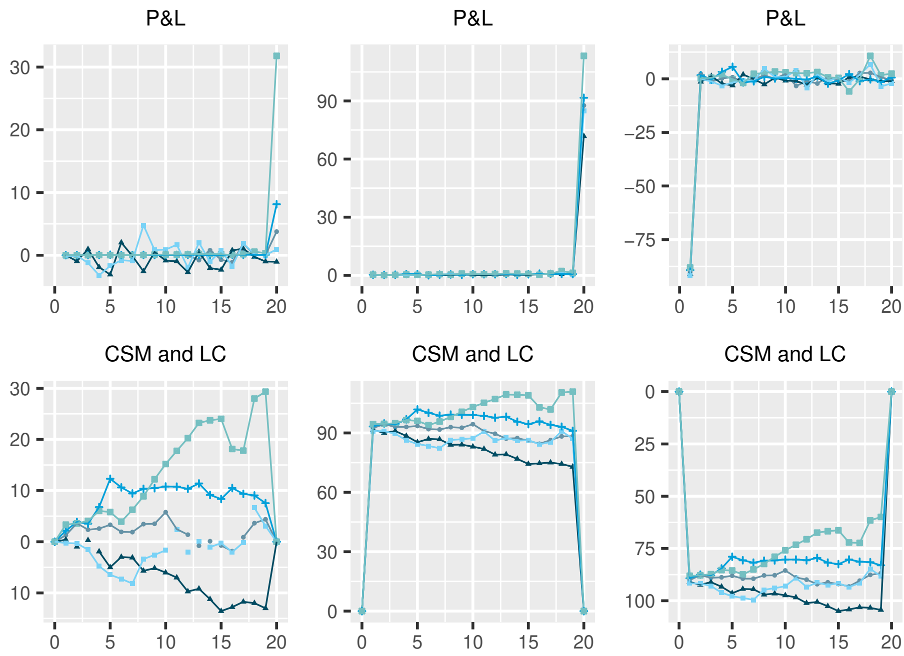

Figure 5: P&L (top), CSM and LC (bottom) for a portfolio of 1000 policy-holders, with margin 0 (left), margin 10% (middle), and margin -10% (right).

any remaining CSM is released into profit or loss. For trajectories where we instead have a positive LC, profit or loss can be both positive and negative, and tends to fluctuate around zero. These patterns are even clearer in the middle figures ($m = 0.1$) and left figures ($m = -0.1$). When the contract group is very profitable it will give rise to a positive CSM during the whole period, hence the insurer will be protected against losses since the CSM acts as a buffer against any adverse developments (as seen from the insurer's perspective) over the lifetime of the contracts. When the contract group is loss-making from its recognition, this loss has to be realised immediately, and any consecutive changes will directly affect profit or loss, leading to higher volatility in the financial performance.

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A Contractual service margin, loss component, and profit or loss as defined in the IFRS 17 Standard

In what follows, all liability cash flows and values are as defined in Section 2.

A.1 Algorithm for calculating CSM and LC

Consider a group of contracts g recognised during reporting period t_0. As the group of contracts is recognised, it will either give rise to a contractual service margin $\mathrm{CSM}^{(t_0,g)}$ or a loss component $\mathrm{LC}^{(t_0,g)}$, with the former being defined as an amount such that the contractual service margin added to the insurance liability value allocated to the group, and any cash flows arising from the contracts at that date, sum to zero (§ 38 in [10]). However, the contractual service margin cannot be negative, hence when the insurance liability value exceeds any cash flows arising from the contracts at that date, we will instead have a loss component, which is part of the liability value (§ 47 in [10]). Thus, at initial recognition,

$$\mathrm{CSM}^{(t_0,g)} = \left(P^{(t_0,g)} - L_{\mathrm{RC}}^{(t_0,g)}\right)^+, \quad \mathrm{LC}^{(t_0,g)} = \left(L_{\mathrm{RC}}^{(t_0,g)} - P^{(t_0,g)}\right)^+.$$

For reporting period t, where $t > t_0$, if $\mathrm{CSM}^{(t-1,g)} \geq 0$ and $\mathrm{LC}^{(t-1,g)} = 0$, to get the closing contractual service margin, the opening contractual service margin needs to be adjusted for any new contracts added to the group in the period, for interest accreted on the contractual service margin in the period, for changes in the liability value relating to future service, and finally for an amount recognised as profit or loss due to the transfer of services in the period. However, the adjustment due to changes in the liability value relating to future service can only be done to the extent that such a change does not exceed the contractual service margin, in which case this exceeding amount instead gives rise to a loss component at the end of the period. Furthermore, the interest accreted on the contractual service margin in the period should be based on the discount rates determined at initial recognition of the group, and the adjustment due to changes in the liability value relating to future service should be based on the liability value measured at the discount rates determined at initial recognition and should exclude the effect of the time value of money. (§ 44, §§ B96-B97, § B72(b) in [10]). It is not specified in what order these adjustments need to be made, apart from that allocating an amount of the contractual service margin to profit or loss has to be the last step, i.e. based on the contractual service margin after all other adjustments have been made. We choose to adjust for interest accreted on the contractual service margin first, followed by the adjustment for changes in the liability value relating to future service and new contracts added.

Adjusting the contractual service margin for interest accreted in the period, based on the discount rates at initial recognition, leads to the adjustment

$$\left(\frac{d_{t_0,t-1}}{d_{t_0,t}} - 1\right) \mathrm{CSM}^{(t-1,g)}.$$

If we let $L_{\mathrm{EC},t_0}^{(t,g)}$ (liability for existing contracts) denote the liability value at time t for group g allocated to cash flows from contracts belonging to the group at time $t-1$, and let $L^{(t,g)}_{\mathrm{NC},t_0}$ (liability for new contracts) denote the liability value allocated to cash flows from new contracts added to the group in period t, with $L^{(t,g)}_{\mathrm{RC},t_0} = L^{(t,g)}_{\mathrm{EC},t_0} + L^{(t,g)}_{\mathrm{NC},t_0}$, where the subscript t_0 denotes that the liability value is measured at the discount rates at initial recognition, then the adjustment for changes in the liability value relating to future service, excluding the effect of the time value of money, is

$$\frac{d_{t_0,t-1}}{d_{t_0,t}} L^{(t-1,g)}_{\mathrm{FS},t_0} - L^{(t,g)}_{\mathrm{EC},t_0}.$$

Finally, the adjustment for new contracts added to the group in the period is

$$P^{(t,g)} - L^{(t,g)}_{\mathrm{NC},t_0}.$$

Consequently, the total adjustment is $\Delta_1 - \mathrm{CSM}^{(t-1,g)}$ in Definition 1, hence $\mathrm{CSM}^{(t,g)} = W^{(g)}_t \Delta_1^+$ and $\mathrm{LC}^{(t,g)} = \Delta_1^-$, where $1 - W^{(g)}_t$ represents the proportion of the unearned profit at time t for group g that is allocated to profit or loss in period t. See Section 3.1 for precise details on the weights $W^{(g)}_t$.

For reporting period t, where $t > t_0$, if $\mathrm{CSM}^{(t-1,g)} = 0$ and $\mathrm{LC}^{(t-1,g)} > 0$, to get the closing loss component, the opening loss component should be adjusted for a proportion of changes in the liability value due to services provided in the period, and the same proportion of the changes in the liability value due to the effect of the time value of money and changes in the time value of money. Furthermore, it should be adjusted for the changes in the liability value relating to future service, until the loss component is zero. If the change in the liability value relating to future service exceeds the loss component remaining after previous adjustments, then this exceeding amount instead gives rise to a contractual service margin at the end of the period. Just like when adjusting the contractual service margin, the change in the liability value relating to future service should be based on the liability value measured at the discount rates determined at initial recognition of the group, and should exclude the effect of the time value of money and changes in the time value of money. The proportion of the changes in the liability value due to services provided and the time value of money that adjusts the loss component should be based on a systematic allocation of these changes between the loss component and the liability value excluding the loss component, and the allocation should ensure that the loss component at the end of the coverage period is equal to zero. (§ 48, §§50-52, § 87 in [10]). For consistency, we also adjust the loss component for any new contracts added to the group in the period.

The change in the liability value relating to services provided in the period is $-\frac{1}{d_{t-1,t}} L^{(t-1,g)}_{\mathrm{SP}}$ (the change due to the effect of the time value of money is excluded, see (§ B123(a)(iv) in [10]). Furthermore, the change in the liability value due to the effect of the time value of money and changes in the time value of money is $\frac{1}{d_{t-1,t}} L_{\mathrm{SP}}^{(t-1,g)} - L_{\mathrm{SP}}^{(t-1,g)} + \frac{1}{d_{t-1,t}} L_{\mathrm{FS},t}^{(t-1,g)} - L_{\mathrm{FS}}^{(t-1,g)}$. Note that the subscript t in $L_{\mathrm{FS},t}^{(t-1,g)}$ denotes that the liability value is measured at the discount rates at time t, hence $\frac{1}{d_{t-1,t}} L_{\mathrm{FS},t}^{(t-1,g)}$ is the liability value at time t for cash flows after t, measured at the t-discount rates, but where all non-financial assumptions are based on the information at time $t-1$. However, this is only the change in the liability value due to the time value of money measured at current discount rates at $t-1$ and t. Since changes in the liability value relating to future service that adjust the loss component and contractual service margin also excludes changes due to the time value of money but is measured at the discount rates determined at initial recognition, an extra adjustment term is needed to ensure that the full effect of the change due to the time value of money is included. This adjustment term is

$$L_{\mathrm{RC}}^{(t,g)} - \frac{1}{d_{t-1,t}} L_{\mathrm{FS},t}^{(t-1,g)} - \Big(L_{\mathrm{RC},t_0}^{(t,g)} - \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} \Big)$$

i.e. the difference between the change in the liability relating to future service measured at the current rate and the change in the liability relating to future service measured at the rate determined at initial recognition. See further § BC275 in [11].

We define the proportion of the changes due to services provided in the period and of the changes due to the effect of time value of money that adjusts the loss component to be the ratio of the loss component to the liability value at $t-1$. For details on this choice of definition, see Section A.3. However, this adjustment must not cause the loss component to become negative. This is in order to ensure that a contractual service margin only arises if changes in the liability value relating to future service exceeds the amount of the loss component after previous adjustments. Hence, the first adjustment to the loss component is

$$\begin{aligned}
&\frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}} \Big(- \frac{1}{d_{t-1,t}} L_{\mathrm{SP}}^{(t-1,g)} + \frac{1}{d_{t-1,t}} L_{\mathrm{SP}}^{(t-1,g)} - L_{\mathrm{SP}}^{(t-1,g)} + \frac{1}{d_{t-1,t}} L_{\mathrm{FS},t}^{(t-1,g)} - L_{\mathrm{FS}}^{(t-1,g)} \\
&\qquad + L_{\mathrm{RC}}^{(t,g)} - \frac{1}{d_{t-1,t}} L_{\mathrm{FS},t}^{(t-1,g)} - \Big(L_{\mathrm{RC},t_0}^{(t,g)} - \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} \Big) \Big) \\
&\quad = \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}} \Big(- L_{\mathrm{RC}}^{(t-1,g)} + L_{\mathrm{RC}}^{(t,g)} - L_{\mathrm{RC},t_0}^{(t,g)} + \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} \Big) \\
&\quad = - \mathrm{LC}^{(t-1,g)} + \mathrm{LC}^{(t-1,g)} \frac{L_{\mathrm{RC}}^{(t,g)}}{L_{\mathrm{RC}}^{(t-1,g)}} - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}} \Big(L_{\mathrm{RC},t_0}^{(t,g)} - \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} \Big),
\end{aligned}$$

as long as this adjustment does not make the loss component negative, i.e. as long as $L^{(t,g)}_{\mathrm{RC}} \geq L^{(t,g)}_{\mathrm{RC},t_0} - \frac{d_{t_0,t-1}}{d_{t_0,t}} L^{(t-1,g)}_{\mathrm{FS},t_0}$. Otherwise, the adjustment is $-\mathrm{LC}^{(t-1,g)}$. Hence the opening loss component after the first adjustment is equal to Δ_2^- in Definition 1.

Furthermore, as when adjusting the contractual service margin, the adjustment for changes in the liability value relating to future service, excluding the effect of the time value of money, is

$$L^{(t,g)}_{\mathrm{EC},t_0} - \frac{d_{t_0,t-1}}{d_{t_0,t}} L^{(t-1,g)}_{\mathrm{FS},t_0}.$$

Finally, the adjustment for new contracts added to the group in the period is

$$L^{(t,g)}_{\mathrm{NC},t_0} - P^{(t,g)}.$$

Consequently, the remaining adjustment is $-\Delta_3 - \Delta_2^-$ with Δ_3 as in Definition 1, hence $\mathrm{CSM}^{(t,g)} = \Delta_3^+$ and $\mathrm{LC}^{(t,g)} = \Delta_3^-$.

A.1.1 Initial recognition of a group of contracts

In IFRS 17, the initial contractual service margin (or for onerous contracts, the initial loss component) for a group should be determined at the time of initial recognition of the group. This is defined as the earliest of the following three dates: the beginning of the coverage period of the group; the date when the first payment from a policyholder in the group becomes due; and, for an onerous group, when the group becomes onerous (§ 25 in [10]).

With the initial contractual service margin or loss component as in Definition 1, the time of initial recognition of a group of contracts issued in period t will always be at time t. In practice, this approximation might be too crude, especially if the reporting frequency is annual. For contracts recognised in the period, there will be no estimate of incurred amounts as seen from the beginning of the period, and no release from risk, despite the fact that there could be actual incurred amounts for these contracts in the period. Furthermore, the discount rates used for the development of the contractual service margin and loss component depend on the date of initial recognition. At the same time, it is generally not practicable for an insurer to value its liabilities daily. A more realistic assumption is that the insurer is able to value its liabilities at the end of each month, and thus some approximation of the date of initial recognition will still be needed.

The algorithm in Definition 1 for calculating the contractual service margin and the loss component in the period when the group is initially recognised can be adjusted in the following way if a better approximation of the time of initial recognition is needed. Calculate the liability value at some intermediate time point $t_0 \in (t-1, t]$, including information on the new contracts issued in the period, and use this liability value to determine the initial contractual service margin and loss component as in Remark 6. Next, use this initial liability value, contractual service margin and loss component instead of $L_{\mathrm{RC}}^{(t-1,g)}$, $\mathrm{CSM}^{(t-1,g)}$ and $\mathrm{LC}^{(t-1,g)}$ in Definition 1 to get the closing contractual service margin and loss component in the period. Profit or loss can still be calculated according to Definition 2, with $\mathrm{CSM}^{(t,g)}$ calculated in this way, and $L_{\mathrm{RC}}^{(t-1,g)} = \mathrm{CSM}^{(t-1,g)} = 0$. This will ensure that we do not have $L_{\mathrm{RC}}^{(t,g)} + \mathrm{CSM}^{(t,g)} = P^{(t,g)}$, i.e. $\mathrm{P\&L}_t^{(g)} = -I_t^{(g)}$ for a profitable group of contracts, which would be the case if the time of initial recognition of the group is at time t.

Hence, the algorithm in Definition 1 can be adjusted in the following way. We assume that the insurer can value its liabilities n times during each reporting period, i.e. at time $s \in \{t - \frac{n-1}{n}, t - \frac{n-2}{n} \dots, t - \frac{1}{n}, t\}$ in reporting period t, and that time of initial recognition is set to $t_0 = t - \frac{n-k}{n}$, where k is chosen to best approximate the actual date of initial recognition in the period.

Let $X_{\mathrm{NCA}}^{(t_0)}$ (New Contracts Added) be the cash flow that corresponds to the outstanding liability as seen from time t_0 when new contracts issued during period t have been added to the total portfolio:

$$X_{\mathrm{NCA}}^{(t_0)} := \sum_{g \in \mathrm{Gr}_{t-1} \cup \mathrm{Gr}_t} \left(I_{t_0+\frac{1}{n}}^{(g)}, \dots, I_{t_0+\tau-\frac{1}{n}}^{(g)}, I_{t_0+\tau}^{(g)} \right).$$

Note that $I_t^{(g)}$ here correspond to the incremental net cash flow for group g in the time interval $(t - \frac{1}{n}, t]$. Let $L_{\mathrm{NCA}}^{(t_0)}$ denote the value of the outstanding liability cash flow $X_{\mathrm{NCA}}^{(t_0)}$ as seen from time t_0, conditional on information on the new contracts recognised between t_0 and t, including the terms of the contracts, but not including information on the actual amounts paid in the period after the date of initial recognition. Let $L_{\mathrm{NCA}}^{(t_0,g)}$ denote the liability value allocated to group g, i.e.

$$L_{\mathrm{NCA}}^{(t_0)} = \sum_{g \in \mathrm{Gr}_{t-1} \cup \mathrm{Gr}_t} L_{\mathrm{NCA}}^{(t_0,g)}.$$

If $g \notin \mathrm{Gr}_{t-1} \cup \mathrm{Gr}_t$, then $L_{\mathrm{NCA}}^{(t_0,g)} = 0$. Let $L_{\mathrm{NCA,SP}}^{(t_0,g)}$ denote the liability value for remaining coverage allocated to services provided between t and $t+1$ for group g, and $L_{\mathrm{NCA,FS}}^{(t_0,g)}$ the liability value allocated to future service (after $t+1$) for group g, as seen from time t_0, but including information on the new contracts recognised between t_0 and t, i.e.

$$L_{\mathrm{NCA,RC}}^{(t_0,g)} = L_{\mathrm{NCA,SP}}^{(t_0,g)} + L_{\mathrm{NCA,FS}}^{(t_0,g)}.$$

Definition 1 can now be adjusted as follows for the period when the group is initially recognised:

If $g \notin \mathrm{Gr}_{t-1}$ and $g \in \mathrm{Gr}_t$, then set

$$\mathrm{CSM}^{(t_0,g)}_{\mathrm{NCA}} = \Big(P^{(t,g)} - L^{(t_0,g)}_{\mathrm{NCA,RC}}\Big)^+, \quad \mathrm{LC}^{(t_0,g)}_{\mathrm{NCA}} = \Big(L^{(t_0,g)}_{\mathrm{NCA,RC}} - P^{(t,g)}\Big)^+.$$

If $\mathrm{CSM}^{(t_0,g)}_{\mathrm{NCA}} \geq 0$ and $\mathrm{LC}^{(t_0,g)}_{\mathrm{NCA}} = 0$, then set

$$\Delta_1 := \frac{1}{d_{t_0,t}} \mathrm{CSM}^{(t_0,g)}_{\mathrm{NCA}} + \frac{1}{d_{t_0,t}} L^{(t_0,g)}_{\mathrm{NCA,FS}} - L^{(t,g)}_{\mathrm{RC},t_0}$$

and set $\mathrm{CSM}^{(t,g)} := W^{(g)}_t \Delta_1^+$ and $\mathrm{LC}^{(t,g)} := \Delta_1^-$.

If $\mathrm{CSM}^{(t_0,g)}_{\mathrm{NCA}} = 0$ and $\mathrm{LC}^{(t_0,g)}_{\mathrm{NCA}} > 0$, then set

$$\begin{aligned}\Delta_2 &:= -\mathrm{LC}^{(t_0,g)}_{\mathrm{NCA}} \frac{L^{(t,g)}_{\mathrm{RC}}}{L^{(t_0,g)}_{\mathrm{NCA,RC}}} + \frac{\mathrm{LC}^{(t_0,g)}_{\mathrm{NCA}}}{L^{(t_0,g)}_{\mathrm{NCA,RC}}}\Big(L^{(t,g)}_{\mathrm{RC},t_0} - \frac{1}{d_{t_0,t}} L^{(t_0,g)}_{\mathrm{NCA,FS}}\Big),\\ \Delta_3 &:= -\Delta_2^- + \frac{1}{d_{t_0,t}} L^{(t_0,g)}_{\mathrm{NCA,FS}} - L^{(t,g)}_{\mathrm{RC},t_0}\end{aligned}$$

and set $\mathrm{CSM}^{(t,g)} := \Delta_3^+$ and $\mathrm{LC}^{(t,g)} := \Delta_3^-$.

Definition 3 (see Section A.2 below) for profit or loss can be adjusted in a similar manner for the period when the group is initially recognised:

If $\mathrm{CSM}^{(t_0,g)}_{\mathrm{NCA}} \geq 0$ and $\mathrm{LC}^{(t_0,g)}_{\mathrm{NCA}} = 0$, then

$$\begin{aligned}\mathrm{P\&L}^{(g)}_t := {} & L^{(t_0,g)}_{\mathrm{NCA,SP}} - \sum_{s=1}^n I^{(g)}_{t-\frac{n-s}{n}} + \Delta_1^+ - \mathrm{CSM}^{(t,g)} - \mathrm{LC}^{(t,g)} + \Big(1 - \frac{1}{d_{t_0,t}}\Big) \mathrm{CSM}^{(t_0,g)}_{\mathrm{NCA}}\\ & + L^{(t_0,g)}_{\mathrm{NCA,FS}} - L^{(t,g)}_{\mathrm{RC}} - \Big(\frac{1}{d_{t_0,t}} L^{(t_0,g)}_{\mathrm{NCA,FS}} - L^{(t,g)}_{\mathrm{RC},t_0}\Big).\end{aligned}$$

If $\mathrm{CSM}^{(t_0,g)}_{\mathrm{NCA}} = 0$ and $\mathrm{LC}^{(t_0,g)}_{\mathrm{NCA}} > 0$, then

$$\begin{aligned}\mathrm{P\&L}^{(g)}_t := {} & \Big(1 - \frac{\mathrm{LC}^{(t_0,g)}_{\mathrm{NCA}}}{L^{(t_0,g)}_{\mathrm{NCA,RC}}}\Big) L^{(t_0,g)}_{\mathrm{NCA,SP}} - \sum_{s=1}^n I^{(g)}_{t-\frac{n-s}{n}} - \mathrm{LC}^{(t_0,g)}_{\mathrm{NCA}} + \mathrm{LC}^{(t_0,g)}_{\mathrm{NCA}} - \mathrm{LC}^{(t,g)} + \Delta_2^+\\ & + \Big(1 - \frac{\mathrm{LC}^{(t_0,g)}_{\mathrm{NCA}}}{L^{(t_0,g)}_{\mathrm{NCA,RC}}}\Big)\Big(L^{(t_0,g)}_{\mathrm{NCA,FS}} - L^{(t,g)}_{\mathrm{RC}} - \Big(\frac{1}{d_{t_0,t}} L^{(t_0,g)}_{\mathrm{NCA,FS}} - L^{(t,g)}_{\mathrm{RC},t_0}\Big)\Big).\end{aligned}$$

In either case $\mathrm{P\&L}^{(g)}_t := L^{(t-1,g)}_{\mathrm{RC}} + \mathrm{CSM}^{(t-1,g)} + P^{(t,g)} - L^{(t,g)}_{\mathrm{RC}} - \mathrm{CSM}^{(t,g)} - \sum_{s=1}^n I^{(g)}_{t-\frac{n-s}{n}}$ still holds, i.e. Definition 2 can be used to determine profit or loss in the period when the group is initially recognised, with $L^{(t-1,g)}_{\mathrm{RC}} = \mathrm{CSM}^{(t-1,g)} = 0$ and $\mathrm{CSM}^{(t,g)}$ computed by the adjusted algorithm above.

For reporting periods after the period when the group is initially recognised, the algorithm follows Definition 1 in and profit or loss can be calculated according to Definition 2. If new contracts are added to an already existing group in later reporting periods, the contractual service margin and loss component can be adjusted in a similar manner if a better approximation of the time when these contracts join the group is needed.

A.2 Alternative definition of profit and loss

In the IFRS 17 Standard (§ 83 and §§ B121-B124 for insurance revenue, § 84 and § 103 (b) for insurance service expenses, and § 87 for insurance finance income and expenses, in [10]) profit or loss for group g of contracts in period t is defined as follows:

If $\mathrm{CSM}^{(t-1,g)} \geq 0$ and $\mathrm{LC}^{(t-1,g)} = 0$, insurance revenue and insurance service expenses in the period are defined as the experience adjustments, the release from risk in the period, and, if $\Delta_1 > 0$, a part of the contractual service margin allocated to profit or loss in the period, otherwise an amount of $-\mathrm{LC}^{(t,g)} = \Delta_1$ is included in profit or loss. Insurance finance income and expenses are defined as any changes in the liability value for the group due to the effect of time value of money and changes in the time value of money. As discussed in Section A.1, this includes an adjustment term which is the difference between the change in the liability value relating to future service measured at current discount rates and and the change in the liability value relating to future service measured at the discount rates determined at initial recognition.

If instead $\mathrm{CSM}^{(t-1,g)} = 0$ and $\mathrm{LC}^{(t-1,g)} > 0$, insurance revenue and insurance service expenses in the period are defined as the experience adjustments and release from risk in the period, excluding a proportion of the change in the liability value due to services provided that is allocated to the loss component, and any increase or reversal of the loss component in the period. Insurance finance income and expenses are defined as any changes in the liability value for the group due to the effect of the time value of money and changes in the time value of money, excluding a proportion which is allocated to the loss component. As explained in Section A.3 we define the proportion allocated to the loss component as the ratio of the loss component to the liability value at time $t-1$, however, this adjustment must not cause the loss component to become negative, in which case the excess if the amount that reduces the loss component to zero, i.e. Δ_2^+, is instead included in profit or loss.

Definition 3 (Profit and loss in IFRS 17)**.** *Profit or loss determined at time*

t for reporting period t and group g of contracts is given by:
If $\mathrm{CSM}^{(t-1,g)} \geq 0$ *and* $\mathrm{LC}^{(t-1,g)} = 0$*, then*

$$\begin{aligned}\mathrm{P\&L}_t^{(g)} := {} & L_{\mathrm{SP}}^{(t-1,g)} - I_t^{(g)} + \Delta_1^+ - \mathrm{CSM}^{(t,g)} - \mathrm{LC}^{(t,g)} + \Big(1 - \frac{d_{t_0,t-1}}{d_{t_0,t}}\Big)\mathrm{CSM}^{(t-1,g)} \\ & + L_{\mathrm{FS}}^{(t-1,g)} - L_{\mathrm{RC}}^{(t,g)} - \Big(\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)}\Big).\end{aligned}$$

If $\mathrm{CSM}^{(t-1,g)} = 0$ *and* $\mathrm{LC}^{(t-1,g)} > 0$*, then*

$$\begin{aligned}\mathrm{P\&L}_t^{(g)} := {} & \Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big) L_{\mathrm{SP}}^{(t-1,g)} - I_t^{(g)} + \mathrm{LC}^{(t-1,g)} - \mathrm{LC}^{(t,g)} + \Delta_2^+ \\ & + \Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big)\Big(L_{\mathrm{FS}}^{(t-1,g)} - L_{\mathrm{RC}}^{(t,g)} - \Big(\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)}\Big)\Big)\end{aligned}$$

Remark 13. *Note that this is under the assumption that all contracts generate cash flows that are independent of financial asset values. If this were not the case, insurance finance income and expenses would also include the change in the liability value due to the effect of financial risk and changes in financial risk (§ 87(b) in [10]). Furthermore, any effect on profit or loss from assets held by the insurer has been disregarded.*

Proposition 1. *Definition 2 and Definition 3 are equivalent.*

Proof. By Definition 3, if $\mathrm{CSM}^{(t-1,g)} \geq 0$ and $\mathrm{LC}^{(t-1,g)} = 0$, then

$$\begin{aligned}\mathrm{P\&L}_t^{(g)} = {} & L_{\mathrm{SP}}^{(t-1,g)} - I_t^{(g)} + \Delta_1^+ - \mathrm{CSM}^{(t,g)} - \Delta_1^- + \Big(1 - \frac{d_{t_0,t-1}}{d_{t_0,t}}\Big)\mathrm{CSM}^{(t-1,g)} \\ & + L_{\mathrm{FS}}^{(t-1,g)} - L_{\mathrm{RC}}^{(t,g)} - \Big(\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)}\Big) \\ = {} & L_{\mathrm{RC}}^{(t-1,g)} - I_t^{(g)} + \frac{d_{t_0,t-1}}{d_{t_0,t}}\mathrm{CSM}^{(t-1,g)} + \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)} + P^{(t,g)} \\ & - \mathrm{CSM}^{(t,g)} + \Big(1 - \frac{d_{t_0,t-1}}{d_{t_0,t}}\Big)\mathrm{CSM}^{(t-1,g)} - L_{\mathrm{RC}}^{(t,g)} - \Big(\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)}\Big) \\ = {} & L_{\mathrm{RC}}^{(t-1,g)} + \mathrm{CSM}^{(t-1,g)} + P^{(t,g)} - L_{\mathrm{RC}}^{(t,g)} - \mathrm{CSM}^{(t,g)} - I_t^{(g)}\end{aligned}$$

By Definition 3, if $\mathrm{CSM}^{(t-1,g)} = 0$ and $\mathrm{LC}^{(t-1,g)} > 0$, then

$$\begin{aligned}
\mathrm{P\&L}_t^{(g)} &= \Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big) L_{\mathrm{SP}}^{(t-1,g)} - I_t^{(g)} + \mathrm{LC}^{(t-1,g)} - \mathrm{LC}^{(t,g)} + \Delta_2^+ \\
&\quad + \Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big)\Big(L_{\mathrm{FS}}^{(t-1,g)} - L_{\mathrm{RC}}^{(t,g)} - \Big(\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)}\Big)\Big) \\
&= \Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big) L_{\mathrm{RC}}^{(t-1,g)} - I_t^{(g)} + \mathrm{LC}^{(t-1,g)} - \mathrm{CSM}^{(t,g)} + \mathrm{CSM}^{(t,g)} - \mathrm{LC}^{(t,g)} \\
&\quad + \Delta_2^+ - L_{\mathrm{RC}}^{(t,g)} + \mathrm{LC}^{(t-1,g)} \frac{L_{\mathrm{RC}}^{(t,g)}}{L_{\mathrm{RC}}^{(t-1,g)}} + \Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big)\Big(L_{\mathrm{RC},t_0}^{(t,g)} - \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)}\Big) \\
&= L_{\mathrm{RC}}^{(t-1,g)} - I_t^{(g)} - \mathrm{CSM}^{(t,g)} + \Delta_3 - L_{\mathrm{RC}}^{(t,g)} + \Delta_2^+ + L_{\mathrm{RC},t_0}^{(t,g)} - \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - \Delta_2 \\
&= L_{\mathrm{RC}}^{(t-1,g)} - I_t^{(g)} - \mathrm{CSM}^{(t,g)} + \Delta_3 - L_{\mathrm{RC}}^{(t,g)} + \Delta_2^+ - \Delta_3 - \Delta_2^- + P^{(t,g)} - \Delta_2 \\
&= L_{\mathrm{RC}}^{(t-1,g)} - I_t^{(g)} - \mathrm{CSM}^{(t,g)} - L_{\mathrm{RC}}^{(t,g)} + \Delta_2 + P^{(t,g)} - \Delta_2 \\
&= L_{\mathrm{RC}}^{(t-1,g)} + \mathrm{CSM}^{(t-1,g)} + P^{(t,g)} - L_{\mathrm{RC}}^{(t,g)} - \mathrm{CSM}^{(t,g)} - I_t^{(g)}
\end{aligned}$$

□

A.3 General allocation between LC and liability value excluding LC

When determining the closing loss component in the period, the opening loss component is adjusted for a proportion of the changes in the liability value due to services provided in the period and due to the effect of the time value of money and changes in the time value of money. The remaining proportion of these changes is included in profit or loss for the period. The proportion should be decided based on a systematic allocation between the loss component and the liability value excluding the loss component, and the allocation should ensure that the loss component at the end of the coverage period is equal to zero (§§ 50-52 in [10]). We have defined this as the ratio of the loss component to the liability value at time $t-1$, as long as this does not lead to an adjustment that causes the loss component to become negative, in which case the excess of the amount that reduces the loss component to zero is instead included in profit or loss. However, other definitions are possible.

Let $u_{t-1}^{(g)} \in [0, 1]$ be the proportion of the changes in the liability value due to services provided and due to the effect of the time value of money and changes in the time value of money that adjusts the loss component for group g in period t. Furthermore, let

$$\Delta_2 := -\,\mathrm{LC}^{(t-1,g)} - u_{t-1}^{(g)}\Big(- L_{\mathrm{RC}}^{(t-1,g)} + L_{\mathrm{RC}}^{(t,g)} - L_{\mathrm{RC},t_0}^{(t,g)} + \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)}\Big).$$

Since a contractual service margin at time t only arises if changes in the liability value relating to future service exceeds the amount of the loss component remaining after previous adjustments, if $\Delta_2 > 0$ this amount should not contribute to the contractual service margin, but instead affect profit or loss. Hence we let

$$\Delta_3 := -\Delta_2^- + \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)} + P^{(t,g)},$$

and $\mathrm{CSM}^{(t,g)} := \Delta_3^+$ and $\mathrm{LC}^{(t,g)} := \Delta_3^-$. Similarly, profit or loss for period t becomes

$$\begin{aligned}\mathrm{P\&L}_t^{(g)} := {} & (1 - u_{t-1}^{(g)}) L_{\mathrm{SP}}^{(t-1,g)} - I_t^{(g)} + \mathrm{LC}^{(t-1,g)} - \mathrm{LC}^{(t,g)} + \Delta_2^+ \\ & + (1 - u_{t-1}^{(g)})\Big(L_{\mathrm{FS}}^{(t-1,g)} - L_{\mathrm{RC}}^{(t,g)} - \Big(\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)}\Big)\Big).\end{aligned}$$

Since the systematic allocation should ensure that the loss component at the end of the coverage period is equal to zero, we need $\Delta_3 \geq 0$ for reporting time τ. Since $L_{\mathrm{RC}}^{(\tau,g)} = L_{\mathrm{FS},t_0}^{(\tau-1,g)} = L_{\mathrm{RC},t_0}^{(\tau,g)} = 0$ and $P^{(\tau,g)} = 0$ per definition, this leads to the condition

$$\mathrm{LC}^{(\tau-1,g)} - u_{\tau-1}^{(g)} L_{\mathrm{RC}}^{(\tau-1,g)} \leq 0$$

to ensure $\mathrm{LC}^{(\tau,g)} = 0$. Letting

$$u_{t-1}^{(g)} := \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}},$$

this condition holds with equality, and we further note that

$$\begin{aligned}\Delta_2 := {} & -\,\mathrm{LC}^{(t-1,g)} - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big(- L_{\mathrm{RC}}^{(t-1,g)} + L_{\mathrm{RC}}^{(t,g)} - L_{\mathrm{RC},t_0}^{(t,g)} + \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)}\Big) \\ = {} & \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big(- L_{\mathrm{RC}}^{(t,g)} + L_{\mathrm{RC},t_0}^{(t,g)} - \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)}\Big)\end{aligned}$$

as in Definition 1, and

$$\begin{aligned}\mathrm{P\&L}_t^{(g)} := {} & \Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big) L_{\mathrm{SP}}^{(t-1,g)} - I_t^{(g)} + \mathrm{LC}^{(t-1,g)} - \mathrm{LC}^{(t,g)} + \Delta_2^+ \\ & + \Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big)\Big(L_{\mathrm{FS}}^{(t-1,g)} - L_{\mathrm{RC}}^{(t,g)} - \Big(\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)}\Big)\Big),\end{aligned}$$

as in Definition 3.

Furthermore, note that with this definition of $u_t^{(g)}$, $\Delta_2 > 0$ requires that $L_{\mathrm{RC}}^{(t,g)} < L_{\mathrm{RC},t_0}^{(t,g)} - \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)}$, i.e. that the liability value measured at current rates is smaller than the change in the liability value relating to future service measured at the rates determined at initial recognition. Under the assumption that $\Delta_2 \leq 0$ we obtain

$$\begin{aligned}\Delta_3 &= \Delta_2 + \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)} + P^{(t,g)} \\ &= -\,\mathrm{LC}^{(t-1,g)} \frac{L_{\mathrm{RC}}^{(t,g)}}{L_{\mathrm{RC}}^{(t-1,g)}} + \Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big)\Big(\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)}\Big) + P^{(t,g)}\end{aligned}$$

and

$$\begin{aligned}\mathrm{P\&L}_t^{(g)} = &\Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big) L_{\mathrm{SP}}^{(t-1,g)} - I_t^{(g)} + \mathrm{LC}^{(t-1,g)} - \mathrm{LC}^{(t,g)} \\ &+ \Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big)\Big(L_{\mathrm{FS}}^{(t-1,g)} - L_{\mathrm{RC}}^{(t,g)} - \Big(\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)}\Big)\Big).\end{aligned}$$

For the case when $\Delta_2 > 0$ we obtain

$$\Delta_3 = \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)} + P^{(t,g)}$$

and

$$\begin{aligned}\mathrm{P\&L}_t^{(g)} = &\Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big) L_{\mathrm{SP}}^{(t-1,g)} - I_t^{(g)} + \mathrm{LC}^{(t-1,g)} - \mathrm{LC}^{(t,g)} + \Delta_2 \\ &+ \Big(1 - \frac{\mathrm{LC}^{(t-1,g)}}{L_{\mathrm{RC}}^{(t-1,g)}}\Big)\Big(L_{\mathrm{FS}}^{(t-1,g)} - L_{\mathrm{RC}}^{(t,g)} - \Big(\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\mathrm{FS},t_0}^{(t-1,g)} - L_{\mathrm{RC},t_0}^{(t,g)}\Big)\Big)\end{aligned}$$

For either case we get $\mathrm{P\&L}_t^{(g)} = L_{\mathrm{RC}}^{(t-1,g)} + \mathrm{CSM}^{(t-1,g)} + P^{(t,g)} - L_{\mathrm{RC}}^{(t,g)} - \mathrm{CSM}^{(t,g)} - I_t^{(g)}$ as shown in Proposition 1.

B Computational details for the numerical illustration

We will now go through some technical details needed in the life-insurance example to illustrate the results of the previous sections.

B.1 Covariances between sizes of contract groups

We need to determine the covariances

$$\mathrm{Cov}(N_{x,t}, N_{y,s}) = \mathbb{E}[\mathrm{Cov}(N_{x,t}, N_{y,s} \mid \mu)] + \mathrm{Cov}(\mathbb{E}[N_{x,t} \mid \mu], \mathbb{E}[N_{y,s} \mid \mu]). \quad (12)$$

Repeated use of iterated expectations yields

$$\mathbb{E}[N_{x,t} \mid \mu] = \mathbb{E}[\mathbb{E}[N_{x,t} \mid N_{x-1,t-1}, \mu] \mid \mu] = \mathbb{E}[N_{x-1,t-1} p_{x-1,t-1} \mid \mu]$$

and ultimately

$$\mathbb{E}[N_{x,t} \mid \mu] = n_{x-t,0} \prod_{i=0}^{t-1} p_{x-t+i,i} = n_{x-t,0} \exp\Big\{ -\sum_{i=0}^{t-1} \mu_{x-t+i,i} \Big\}$$

Taylor approximation around the mean yields the approximation

$$\mathbb{E}[N_{x,t} \mid \mu] \approx n_{x-t,0} \exp\Big\{ -\sum_{i=0}^{t-1} \mathbb{E}[\mu_{x-t+i,i}] \Big\} \Big(1 - \sum_{i=0}^{t-1} \big(\mu_{x-t+i,i} - \mathbb{E}[\mu_{x-t+i,i}] \big) \Big).$$

Consequently,

$$\begin{aligned} \mathrm{Cov}(\mathbb{E}[N_{x,t} \mid \mu], \mathbb{E}[N_{y,s} \mid \mu]) \approx n_{x-t,0} n_{y-s,0} \exp\Big\{ -\sum_{i=0}^{t-1} \mathbb{E}[\mu_{x-t+i,i}] - \sum_{j=0}^{s-1} \mathbb{E}[\mu_{y-s+j,j}] \Big\} \\ \times \sum_{i=0}^{t-1} \sum_{j=0}^{s-1} \mathrm{Cov}(\mu_{x-t+i,i}, \mu_{y-s+j,j}), \end{aligned}$$

We now turn to the first term in the covariance decomposition (12). Notice that

$$\begin{aligned} \mathcal{L}\Big(N_{x,t} \mid \mu\Big) &= \mathrm{Bin}\Big(n_{x-t,0}, \prod_{i=0}^{t-1} p_{x-t+i,i} \Big), \\ \mathcal{L}\Big(N_{x+h,t+h} \mid N_{x,t}, \mu\Big) &= \mathrm{Bin}\Big(N_{x,t}, \prod_{i=t}^{t+h-1} p_{x-t+i,i} \Big). \end{aligned}$$

Conditional on μ, $N_{x,t}$ and $N_{y,s}$ are independent when $x - t \neq y - s$, i.e. when $N_{y,s}$ is not the number of active contracts at time s for individuals of age $x - t$ periods at time 0. In particular,

$$\mathrm{Cov}(N_{x,t}, N_{y,s} \mid \mu) = 0 \quad \text{if } x - t \neq y - s.$$

For $s > t$, let $s - t = h$. For $y = x - t + s = x + h$:

$$\mathrm{Cov}(N_{x,t}, N_{x+h,t+h} \mid \mu) = \mathbb{E}[N_{x,t} N_{x+h,t+h} \mid \mu] - \mathbb{E}[N_{x,t} \mid \mu] \mathbb{E}[N_{x+h,t+h} \mid \mu].$$

Moreover,

$$
\begin{aligned}
&\mathbb{E}[N_{x,t}N_{x+h,t+h} \mid \mu] = \mathbb{E}[N_{x,t}\mathbb{E}[N_{x+h,t+h} \mid N_{x,t}, \mu] \mid \mu] \\
&\quad = \mathbb{E}\Bigg[N_{x,t}^2 \prod_{i=t}^{t+h-1} p_{x-t+i,i} \mid \mu\Bigg] = \prod_{i=t}^{t+h-1} p_{x-t+i,i}\Big(\mathrm{Var}(N_{x,t} \mid \mu) + \mathbb{E}[N_{x,t} \mid \mu]^2\Big) \\
&\quad = \prod_{i=t}^{t+h-1} p_{x-t+i,i}\Bigg(n_{x-t,0}\prod_{i=0}^{t-1} p_{x-t+i,i}\Bigg(1 - \prod_{i=0}^{t-1} p_{x-t+i,i}\Bigg) + n_{x-t,0}^2 \prod_{i=0}^{t-1} p_{x-t+i,i}^2\Bigg) \\
&\quad = n_{x-t,0} \prod_{i=0}^{t+h-1} p_{x-t+i,i}\Bigg(1 - \prod_{i=0}^{t-1} p_{x-t+i,i} + n_{x-t,0}\prod_{i=0}^{t-1} p_{x-t+i,i}\Bigg).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
&\mathrm{Cov}(N_{x,t}, N_{x+h,t+h} \mid \mu) \\
&\quad = n_{x-t,0} \prod_{i=0}^{t+h-1} p_{x-t+i,i}\Bigg(1 - \prod_{i=0}^{t-1} p_{x-t+i,i} + n_{x-t,0}\prod_{i=0}^{t-1} p_{x-t+i,i}\Bigg) \\
&\qquad - n_{x-t,0}^2 \prod_{i=0}^{t-1} p_{x-t+i,i} \prod_{i=0}^{t+h-1} p_{x-t+i,i} \\
&\quad = n_{x-t,0} \prod_{i=0}^{t+h-1} p_{x-t+i,i}\Bigg(1 - \prod_{i=0}^{t-1} p_{x-t+i,i}\Bigg) \\
&\quad = n_{x-t,0} \exp\Bigg\{-\sum_{i=0}^{t+h-1} \mu_{x-t+i,i}\Bigg\}\Bigg(1 - \exp\Bigg\{-\sum_{i=0}^{t-1} \mu_{x-t+i,i}\Bigg\}\Bigg) \\
&\quad = n_{x-t,0}\Bigg(\exp\Bigg\{-\sum_{i=0}^{t+h-1} \mu_{x-t+i,i}\Bigg\} - \exp\Bigg\{-\sum_{i=0}^{t+h-1} \mu_{x-t+i,i} - \sum_{i=0}^{t-1} \mu_{x-t+i,i}\Bigg\}\Bigg).
\end{aligned}
$$

Taylor approximation around the mean yields

$$
\begin{aligned}
&\mathbb{E}[\mathrm{Cov}(N_{x,t}, N_{x+h,t+h} \mid \mu)] \\
&\quad \approx n_{x-t,0}\Bigg(\exp\Bigg\{-\sum_{i=0}^{t+h-1} \mathbb{E}[\mu_{x-t+i,i}]\Bigg\}\Bigg(1 - \exp\Bigg\{-\sum_{i=0}^{t-1} \mathbb{E}[\mu_{x-t+i,i}]\Bigg\}\Bigg)\Bigg).
\end{aligned}
$$

Putting the pieces together,

$$
\begin{aligned}
&\mathrm{Cov}(N_{x,t}, N_{y,s}) \\
&\quad \approx n_{x-t,0}n_{y-s,0} \exp\Bigg\{-\sum_{i=0}^{t-1} \mathbb{E}[\mu_{x-t+i,i}] - \sum_{j=0}^{s-1} \mathbb{E}[\mu_{y-s+j,j}]\Bigg\} \sum_{i=0}^{t-1}\sum_{j=0}^{s-1} \mathrm{Cov}(\mu_{x-t+i,i}, \mu_{y-s+j,j}) \\
&\qquad + \mathbb{1}_{\{x-t=y-s\}} n_{x-t,0}\Bigg(\exp\Bigg\{-\sum_{i=0}^{M-1} \mathbb{E}[\mu_{x-t+i,i}]\Bigg\}\Bigg(1 - \exp\Bigg\{-\sum_{i=0}^{m-1} \mathbb{E}[\mu_{x-t+i,i}]\Bigg\}\Bigg)\Bigg),
\end{aligned}
$$

where $m = \min(s,t)$ and $M = \max(s,t)$.

It remains to determine the expectations $\mathbb{E}[\mu_{x,t}]$ and covariances $\mathrm{Cov}(\mu_{x,t}, \mu_{y,s})$ for $s, t \geq 1$. Notice that

$$\begin{aligned}\log(\mu_{x,t}) &= \alpha_x + \beta_x \kappa_t = \alpha_x + \beta_x (t\delta + \kappa_0 + \sum_{i=1}^{t} \xi_i) \\ &\sim N(\alpha_x + \beta_x(t\delta + \kappa_0), \beta_x^2 t \sigma_\kappa^2).\end{aligned}$$

Hence,

$$\mathbb{E}[\mu_{x,t}] = \exp\left\{ \alpha_x + \beta_x(t\delta + \kappa_0) + \frac{1}{2}\beta_x^2 t \sigma_\kappa^2 \right\}.$$

Notice also that $\log(\mu_{x,t}), \log(\mu_{y,s})$ are jointly normally distributed with covariance

$$C_{(x,t),(y,s)} := \mathrm{Cov}(\log(\mu_{x,t}), \log(\mu_{y,s})) = \beta_x \beta_y \min(s,t) \sigma_\kappa^2.$$

Therefore,

$$\begin{aligned}\mathrm{Cov}(\mu_{x,t}, \mu_{y,s}) = \exp&\left\{ \alpha_x + \beta_x(t\delta + \kappa_0) + \alpha_y + \beta_y(s\delta + \kappa_0) \right\} \\ &\times \exp\left\{ \frac{1}{2}(C_{(x,t),(x,t)} + C_{(y,s),(y,s)}) \right\} \Big(\exp\{C_{(x,t),(y,s)}\} - 1 \Big).\end{aligned}$$

C Euler allocation of the multi-period value

Proposition 2. *The allocation* Λ *in* (10) *satisfies*

$$\sum_{k=1}^{n} \Lambda(R_k, R) = \Lambda(R, R), \quad \Lambda(R_k, R) \leq \Lambda(R_k, R_k).$$

Proof of Proposition 2. Let $\mathcal{V}$ denote the linear Gaussian vector space spanned by the components of the Gaussian process $(G_t)_{t=1}^{\tau}$ generating the filtration $(\mathcal{H}_t)_{t=0}^{\tau}$. Define $\varphi : \mathcal{V} \to \mathbb{R}$ by

$$\varphi(Y) = \mathbb{E}[Y \mid \mathcal{H}_0] + \sum_{t=0}^{\tau-1} c_t \Big(\mathrm{Var}\Big(Y \mid \mathcal{H}_t\Big) - \mathrm{Var}\Big(Y \mid \mathcal{H}_{t+1}\Big)\Big)^{1/2}$$

and notice that $\varphi(R) = V_0$. Moreover, for any $X, Y \in \mathcal{V}$, the limit

$$\Lambda(X,Y) := \lim_{\varepsilon \to 0} \frac{\varphi(Y + \varepsilon X) - \varphi(Y)}{\varepsilon}$$

exists and can be computed using l'Hospital's rule. Toward this end, notice that

$$\begin{aligned}\frac{\partial}{\partial\varepsilon}\varphi(Y+\varepsilon X) &= \mathbb{E}[X] + \varepsilon\sum_{t=0}^{\tau-1} c_t \frac{\mathrm{Var}\left(X \mid \mathcal{H}_t\right) - \mathrm{Var}\left(X \mid \mathcal{H}_{t+1}\right)}{\Big(\mathrm{Var}\big(Y+\varepsilon X \mid \mathcal{H}_t\big) - \mathrm{Var}\big(Y+\varepsilon X \mid \mathcal{H}_{t+1}\big)\Big)^{1/2}} \\ &\quad + \sum_{t=0}^{\tau-1} c_t \frac{\mathrm{Cov}\left(X, Y \mid \mathcal{H}_t\right) - \mathrm{Cov}\left(X, Y \mid \mathcal{H}_{t+1}\right)}{\Big(\mathrm{Var}\big(Y+\varepsilon X \mid \mathcal{H}_t\big) - \mathrm{Var}\big(Y+\varepsilon X \mid \mathcal{H}_{t+1}\big)\Big)^{1/2}} \\ &\to \mathbb{E}[X] + \sum_{t=0}^{\tau-1} c_t \frac{\mathrm{Cov}\left(X, Y \mid \mathcal{H}_t\right) - \mathrm{Cov}\left(X, Y \mid \mathcal{H}_{t+1}\right)}{\Big(\mathrm{Var}\big(Y \mid \mathcal{H}_t\big) - \mathrm{Var}\big(Y \mid \mathcal{H}_{t+1}\big)\Big)^{1/2}}\end{aligned}$$

as $\varepsilon \to 0$. Since $R = \sum_{k=1}^n R_k$ and the covariance is linear in each of its two arguments, the property $\sum_{k=1}^n \Lambda(R_k, R) = \Lambda(R, R)$ follows.

The property $\Lambda(R_k, R) \leq \Lambda(R_k, R_k)$ is shown using covariance decomposition together with properties of conditional covariances for the multivariate normal distribution as follows. Since

$$\begin{aligned}\mathrm{Cov}\left(X, Y \mid \mathcal{H}_t\right) &= \mathbb{E}\Big[\mathrm{Cov}\left(X, Y \mid \mathcal{H}_{t+1}\right) \mid \mathcal{H}_t\Big] + \mathrm{Cov}\left(\mathbb{E}[X \mid \mathcal{H}_{t+1}], \mathbb{E}[Y \mid \mathcal{H}_{t+1}] \mid \mathcal{H}_t\right) \\ &= \mathrm{Cov}\left(X, Y \mid \mathcal{H}_{t+1}\right) + \mathrm{Cov}\left(\mathbb{E}[X \mid \mathcal{H}_{t+1}], \mathbb{E}[Y \mid \mathcal{H}_{t+1}] \mid \mathcal{H}_t\right)\end{aligned}$$

it follows that

$$\begin{aligned}&\mathrm{Cov}\left(X, Y \mid \mathcal{H}_t\right) - \mathrm{Cov}\left(X, Y \mid \mathcal{H}_{t+1}\right) \\ &\quad = \mathrm{Cov}\left(\mathbb{E}[X \mid \mathcal{H}_{t+1}], \mathbb{E}[Y \mid \mathcal{H}_{t+1}] \mid \mathcal{H}_t\right) \\ &\quad \leq \mathrm{Var}\big(\mathbb{E}[X \mid \mathcal{H}_{t+1}] \mid \mathcal{H}_t\big)^{1/2} \mathrm{Var}\big(\mathbb{E}[Y \mid \mathcal{H}_{t+1}] \mid \mathcal{H}_t\big)^{1/2}.\end{aligned}$$

Variance decomposition together with properties of conditional variances for the multivariate normal distribution gives

$$\begin{aligned}\mathrm{Var}\big(\mathbb{E}[X \mid \mathcal{H}_{t+1}] \mid \mathcal{H}_t\big) &= \mathrm{Var}\big(X \mid \mathcal{H}_t\big) - \mathbb{E}\Big[\mathrm{Var}\big(X \mid \mathcal{H}_{t+1}\big)\mathcal{H}_t\Big] \\ &= \mathrm{Var}\big(X \mid \mathcal{H}_t\big) - \mathrm{Var}\big(X \mid \mathcal{H}_{t+1}\big)\end{aligned}$$

and similarly for $\mathrm{Var}\big(\mathbb{E}[Y \mid \mathcal{H}_{t+1}] \mid \mathcal{H}_t\big)$. Hence,

$$\frac{\mathrm{Cov}\left(X, Y \mid \mathcal{H}_t\right) - \mathrm{Cov}\left(X, Y \mid \mathcal{H}_{t+1}\right)}{\Big(\mathrm{Var}\big(Y \mid \mathcal{H}_t\big) - \mathrm{Var}\big(Y \mid \mathcal{H}_{t+1}\big)\Big)^{1/2}} \leq \Big(\mathrm{Var}\big(X \mid \mathcal{H}_t\big) - \mathrm{Var}\big(X \mid \mathcal{H}_{t+1}\big)\Big)^{1/2}$$

from which the property $\Lambda(R_k, R) \leq \Lambda(R_k, R_k)$ immediately follows. $\square$

Remark 14. *It is clear that φ in the proof of Proposition 2 is positively homogeneous and shown in Proposition 9 in [7] that φ is subadditive on $\mathcal{V}$. The claim can therefore be shown by combining Theorems 4.2 and 4.3 in [14] after minor adjustments and clarifications. However, we prefer to present a direct proof.*