



On the mean and variance of the estimated tangency portfolio weights for small samples

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Abstract

In this paper, we consider the sample estimator of the tangency portfolio (TP) weights, where the inverse of the sample covariance matrix plays an important role. We assume that the number of observations is less than the number of assets in the portfolio, and the returns are independent and identically multivariate normally distributed. Under these assumptions, the sample covariance matrix follows a singular Wishart distribution and, therefore, the regular inverse cannot be taken. This paper delivers bounds and approximations for the first two moments of the estimated TP weights, as well as exact results when the population covariance matrix is equal to the identity matrix, employing the Moore-Penrose inverse. Moreover, exact moments based on the reflexive generalized inverse are provided. The properties of the bounds are investigated in a simulation study, where they are compared to the sample moments. The difference between the moments based on the reflexive generalized inverse and the sample moments based the Moore-Penrose inverse is also studied.

Keywords: Tangency portfolio, Singular inverse Wishart, Moore-Penrose inverse, Reflexive generalized inverse, Estimator moments.

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