Is the empirical out-of-sample variance an informative risk measure for high-dimensional portfolios?

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Abstract

The main contribution of this paper is the derivation of the asymptotic behaviour of the out-of-sample variance, the out-of-sample relative loss, and of their empirical counterparts in the high-dimensional setting, i.e., when both ratios p/n and p/m tend to some positive constants as $m \to \infty$ and $n \to \infty$, where p is the portfolio dimension, while n and m are the sample sizes from the in-sample and out-of-sample periods, respectively. The results are obtained for the traditional estimator of the global minimum variance (GMV) portfolio, for the two shrinkage estimators introduced by Frahm and Memmel (2010) and Bodnar et al. (2018), and for the equally-weighted portfolio, which is used as a target portfolio in the specification of the two considered shrinkage estimators. We show that the behaviour of the empirical out-of-sample variance may be misleading is many practical situations. On the other hand, this will never happen with the empirical out-of-sample relative loss, which seems to provide a natural normalization of the out-of-sample variance in the high-dimensional setup. As a result, an important question arises if this risk measure can safely be used in practice for portfolios constructed from a large asset universe.