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Claims reserving using separate exposure for claims with and without a case reserve

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Abstract

In the traditional Chain Ladder for non-life claims reserving, the exposure measure (driver) for claims development is cumulative paid or incurred claims. Dahms (2008) suggested replacing this by the outstanding claims amounts, i.e. the sum of the case reserves. For outstanding claims, this is obviously a good alternative. For the unknown claims and for future costs for reopening of closed claims, though, the correlation to the outstanding amounts can be expected to be weak. We suggest that the development of outstanding claims (reported and still open with a non-zero case reserve) is separated from the development on non-outstanding claims (unknown or having case reserve zero). For the latter, we use some volume measure, such as the premium, as exposure. This idea is inspired by Schnieper (1991), who treated unknown claims this way.

Dahms' method has the further advantage of giving consistent reserves based on paid claims and incurred claims, respectively. This problem was previously addressed, but not completely solved, by Quarg & Mack (2004) in the Munich Chain Ladder method. This property carries over to our method.

The new method has several other advantages from an applied perspective, as discussed at end of the paper, were we present an application to personal accident insurance, taken from our own practice.

Key Words: Claims reserving, Case reserves, Development factors methods, Exposure measure.

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1 Introduction

Many methods for non-life claims reserving are based on the idea that the expected changes in a period are (to some extent) proportional to some *exposure measure*, or *driver*, and an important task should be to identify the best available such exposure. For example, a traditional Chain Ladder (CL) on paid amounts is proper if we believe that future payments are proportional to the observed cumulative payments. In a CL on incurred claims, future changes are assumed to be proportional to the present incurred claims.

Dahms (2008) suggested changing the CL exposure to *outstandings*, i.e. the sum of the case reserves, for both paid and incurred. This seems to be an obvious choice of exposure, as long as we are talking about *outstanding claims*, by which we mean reported claims that are still open, *with a non-zero case reserve*. Indeed, if the amount of cumulative payments is larger than normal, but there is very little outstandings, then why should we expect future payments to be larger than normal, as we do in the CL? In the extreme case when the outstandings are zero, it is likely that there will be no more payments, but this fact is not recognized by the CL.

For unknown claims, the situation is different: here the outstandings are less relevant and we would rather use some measure of the business volume as main driver. This calls for treating unknown claims separately. A third category is *reopened claims*, or rather payments on reported claims in spite of the case reserve being zero. Here, outstandings is obviously not the best exposure. For the time being, we will handle these claims together with unknown claims, and then return to the subject in Section 4.3.) So we separate out the outstanding claims from those that are non-outstanding, i.e. are unreported or reported with case reserve zero.

Schnieper (1991) suggested splitting the *incurred* claims triangle in two parts, allowing for separate treatment of IBNYR (the reserve for unreported claims) and IBNER (changes to the incurred claims for reported claims). For IBNYR, Schnieper used an exposure related to volume, such as earned premium. This can often be a more relevant exposure than the cumulative claims used in the CL method. For IBNER, on the other hand, Schnieper suggested sticking to the CL exposure, i.e. the latest cumulative incurred claims.

Following discussions with René Dahms during the on-line 2021 ASTIN Colloquium, we suggest a combination of elements from the approaches by Dahms (2008) and Schnieper (1991), using outstandings as exposure for reported claims with a non-zero case reserve, i.e. outstanding claims, and using the premium, or some other volume measure, as exposure for the future incurred claims relating to the rest of the claims, including unknown claims.

An analogue model is suggested for paid claims, where the outstandings are then used as exposure for future payments on outstanding claims, and the volume measure as exposure for any other claims. In this method, as in Dahms (2008), paid and incurred claims should be analysed simultaneously.

An important property of the method of Dahms (2008) is that it closes the gap between reserving on paid claims and incurred claims. The difference between these approaches may often be substantial, as noted by Quarg & Mack (2004), who suggested the *Munich Chain Ladder* (MCL) to resolve this problem. The MCL reduces the gap substantially, with the aid of estimated correlations between paid and incurred. In contrast, Dahms (2008) method always gives exactly the same claims reserve for both paid and incurred, see Theorem 3.2 there, simply by using a more relevant exposure. We will show in Proposition 4.1 below that this property carries over to the method suggested here.

Dahms (2012) introduced the term Linear Stochastic Reserving Methods (LSRM) for procedures that assume expected changes in a period to be proportional to some exposure measure, based on one or several observed triangles. The method suggested here is within this class, so that the results in Dahms (2012) are valid. In particular, these results may be used to find the MSEP (Mean Squared Error of Predicion) of the reserve estimates.

Being a special case of LSRM, the method presented here is not new from a mathematical point of view. However, from an applied perspective, it gives a novel way to estimate the claims reserve, with advantages over standard methods, as discussed below.

2 Splitting the incurred claims triangle

We start with an ordinary cumulative data triangle of incurred claims C_{ij}, with accident years $i = 1, \ldots, I$ and development years $i = 0, \ldots, J$. We now split this triangle into two, in a way that is close to, but not identical to the split suggested by Schnieper (1991).

First we present the D triangle in Table 2.1 (D for *development*). Here D_{ij} is the one-year change to the reported amount of incurred for claims that had a non-zero case reserve at the opening of the present development year.

We next compute the N triangle, where N_{ij} is the incurred claim cost for any claims not included in the D_{ij}. In particular, N_{ij} includes incurred claims for the accident year i, that are reported in development year j, but any cost on claims that have a non-zero case reserve at the opening of the year is included. If there are no changes to reported claims with zero outstandings

Accident year	*Development year* 0	1	2	$\cdots$	$J-1$	J
1	$D_{1,0}$	$D_{1,1}$	$D_{1,2}$	$\cdots$	$D_{1,J-1}$	$D_{1,J}$
2	$D_{2,0}$	$D_{2,1}$	$D_{2,2}$	$\cdots$	$D_{2,J-1}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
$I-1$	$D_{I-1,0}$	$D_{I-1,1}$				
I	$D_{I,0}$					

Table 2.1: The D triangle.

(and in particular no reopened claims), then the definitions of D and N here coincide with those by Schnieper (1991).

By definition, we have $D_{i,0} \equiv 0$, for all i and hence $N_{i0} \equiv C_{i0}$. For the subsequent columns of the N-triangle, we may use the obvious relation

$$C_{ij} = C_{i,j-1} + D_{ij} + N_{ij} \qquad \Leftrightarrow \quad N_{ij} = C_{ij} - C_{i,j-1} - D_{ij}\,; \\ i = 1, 2. \ldots, I; \quad j = 1, 2. \ldots, J. \tag{2.1}$$

The triangle is shown in Table 2.2.

Accident year	*Development year* 0	1	2	$\cdots$	$J-1$	J
1	$N_{1,0}$	$N_{1,1}$	$N_{1,2}$	$\cdots$	$N_{1,J-1}$	$N_{1,J}$
2	$N_{2,0}$	$N_{2,1}$	$N_{2,2}$	$\cdots$	$N_{2,J-1}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
$I-1$	$N_{I-1,0}$	$N_{I-1,1}$				
I	$N_{I,0}$					

Table 2.2: The new claims triangle.

Unless otherwise stated, we will make the simplifying assumption $I = J + 1$. In practice we often encounter other structures than this symmetric triangle, but our method can, without loss of generality, be presented in terms of triangles.

Note that the N and D triangles are incremental, rather than cumulative. Counterparts to these triangles for *paid* claims are discussed in the next section.

3 Schnieper's method

Here we briefly recapitulate the method by Schnieper (1991), under the assumption that we have no claims reopening, so that our D and N triangles are the same as his. For accident year i, denote the collection of observed variables up to and including development period j by $\mathcal{H}_{ij}$,

$$\mathcal{H}_{ij} = \{N_{i.0}, N_{i,1}, \ldots, N_{ij}; D_{i,0}, D_{i,1}, \ldots, D_{ij}\},$$

Note that the corresponding information on C_{ij} is implicitly included in this set by (2.1).

We assume that we have some exposure e_i for each accident year i, which we treat as non-stochastic. For simplicity, we can think of e_i as the earned premium, but it may be sum insured, the number of insurances or some other measure of the business volume.

The basic assumptions in Schnieper (1991) are given here in a form that is closer to the one in Wüthrich & Merz (2008). We use the convention that $\mathcal{H}_{i,-1} = \emptyset$.

(A1) For some parameters $\lambda_j \geq 0$,

$$E[N_{ij}|\mathcal{H}_{i,j-1}] \quad = \quad e_i\lambda_j\,; \quad i = 1, 2. \ldots, I; \quad j = 0, 1, 2, \ldots, J.$$

(A2) For some parameters δ_j,

$$E[D_{ij}|\mathcal{H}_{i,j-1}] = C_{i,j-1}\,\delta_j\,; \quad i = 1, 2. \ldots, I; \quad j = 1, 2, \ldots, J.$$

(A3) The random variables (i.e. N_{ij} and D_{ij}) from different accident years are independent.

Note that, by (A3), the conditioning in (A1) and (A2) could just as well be made on on a triangle containing $\mathcal{H}_{i,j-1}$ as is done in Schnieper (1991). The present conditioning is more like that in Mack (1993).

It follows from (2.1) that for $i = 1, 2. \ldots, I$ and $j = 1, 2, \ldots, J$,

$$E[C_{ij}|\mathcal{H}_{i,j-1}] = E[C_{i,j-1} + D_{ij} + N_{ij}|\mathcal{H}_{i,j-1}] = C_{i,j-1}\,(1 + \delta_j) + e_i\lambda_j. \quad (3.1)$$

For $j = 0$ we get $E[C_{i0}] = E[N_{i0}] = e_i\lambda_0$. This is useful when trying to estimate a loss ratio, but will not be discussed further in this paper on reserving.

Note that (A1) means that the expected cost for new claims is not influenced by the observed claims so far, but only by the volume measure e_i. This is in contrast to a CL on incurred claims where, in our notation,

$$E[C_{i,j-1} + N_{ij} + D_{ij}|\mathcal{H}_{i,j-1}] = C_{i,j-1}\,f_j$$

so that the expected value of $N_{ij} + D_{ij}$ is dependent on $C_{i,j-1}$.

Schnieper (1991) presents estimators of the parameters and the ultimate claim cost. He also derives estimators of an approximation to the mean squared error of these estimators. Exact formulas for the mean squre error of prediction are given by Wüthrich & Merz (2008), following Liu and Verrall (2007). It may be noted that Schnieper's ideas were adapted by Mack (1993) to derive his famous result on the MSE of the CL.

4 The new method

We return now to Dahms (2008) and the method there called Extended Complimentary Loss Ratio method (ECLR) where both "payments and adjustments to the reported amount during the year are assumed to be proportional to the opening reserves". We will split the triangles as in Section 2) and apply Dahms idea to the D-triangle while retaining Schnieper's assumption (A1) on the N-triangle.

Let the cumulative values be denoted C^P_{ij} for paid and C^I_{ij} for incurred claims. Further, let R_{ij} be the outstandings for accident year i at development year j. Then $R_{ij} = C^I_{ij} - C^P_{ij}$. Denote the N-triangle in Table 2.2 by N^I_{ij} and introduce another N-triangle for paid amounts N^P_{ij}, containing the payments made on claims reported during the year. Similarly, let the D-triangle in Table 2.1 be denoted by D^I_{ij}, and introduce another D-triangle for paid amounts D^P_{ij}, with the payments made during the year for claims that had a non-zero case reserve at the beginning of the year.

We have the following relations between these variables. First, the relation in (2.1) is of course still valid, now in two versions:

$$C^P_{ij} = C^P_{i,j-1} + D^P_{ij} + N^P_{ij} \quad \text{and} \quad C^I_{ij} = C^I_{i,j-1} + D^I_{ij} + N^I_{ij}, \tag{4.1}$$

with $D^P_{i,0} = D^I_{i,0} = 0$, $C^I_{i0} = N^I_{i0}$ and $C^P_{i0} = N^P_{i0}$.

The first column of outstandings is, of course, given by $R_{i0} = N^I_{i0} - N^P_{i0}$. The subsequent change in outstandings is a function of the changes in paid and incurred:

$$R_{ij} - R_{i,j-1} = D^I_{ij} - D^P_{ij} + N^I_{ij} - N^P_{ij}. \tag{4.2}$$

The cumulative amounts are, of course, the sum of the incremental changes;

$$C^P_{ij} = \sum_{k=0}^{j}(D^P_{i,k} + N^P_{i,k}) \qquad C^I_{ij} = \sum_{k=0}^{j}(D^I_{ij} + N^I_{ij}), \tag{4.3}$$

remembering that $D^P_{i,0} \equiv 0$ and $D^I_{i,0} \equiv 0$.

The target of claims reserving is the sum of the future cash flows. For accident year, i by the end of calendar year I, this is:

$$\sum_{j=I-i+1}^{J} (D_{ij}^P + N_{ij}^P). \tag{4.4}$$

Alternatively, we may base the reserve on incurred claims and set the target as the (known) outstandings plus the future changes to incurred:

$$R_{i,I-i+1} + \sum_{j=I-i+1}^{J} (D_{ij}^I + N_{ij}^I). \tag{4.5}$$

If the run-off is completed by development year J, these two ways of expressing the "theoretical" reserv, i.e. the target of our estimation procedure, are of course equivalent. This can be seen as follows: A full run-off implies that $R_{iJ} \equiv 0$, and by summing over (4.2) and rearranging the terms, we get

$$\begin{aligned}
\sum_{j=I-i+1}^{J} (D_{ij}^P + N_{ij}^P) &= -\sum_{j=I-i+1}^{J} (R_{ij} - R_{i,j-1}) + \sum_{j=I-i+1}^{J} (D_{ij}^I + N_{ij}^I) \\
&= -(R_{iJ} - R_{i,I-i+1}) + \sum_{j=I-i+1}^{J} (D_{ij}^I + N_{ij}^I) \\
&= R_{i,I-i+1} + \sum_{j=I-i+1}^{J} (D_{ij}^I + N_{ij}^I) \qquad (4.6)
\end{aligned}$$

While this equivalence is rather obvious, a corresponding equivalence is usually not found in the case when paid and incurred amounts are estimated by the CL. On the other hand, in Proposition 4.1 below we show that equivalence does hold true when the unknown quantities are estimated by our method, a property inherited from the ECLR of Dahms (2008).

We use the Schnieper assumption (A1), but now for both paid and incurred amounts. As before, e_i is an exposure measure such as earned premium. For simplicity, we use the same exposure for paid and incurred. (Recall that $\mathcal{H}_{i,-1} = \emptyset$.)

(B1) For some parameters $\lambda_j^P \geq 0$,

$$E[N_{ij}^P | \mathcal{H}_{i,j-1}] = e_i \lambda_j^P\,; \quad i = 1, 2 \ldots, I; \quad j = 0, 1, 2, \ldots, J.$$

(B2) For some parameters $\lambda_j^I \geq 0$,

$$E[N_{ij}^I | \mathcal{H}_{i,j-1}] = e_i \lambda_j^I\,; \quad i = 1, 2 \ldots, I; \quad j = 0, 1, 2, \ldots, J.$$

Next we turn to the assumptions for the development of earlier reported claims, changing the exposure C_{ij} in assumption (A2) to the outstandings as suggested by Dahms (2008).

(B3) For some parameters δ_j^P,

$$E[D_{ij}^P|\mathcal{H}_{i,j-1}] = R_{i,j-1}\,\delta_j^P\,; \quad i = 1, 2\ldots, I; \quad j = 1, 2, \ldots, J.$$

(B4) For some parameters δ_j^I,

$$E[D_{ij}^I|\mathcal{H}_{i,j-1}] = R_{i,j-1}\,\delta_j^I\,; \quad i = 1, 2\ldots, I; \quad j = 1, 2, \ldots, J.$$

Finally, we make the standard assumption (B5).

(B5) The random variables (i.e. N_{ij}^P, N_{ij}^I, D_{ij}^P and D_{ij}^I) from different accident years i are independent.

It follows from (4.2) that

$$E[R_{ij}|\mathcal{H}_{i,j-1}] = R_{i,j-1}(1 + \delta_j^I - \delta_j^P) + e_i(\lambda_j^I - \lambda_j^P). \tag{4.7}$$

4.1 Estimation

With x in turn replaced by P or I in each equation, the natural estimators are,

$$\hat{\lambda}_j^x = \frac{\sum_i w_i\,\hat{\lambda}_{ij}^x}{\sum_i w_i}\,, \quad \text{where} \quad \hat{\lambda}_{ij}^x = \frac{N_{ij}^x}{e_i} \qquad j = 0, 1, \ldots, J. \tag{4.8}$$

$$\hat{\delta}_j^x = \frac{\sum_i w_i\,\hat{\delta}_{ij}^x}{\sum_i w_i}\,, \quad \text{where} \quad \hat{\delta}_{ij}^x = \frac{D_{ij}^x}{R_{i,j-1}} \qquad j = 1, 2, \ldots, J. \tag{4.9}$$

Here w_i is a set of non-negative weights. The weights can either be deterministic or stochastic, but should be known at the time for estimation, i.e. $\mathcal{H}_{i,j-1}$-measurable. The weights can be different from estimator to estimator, in spite of our simplified notation here. We do not specify the index sets of the summations – it is part of the reserving actuary's daily work to choose the relevant data and weights for each estimator. Examples include a simple mean, with all $w_i = 1$, a time-weighted mean with $w_i = 1, 2, 3, 4, 5$ if five years are included, and a traditionally weighted mean, with $w_i = e_i$ and $w_i = R_{i,j-1}$, respectively. In the latter case, we get estimators of the same type as in Schnieper's method and Chain Ladder,

$$\hat{\lambda}_j^x = \frac{\sum_i N_{ij}^x}{\sum_i e_i}; \qquad \hat{\delta}_j^x = \frac{\sum_i D_{ij}^x}{\sum_i R_{i,j-1}}. \tag{4.10}$$

As usual the performance of these estimators is measured conditionally on the observations up to development year $j-1$. Introduce the collection of variables up to and including calendar year k in the observed triangle,

$$\mathcal{B}_k = \{N^I_{ij}, D^P_{ij}, D^I_{ij}; i+j \leq I; j \leq k\}.$$

There is an implicit dependence of I in $\mathcal{B}_k$; we follow the convention not to write out the index I, cf. equation (2.6) in Wüthrich and Merz (2008).

It follows directly from (B1)–(B5) that the four estimators are conditionally unbiased, given $\mathcal{B}_{j-1}$, with the convention that $\mathcal{B}_{-1} = \emptyset$. Here we have used the fact that by (B5) we have $E[N^P_{ij}|\mathcal{B}_{j-1}] = E[N^P_{ij}|\mathcal{H}_{i,j-1}]$, and similar for the other three estimators.

Next we look for estimators (predictors) of the unobserved values in the lower triangle, which will then give the estimated reserve. For any i, we get by the "plug-in principle", that for $j = I-i+1, I-i+2, \dots, J$,

$$\hat{N}^P_{ij} = e_i\,\hat{\lambda}^P_j; \qquad \hat{N}^I_{ij} = e_i\,\hat{\lambda}^I_j. \tag{4.11}$$

The unknown D-variables, on the other hand, are dependent on future outstandings, so we have to start by estimating these. Equation (4.7) suggests the following recursive estimators, for $j = I-i+1, I-i+2, \dots, J$,

$$\hat{R}_{ij} = \hat{R}_{i,j-1}(1+\hat{\delta}^I_j - \hat{\delta}^P_j) \quad + \quad e_i(\hat{\lambda}^I_j - \hat{\lambda}^P_j). \tag{4.12}$$

When $j = I-i+1$, the estimator $\hat{R}_{i,j-1}$ in the right-hand side of (4.12) should be read as the observation $R_{i,I-i}$.

Again, by plugging in the above estimates, we get for $j = I-i+1, I-i+2, \dots, J$,

$$\hat{D}^P_{ij} = \hat{R}_{i,j-1}\,\hat{\delta}^P_j, \quad \hat{D}^I_{ij} = \hat{R}_{i,j-1}\,\hat{\delta}^I_j. \tag{4.13}$$

The total claims reserve based on paid claims, for accident year i by the balance day, i.e. by development year $j = I-i$, is the sum of all expected future payments, i.e.

$$\sum_{j=I-i+1}^{J} (\hat{D}^P_{ij} + \hat{N}^P_{ij}). \tag{4.14}$$

The claims reserve based on incurred claims is the present outstanding claims reserve $R_{i,I-i}$ plus the expected changes to incurred claims on existing and late reported claims, i.e.

$$R_{i,I-i} + \sum_{j=I-i+1}^{J} (\hat{D}^I_{ij} + \hat{N}^I_{ij}) \tag{4.15}$$

Next we show that for our method, these two representations of the reserve are equal. First we note that for a single claim, if we to pay out a greater amount than we have in the case reserve, the incurred claims must be increased by the difference. This implies on a portfolio level that

$$R_{i,j-1} + D^I_{ij} - D^P_{ij} \geq 0 \qquad \text{and} \qquad N^I_{ij} - N^P_{ij} \geq 0 \tag{4.16}$$

The following proposition is a counterpart to Theorem 3.2 on the ECLR method in Dahms (2008).

Proposition 4.1 *Assume that there is no tail, i.e. the claims development is completed by development year J. This implies that $R_{iJ} \equiv 0$ for all i. Then the reserve based on paid is equal to that based on incurred claims. That is, for an origin year i where we have observations for development years $j = 0, \ldots, I - i$, we have*

$$\sum_{j=I-i+1}^{J} (\hat{D}^P_{ij} + \hat{N}^P_{ij}) = R_{i,I-i} + \sum_{j=I-i+1}^{J} (\hat{D}^I_{ij} + \hat{N}^I_{ij}) \tag{4.17}$$

Proof. We consider a fixed but arbitrary accident year i. By (4.2) and the mention facts in (4.16), $R_{iJ} \equiv 0$ implies that $R_{iJ} + D^I_{iJ} - D^P_{iJ} = 0$ and $N^I_{iJ} - N^P_{iJ} = 0$. Hence,

$$1 + \hat{\delta}^I_J - \hat{\delta}^P_J = \frac{\sum_i R_{i,J-1} + \sum_i D^I_{i,J} - \sum_i D^P_{i,J}}{\sum_i R_{i,J-1}} = 0$$

Similarly, $\hat{\lambda}^I_J - \hat{\lambda}^P_J = 0$, and we conclude from (4.12) that $\hat{R}_{iJ} = 0$ for all relevant i.

By (4.11), (4.12) and (4.13)

$$\hat{R}_{ij} - \hat{R}_{i,j-1} = \hat{D}^I_{ij} - \hat{D}^P_{ij} + \hat{N}^I_{ij} - \hat{N}^P_{ij} \tag{4.18}$$

with $\hat{R}_{i,j-1}$ interpreted as the observed $R_{i,j-1}$ when $j = I-i+1$. Reorganising this and summing over all future development years we get

$$\sum_{j=I-i+1}^{J} (\hat{D}^I_{ij} + \hat{N}^I_{ij}) - \sum_{j=I-i+1}^{J} (\hat{D}^P_{ij} + \hat{N}^P_{ij}) =$$
$$\sum_{j=I-i+1}^{J} (\hat{R}_{ij} - \hat{R}_{i,j-1}) = \hat{R}_{iJ} - R_{i,I-i} = -R_{i,I-i}$$

which completes the proof of (4.17). □

The importance of this proposition is that we get an estimate of future cash flows, based on paid claims, that is consistent with the development of incurred claims. This is typically not the case if the Chain Ladder estimates are calculated for paid and incurred.

With our method, as with the ECLR, it is no longer a question if we should base the reserving on paid or incurred. It is rather the case that we do a simultaneous, and consistent, analysis of the two. In this process, the series of $\hat{D}^P_{ij} + \hat{N}^P_{ij}$ for all future $j \leq J$ gives the expected future cash flows that are required for discounting.

Even if we should focus on paid claims, the estimation of incurred claims is necessary to get updated outstanding amounts at all future points in time, which are required in the estimation process for paid claims. Equivalently, we could leave out incurred claims and instead estimate outstandings directly, by introducing the variables D^O_{ij} and N^O_{ij}, with obvious meaning, and the following parameters:

$$\delta^O_j = \delta^I_j - \delta^P_j; \qquad \lambda^O_j = \lambda^I_j - \lambda^P_j; \quad j = 1, 2, \ldots, J. \tag{4.19}$$

Then we can rewrite (4.7) as

$$E[R_{ij} - R_{i,j-1}|\mathcal{H}_{i,j-1}] = R_{iJ}\,\delta^O_j + e_i\lambda^O_j. \tag{4.20}$$

It is readily seen that (4.8) and (4.9) can be extended to $\hat{\lambda}^O_j$ and $\hat{\delta}^O_j$.

4.2 Tail estimates

If the claims are not fully developed by year J, there is need for a tail estimate. Denote by M the development year by which all claims are settled, where in this case $M > J$. Settlement means that for any i, we have no outstandings left, i.e. $R_{i,M} \equiv 0$, and no new claims are reported for $j > M$.

Our aim is to construct a tail by defining proper λ's and δ's for $j = J+1, \ldots, M$. These are used in (4.11), (4.12) and (4.13) to get the estimated amounts we need. The claims reserve is then obtained from (4.14) or (4.15) with M replacing J. In the search for tail estimates, it is natural to require that $\hat{R}_{i,M} = 0$ for all relevant i. An implication of this is that the proof of Proposition 4.1 can now be repeated, with M replacing J, from (4.18) and onwards. We thus retain the property that paid and incurred give the same claims reserve, if the mentioned requirement is met.

So the claims reserve in (4.14) based on paid is still equal to that based on incurred claims in (4.15) under the requirement $\hat{R}_{i,M} = 0$. From (4.18) we find

$$\begin{aligned} 0 &= \hat{R}_{iM} = \hat{R}_{i,M-1} + \hat{D}^I_{iM} - \hat{D}^P_{iM} + \hat{N}^I_{iM} - \hat{N}^P_{iM} \\ &= \hat{R}_{i,M-1}(1 + \hat{\delta}^I_M - \hat{\delta}^P_M) + e_i(\hat{\lambda}^I_M - \hat{\lambda}^P_M), \end{aligned}$$

From now on we require that $1 + \hat{\delta}^I_M - \hat{\delta}^P_M = 0$ and $\hat{\lambda}^I_M - \hat{\lambda}^P_M = 0$, by which $\hat{R}_{i,M} = 0$ is automatically fulfilled.

Tail estimation is by necessity a question of expert judgement, since no data are available. In this, we start by inspecting the λ^I_j's for $j \leq J$ and then make some assumptions on the further development of these factors for $j = J+1, \dots, M$, tantamount to what we would do in a CL. One possibility here is to fit some kind of regression curve and extrapolate it for $j > J$.

As for the D triangle, having no data means that we have no reason to adjust the case by case estimates of incurred claims, so we let $\hat{\delta}^I_j = 0$ for $j > J$ from now on. With λ^I_j and $\hat{\delta}^I_j$ set in this way, the claims reserve is given by (4.15) with M replacing J. The rest of the tail parameters only concern the timing of the last payments, so that they only affect the discounting of the reserve.

Note that $\hat{\delta}^P_M = 1$, by the requirement $1 + \hat{\delta}^I_M - \hat{\delta}^P_M = 0$. As for the other payments of reported claims, the most conservative discounting is found be letting $\hat{\delta}^P_j = 1$ for $J < j \leq M$. The term "conservative" is used here in the meaning that it minimizes the effect of discounting, by assuming that all outstandings are paid out during the year.

Turning to the payments on new reported claims, we have assumed that $\hat{\lambda}^P_M = \hat{\lambda}^I_M$, by which $\hat{\lambda}^P_j$ is set for $j = M$. For $J < j < M$, the most conservative choice is to let $\hat{\lambda}^P_j = \hat{\lambda}^I_j$ for all j.

There are of course other possibilities when constructing the tail values of these parameters. Since by definition we have no data, the choice of method must be based on knowledge of the line of business in question.

4.3 Reopened claims

In practice, it may happen that a claim that is closed, and thus supposedly settled, is reopened to generate new payments. In our context, it is not important whether the claims handlers have recorded a claim as closed or not; what matters is if we get future payments in spite of the claim having case reserve zero. So our definition of a reopening here is that we get a change to paid claims or outstandings, in spite of the claim having case reserve zero at the beginning of the year.

Obviously, the assumption of the changes in paid and incurred being proportional to last years outstandings is not realistic for such claims. In some cases, reopenings can in fact be expected to behave more like unreported claims. An example from childrens personal accident insurance is when you have to wait until the child is grown up to determine a disability. If we have already refunded some initial medical expenses and do not expect any more payments, this is a reopening, but if not, it may be a late reported claim. Often the initial payments for expenses give little information on the risk for disability later in life.

In other cases, adjustments to paid amounts for closed claims should rather be expected to be proportional to the cumulative amounts paid on closed claims. However, introducing yet another dynamic (stochastic) exposure for this relatively limited phenomenon seems unnecessarily complicated. A good proxy could be the cumulative amounts paid.

So, if the cost for reopened claims is not negligible, we have two main alternatives.

1. Set up a third type of triangle, besides N and D, for this years reopened claims, for both paid and incurred. Here the cumulative paid or incurred amounts could be a good enough exposure.

2. Add this years reopened claims to the two N triangles, modelled as proportional to the exposure e_i.

Note that premium, or whatever non-stochastic exposure we use as e_i, should be a reasonable exposure also for reopened claim amounts, even in the case where the cumulative amounts paid on closed claims is optimal. The choice between the alternatives above must be based on properties of the particular line of business, but we tend to prefer alternative 2 and use this is the case treated in this paper.

4.4 Further properties of the estimators

The method described above is a Linear Stochastic Reserving Method (LSRM), as defined in Dahms (2012). This fact may be used to derive some mathematical properties of the estimators. For simplicity, we consider the case without reopenings and where there is no need for a tail estimate.

For our method to be an LSRM, Assumption 2.1 in Dahms (2012) must be fulfilled. By our assumption of independent rows in (B5), we can use Dahms' Remark 2.2 and condition only on the observations in row i only, i.e. on $\mathcal{H}_{i,j-1}$ and not on the entire observed triangle. The LSRM assumptions on expected values are now given by (B1)–(B4) above. For the second order moments, we make the follow assumptions.

(B6) For any combination of P and I replacing x and y, N^x_{ij} and D^y_{ij} are independent, given $\mathcal{H}_{i,j-1}$.

(B7) For any combination of P and I replacing x and y, there exists parameters $\sigma^{x,y}_j > 0$ such that

$$\mathrm{Cov}[N^x_{ij}, N^y_{ij}|\mathcal{H}_{i,j-1}] \;=\; e_i\,\sigma^{x,y}_j\,; \quad i = 1,2.\ldots,I; \quad j = 1,2,\ldots,J.$$

(B8) For any combination of P and I replacing x and y, there exists parameters $\tau^{x,y}_j > 0$ such that

$$\mathrm{Cov}[D^x_{ij}, D^y_{ij}|\mathcal{H}_{i,j-1}] = R_{i,j-1}\,\tau^{x,y}_j\,; \quad i = 1,2.\ldots,I; \quad j = 1,2,\ldots,J.$$

The assumption that new reported claims are independent of the development of existing claims in (B6) is a natural generalization of the first of the Model Assumptions 10.7 in Wüthrich and Merz (2008). For $x = y$, (B7) and (B8) are the variance assumptions of Schnieper (1991) as well as Wüthrich and Merz (2008), with the alteration that we have the outstandings as exposure in (B8).

For $x \neq y$, however, (B7) introduces covariances between the N-variables for paid and incurred and (B8) does the same for the D-variables. It is not likely that these variables are uncorrelated when belonging to the same row and column.

By Remark 3.2 in Dahms (2012), when the parameter estimators in (4.8)-(4.9) have minimum variance in the class of unbiased estimators that are linear combinations of the $\hat{\lambda}^x_j$'s and $\hat{\delta}^x_j$'s, respectively. Note that Remark 3.2 only considers the case when the estimators use all data in the triangles.

Under assumptions (B1)-(B8), estimates of the mean squared error of prediction (MSEP) could in principle be derived using the results in Section 4 of Dahms (2012). This extensive derivation is left to future work.

5 Numerical example

Here we present an application to personal accident insurance at Länsförsäkringar Alliance, Sweden. The cover includes income protection for disability, in the form of a lump sum, which gives the portfolio a very long tail (a large number of development years). For confidentiality reasons we do not reveal which of the 23 local mutuals in the alliance the data is taken from; neither do we disclose which particular segment of the portfolio that is used for this illustration. The data, and hence the results, are converted to an unknown currency, i.e. they are all multiplied by a fix constant, which is not disclosed. The data is restricted to the origin years 1991-2015. The largest claims are treated separately and are not included here. After the 25 years of development, there is still some small activity in the portfolio, but we assume in this example that all claims are closed in the year after that, so that the comparisons made here are not disturbed by tail estimation.

As is often the case in practice, there is some trend in the individual development factors $\hat{\lambda}_{ij}^x$ and $\hat{\delta}_{ij}^x$. Therefore our estimators, on the from (4.8)-(4.9), will use only the latest six of these factors, with time-weights $w_i = 1, 2, 3, 4, 5, 6$ in the average. The exposure e_i is based on the sum insured for disability.

As mentioned above, there is a choice of using paid claims in combination with incurred claims or outstandings. The choice here is outstandings, which we have found easier to handle in practice. Figure 5.1 shows the estimates of the λ-parameters for paid claims and outstandings. In practice, we would consider using smoothed values, but here we use the original time-weighted estimates, to keep the illustration simple and clear.

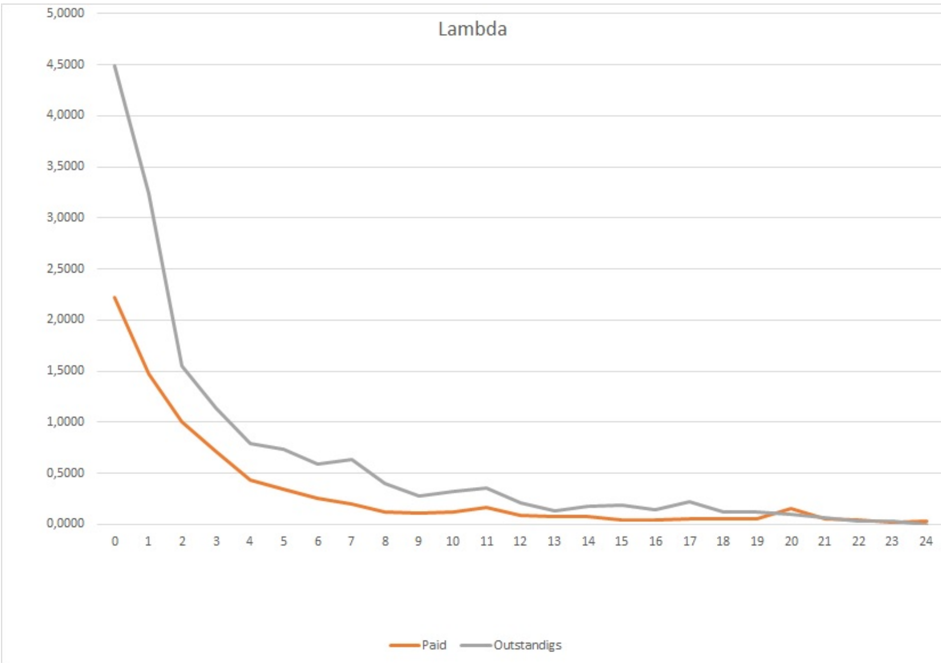

Figure 5.1: The estimated λ-parameters for paid (orange) and outstandings (grey), for development years 0–24. Note that λ_0 is not used in the reserving.

Note that the curve for incurred claims would be the sum of these two curves. The late reported claim cost is substantial rather far out in the tail: after $j = 12$, still some 10 % of the sum of the incurred λ's remains to be reported. During years 0–19, the amount paid is less than half of the incurred amount, as seen by noting that the Paid curve lies below the Outstandings.

Figure 5.2 shows the corresponding estimates of the δ-parameters for paid claims and outstandings.

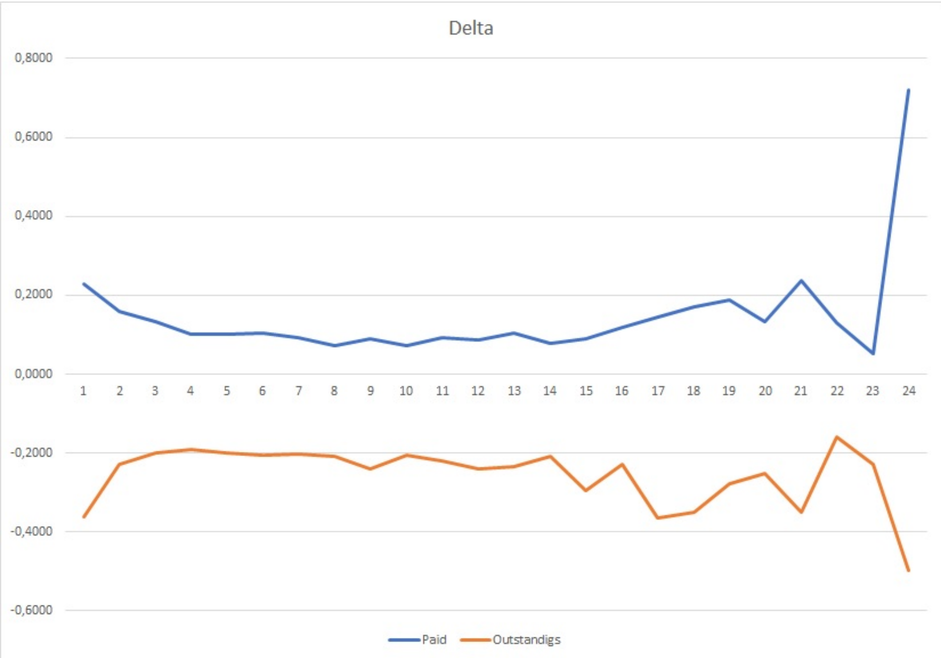

Figure 5.2: The estimated δ-parameters for paid (blue) and oustandings (orange), for development years 1–24.

After a few initial years of somewhat larger payments, for costs like immediate health care, the paid curve stabilizes at or below 10 % for several years. Then the convergence towards 100 % starts – eventually all outstandings must be paid out, but we have not reached that point here. (In practice, we would set a tail here, but for the example, we ad just one development year in which everything is paid out, not shown in the figure.) As we come further to the right, the outstanding amounts are quite small, which increases the random fluctuation in these parameter estimates, but on the other hand the impact on this on the total reserve is, of course, also small. Again, we would use smoothed estimates in practice.

The outstandings curve is much like a mirror of paid, on the negative side.

If we would make no changes to the incurred values, the mirroring would be perfect. However, here the outstandings δ-values are larger in absolute value than the paid amounts, so the incurred claims curve, being the sum of the two, would be negative. This means that the initial incurred claims are overestimated by the claims handlers. This is valuable information that we would not as easily obtain by using a standard Chain Ladder.

For completeness, Table 5.1 gives the resulting claims reserve, in terms of outstandings plus IBNR. As a benchmark, we also include the IBNR of a Chain Ladder on paid and incurred, respectively. For comparability, the same choice that we made in our method, time-weighted averages of six individual developments, is made for the Chain ladder estimates here.

The differences between CL paid and CL incurred are very large, as is not unusual for long-tailed business. Note that the fact that our method gives consistent estimation for paid and incurred does not mean that the resulting reserve necessarily lies between the ones for the CL on paid and incurred, since this is a completely different method with other exposure than the CL. However, for a majority of the years, including the most important last six years, our method gives a reserve between CL paid and CL incurred.

It should be noted that in practice one would make adjustments to the Chain Ladder estimates, such as using a Bornhuetter-Ferguson method for the first year(s). Indeed, there are many possible adjustments that can be made to any of the three methods, which should presumably make the differences smaller. This example is just an illustration, with Chain ladder as a benchmark. Nevertheless, it indicates that the choice of method can be very important.

Our choice of method is mainly a result of judgement of what the main drivers for the claim cost is, based on knowledge of the particular type of insurance. But our preference for the mentioned method is also a result of its practical advantages. Some of these have been mentioned above, and in the next section we discuss a few more.

We have also made an empirical comparison of how well the method applied to 1991-2014 predicts the next paid diagonal, i.e calendar year 2015, as compared to a CL on paid. Our method had 53 % of the root mean square error of prediction (Root MSEP) as compared to the CL. A similar comparison for the change in incurred claims as compared to a CL on incurred gave a reduction to 69 % in Root MSEP. This is of course a very limited study, but the substantial reduction in Root MSEP indicates that our method is more efficient for this particular portfolio.

6 Discussion

The method suggested here enables us to make proper choices of the drivers (exposures) for future development of claims paid and incurred, separately for outstandings and new claims (including reopenings). For the first of these two classes, we use the outstanding amounts as exposure, with the side-effect that the reserve derived from paid claims is equal to that derived from incurred claims. We shall now describe three further advantages of the method.

1. The Bornhuetter-Ferguson (BF) method is sometimes used as a robust alternative to the Chain Ladder, in the first year(s) of the claims development, If there is a large amount of unknown claims, using a Chain Ladder would mean that we apply a very large factor f_1 on a small amount C_{i0}, which results in a very volatile estimator. BF is instead based on multiplying an exposure (typically the premium) by a ratio (typically a loss ratio). Schnieper's method is similar as comes to new reported claims, multiplying an exposure e_i by λ_j, which is a sort of a "one year loss ratio". Here Schnieper's method offers a smooth transition from something that is not too far from a BF, in the early years, to a CL on incurred claims later on. Indeed, when all claims are reported, it is equivalent to a CL on incurred claims. This means that we can use one unified method for all years, instead of jumping from a BF for the first development years(s) to a CL later on. This advantage carries over to our method, but the smooth transition is, of course, to the ECLR of Dahms (2008).

2. For well developed, older accident years, it is in practice often necessary to adjust the CL estimate for its discrepancy to the remaining outstandings. For example, it might be the case that all claims are closed, but the CL still gives a substantial reserve, or the other way around: CL gives a reserve estimate that is much lower than the outstandings, to an extent that is not realistic. An *ad hoc* solution is to make some kind of "transition to incurred claims" for old accident years, i.e. manually forcing the IBNR to decrease to zero. With the ECLR of Dahms (2008), such a transition is no longer necessary, since the IBNER reserve is driven by the outstandings. This property is inherited by our method.

3. Sometimes we need to allocate the claims reserve to a number of small segments, in which data is scarce. This may be achieved by using estimated parameters from the entire portfolio, applied to the observed

exposure in each segment. With a traditional Chain Ladder, this may again lead to unrealistic reserves when compared to the outstandings in the segments. Our method eliminates this problem, since the outstandings in the small segments is used as exposure for reported claims. Furthermore, a small segment in a long-tailed business sometimes has disproportionate reported claim amounts in the first years. By using this amount as exposure, CL gives a volatile estimate, while in our method we get a stable estimate of unknown claims based on the the segment's part of e_i.

The reduced need for adjustments is, in our opinion, an indication of the soundness of the method. The mentioned properties are inherited from Schnieper (1998) or Dahms (2008), but our combination of their ideas is necessary to get a method with all the mentioned properties.

Acknowledgement

As mentioned in the introduction, the basic idea of this paper grew out of discussions with René Dahms. Without his contribution, the method described here had never seen daylight. In spite of this, any errors in the model described above are of course the responsibility of the authors only.

Acc. yr	Outstandings	Our method IBNR	CL on Paid IBNR	CL on Incurred IBNR
1991	252	0	-252	0
1992	1 183	310	-602	223
1993	1 314	163	-592	19
1994	1 479	292	-392	94
1995	2 894	264	-1 006	32
1996	2 487	789	459	203
1997	3 810	1 000	-74	222
1998	4 265	1 210	1 146	-384
1999	7 069	677	-147	-1 290
2000	6 924	1 750	1 918	-1 716
2001	9 471	711	902	-3 238
2002	12 236	59	511	-4 164
2003	18 044	-2 090	-3 510	-5 558
2004	16 189	-255	3 022	-7 711
2005	20 658	-1 164	429	-7 765
2006	19 036	1 023	5 259	-8 573
2007	26 290	-1 967	4 342	-11 923
2008	25 374	407	7 282	-12 534
2009	33 332	-54	8 345	-13 343
2010	35 618	2 932	14 909	-12 474
2011	38 037	7 575	18 458	-9 746
2012	46 445	11 163	26 306	-6 836
2013	46 487	23 863	49 208	3 085
2014	44 105	41 895	66 709	17 719
2015	29 979	76 668	129 531	46 501
Sum	*452 978*	*167 223*	*332 162*	*-39 157*

Table 5.1: IBNR, i.e. the claims reserves on top of the outstandings, with our method, a Chain Ladder on paid, and dito on incurred claims.

7 References

Dahms, R. (2008). A Loss Reserving Method for Incomplete Claim Data. Paper presented at the ASTIN Colloquium in Manchester 2008.

Dahms, R. (2012). Linear stochastic reserving methods. ASTIN Bulletin, 42(1), 1–34.

Liu, H. and Verrall, R.J. (2007). Predictive Distributions for Reserves which Separate True IBNR and IBNER Claims. Conference paper. 37th ASTIN Colloquium Orlando, USA.

Mack, T. (1993). Distribution free calculation of the standard error of chain-ladder reserve estimates. ASTIN Bulletin 23(2), 213–225.

Quarg, G. & Mack, T. (2008). Munich Chain Ladder: A Reserving Method that Reduces the Gap between IBNR Projections Based on Paid Losses and IBNR Projections Based on Incurred Losses. Variance 2:2, 266–299.

Schnieper, R. (1991). Separating true IBNR and IBNER claims. ASTIN Bulletin 21(1), 111–127.

Wüthrich, M. & Merz, M. (2008). Stochastic Claims Reserving Methods in Insurance. Wiley.