Two is better than one: Regularized shrinkage of large minimum variance portfolios

Taras Bodnar[1], Nestor Parolya[2], and Erik Thorsén[1]

[1]Department of Mathematics, Stockholm University, Roslagsvägen 101, SE-10691 Stockholm, Sweden
[2]Department of Applied Mathematics, Delft University of Technology, Mekelweg 4, 2628 CD Delft, The Netherlands

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Abstract

In this paper we construct a shrinkage estimator of the global minimum variance (GMV) portfolio by a combination of two techniques: Tikhonov regularization and direct shrinkage of portfolio weights. More specifically, we employ a double shrinkage approach, where the covariance matrix and portfolio weights are shrunk simultaneously. The ridge parameter controls the stability of the covariance matrix, while the portfolio shrinkage intensity shrinks the regularized portfolio weights to a predefined target. Both parameters simultaneously minimize with probability one the out-of-sample variance as the number of assets p and the sample size n tend to infinity, while their ratio p/n tends to a constant $c > 0$. This method can also be seen as the optimal combination of the well-established linear shrinkage approach of Ledoit and Wolf (2004) and the shrinkage of the portfolio weights by Bodnar et al. (2018). No specific distribution is assumed for the asset returns except of the assumption of finite $4 + \varepsilon$ moments. The performance of the double shrinkage estimator is investigated via extensive simulation and empirical studies. The suggested method significantly outperforms its predecessor (without regularization) and the nonlinear shrinkage approach in terms of the out-of-sample variance, Sharpe ratio and other empirical measures in the majority of scenarios. Moreover, it obeys the most stable portfolio weights with uniformly smallest turnover.

Keywords: Shrinkage estimator; high-dimensional covariance matrix; random matrix theory; minimum variance portfolio; parameter uncertainty; ridge regularization
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