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Extending SIRS epidemics to allow for gradual waning of immunity

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Abstract

SIRS epidemic models assume that individual immunity (from vaccination as well as infection) wanes in one big leap, from complete immunity to complete susceptibility. For many diseases immunity on the contrary wanes gradually, something which has become even more evident during the COVID-19 pandemic where also recently infected have a risk for reinfection, and where booster vaccines are given to increase immunity. This paper considers an epidemic model allowing for such gradual waning of immunity (either linear or exponential waning) thereby extending SIRS epidemics, and also incorporates vaccination. The two versions for gradual waning of immunity are compared with the classic SIRS epidemic, where the three models are calibrated by having the same *average cumulative immunity*. All models are shown to have identical basic reproduction number R_0. However, if no prevention is put in place, the model with exponential waning has highest long term prevalence (endemic level) and the classic SIRS model has lowest. Further, the critical amount of vaccine supply needed to reach and maintain herd immunity, is highest for the model with exponential decay of immunity and lowest for the classic SIRS epidemic model. As a consequence, if reality lies close to exponential (or linear) decay of immunity, and expressions are based on the SIRS epidemic, then these will underestimate the endemic level and the critical vaccine supply will not be sufficient to reach and maintain herd immunity. For parameter choices fitting to COVID-19, the critical amount of vaccine supply is about 50% higher if immunity wanes linearly, and more than 150% higher in case immunity wanes exponentially (the most reasonable assumption), as compared to the classic SIRS epidemic model.

Keywords: SIRS epidemic, immunity waning, vaccination, herd immunity.

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