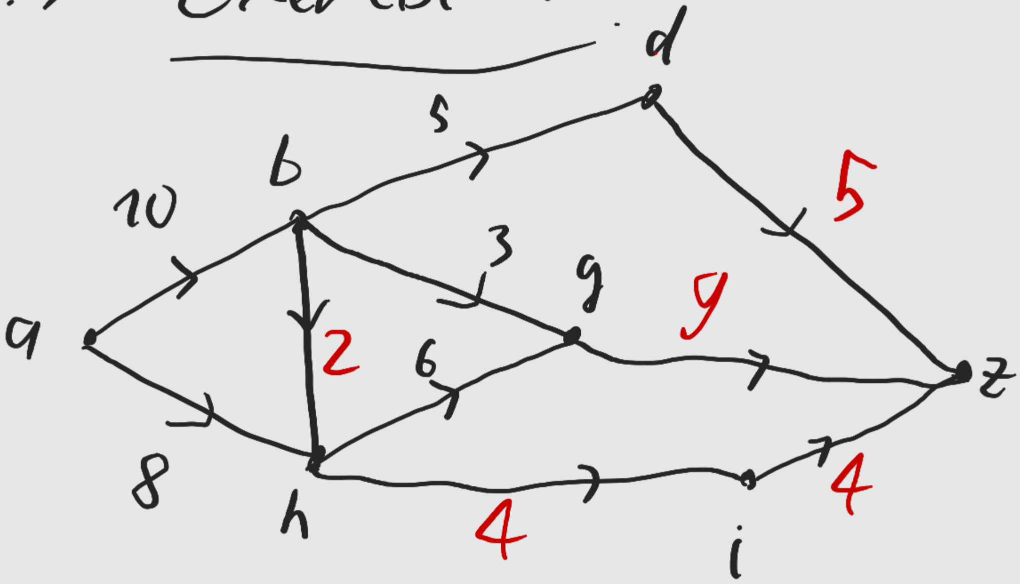
13.3 Exercise 7



a) Find the value of the flow on the remaining edges

b) What is the value of this flow

c) Find three cuts with capacity 30.

Solution:

a) See the graph, the value of incoming flow must equal the value of outgoing flow at each vertex.

b) We can look at the outgoing flow from a, giving a value of $18 = 10 + 8$.

c) Such cuts are for example

$$P_1 = \{a, b, d, g, h, i\}, \quad \overline{P_1} = \{z\}$$

$$P_2 = \{a, b, g, h\}, \quad \overline{P_2} = \{d, i, z\}$$

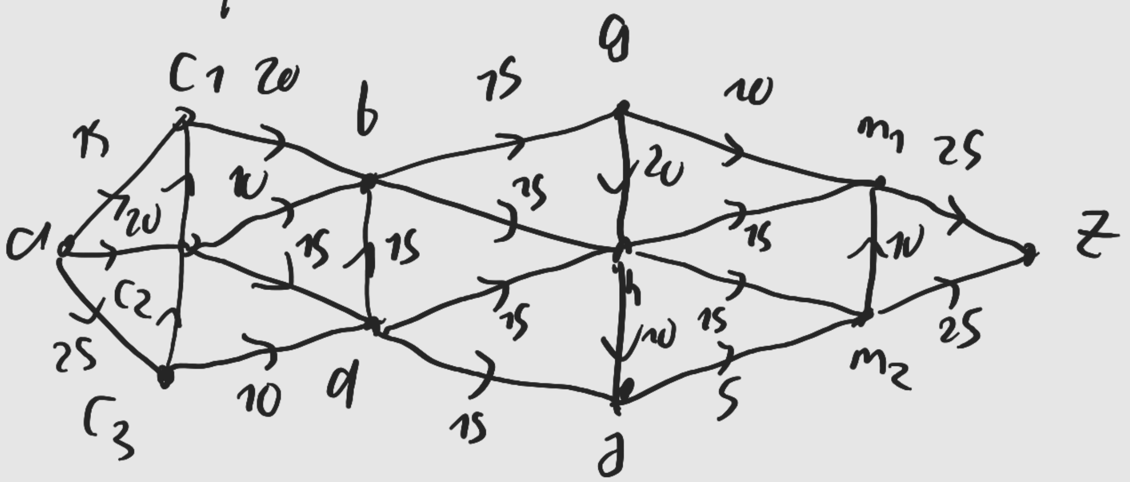
$$P_3 = \{a, h\}, \quad \overline{P_3} = \{b, d, g, i, z\}$$

Exercise 4:

Find a max-flow (and min-cut) in the following graphs

Solution:

- Example 12:

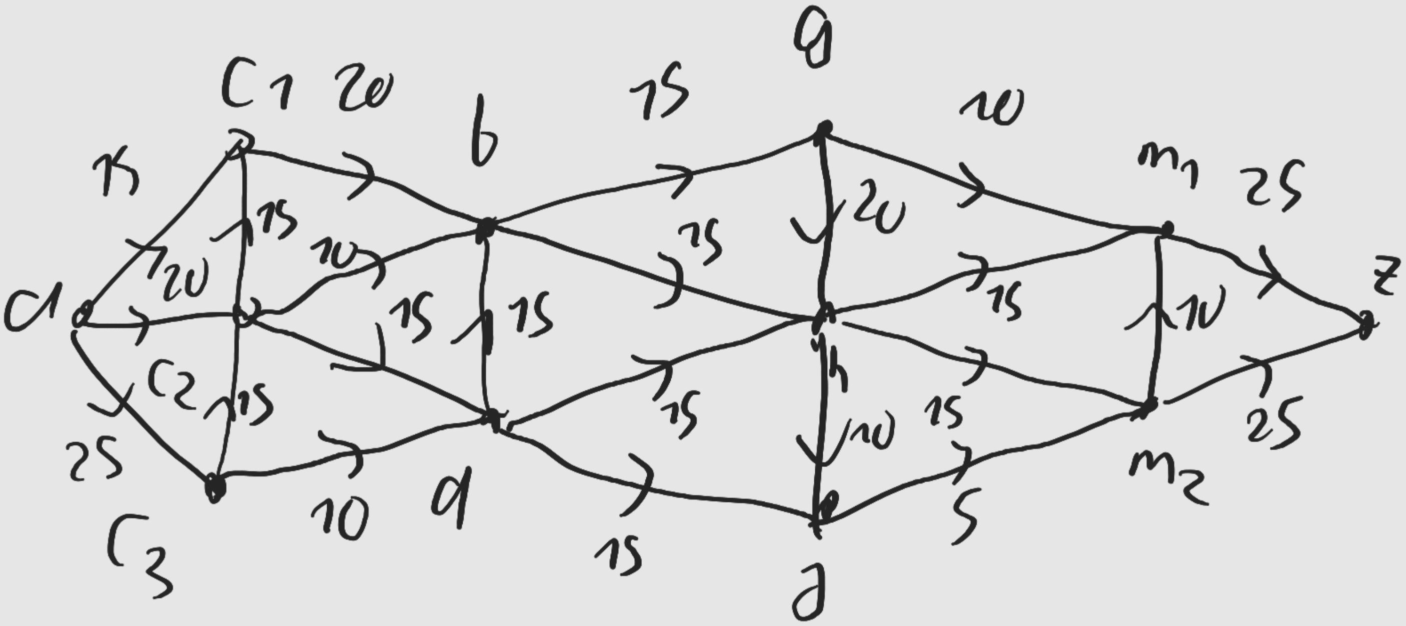


Step 1:

Initial flow is $f(e)=0 \ \forall e$

Step 2:

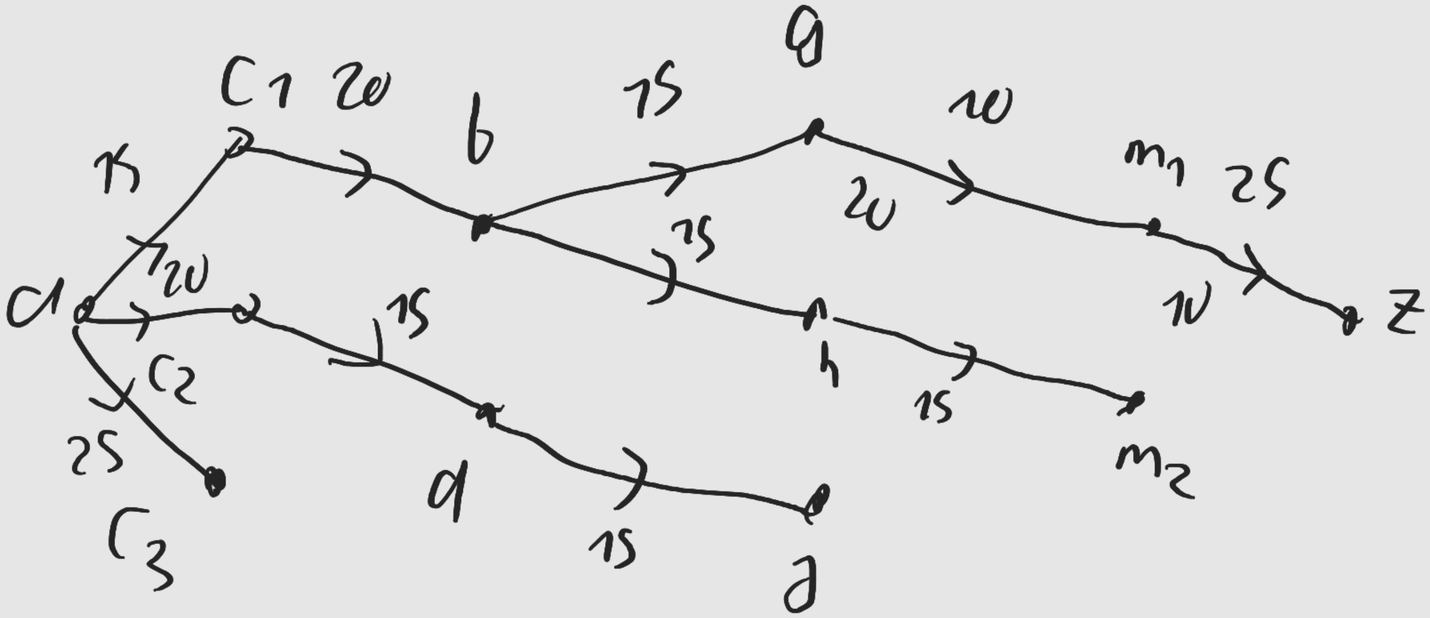
Determine an f-augmenting path with Edmonds-Karp:



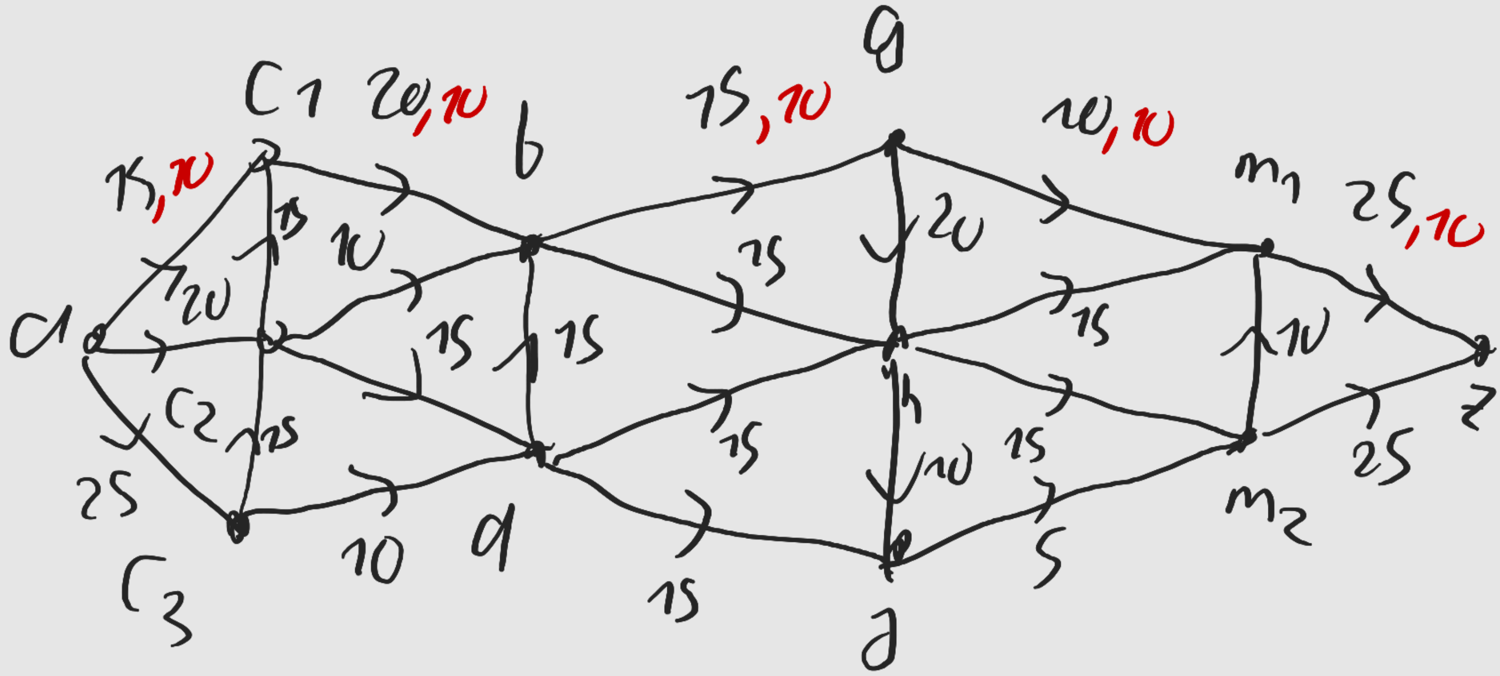
$a:(-,1)$ $c_1:(a,2)$ $c_2:(a,3)$ $c_3:(a,4)$

$b:(c_1,5)$ $d:(c_2,6)$ $g:(b,7)$ $h:(b,8)$

$j:(d,9)$ $m_1:(g,10)$ $m_2:(h,11)$ $z:(m_1,12)$

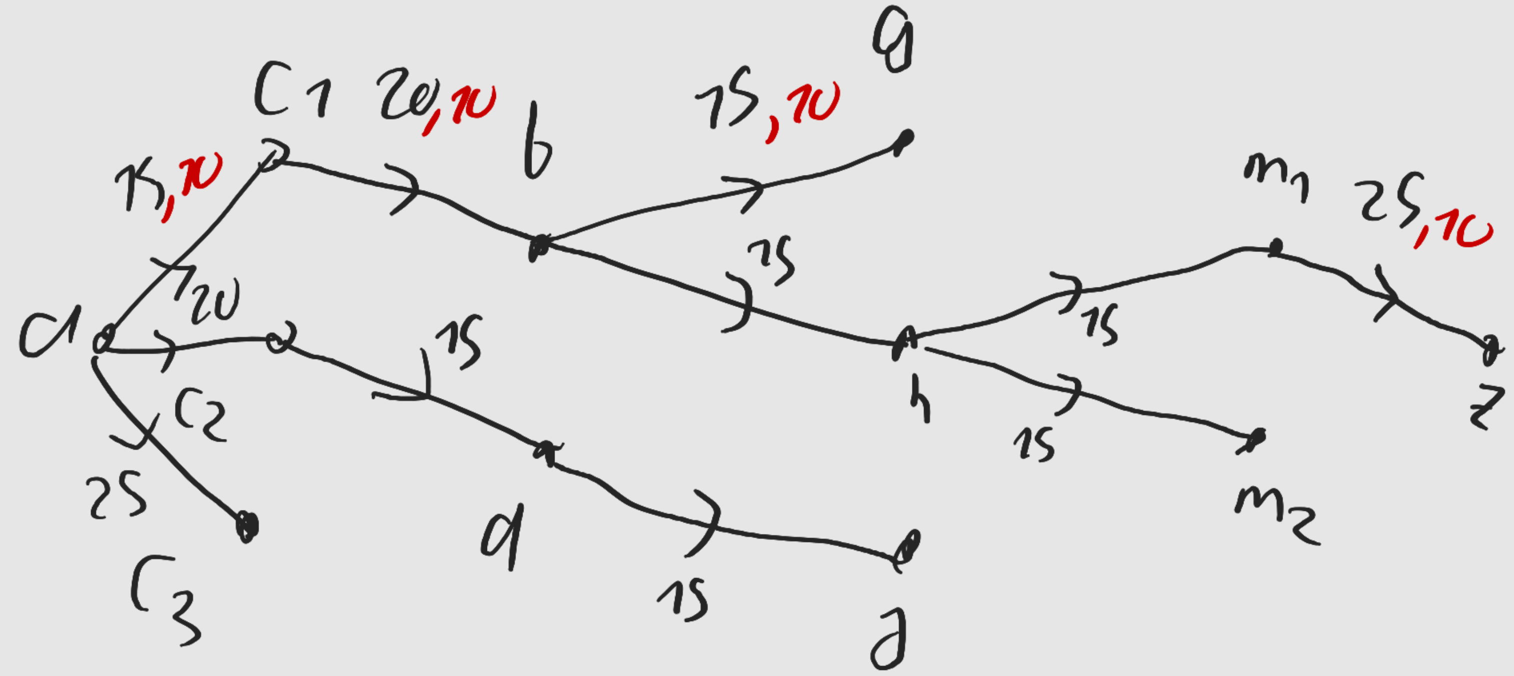


We have an augmenting path from a to z, with capacity 10:

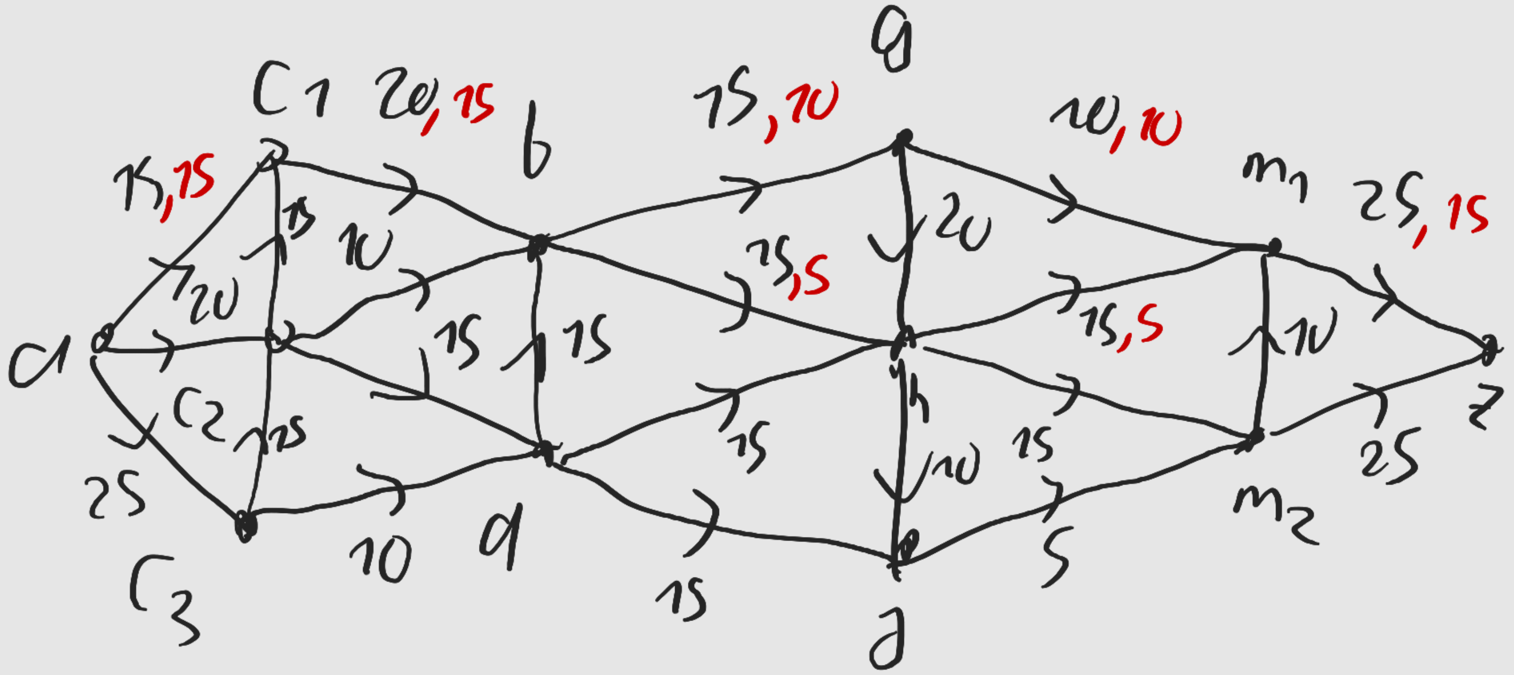


<u>Step 2</u>:

Edmonds-Karp:

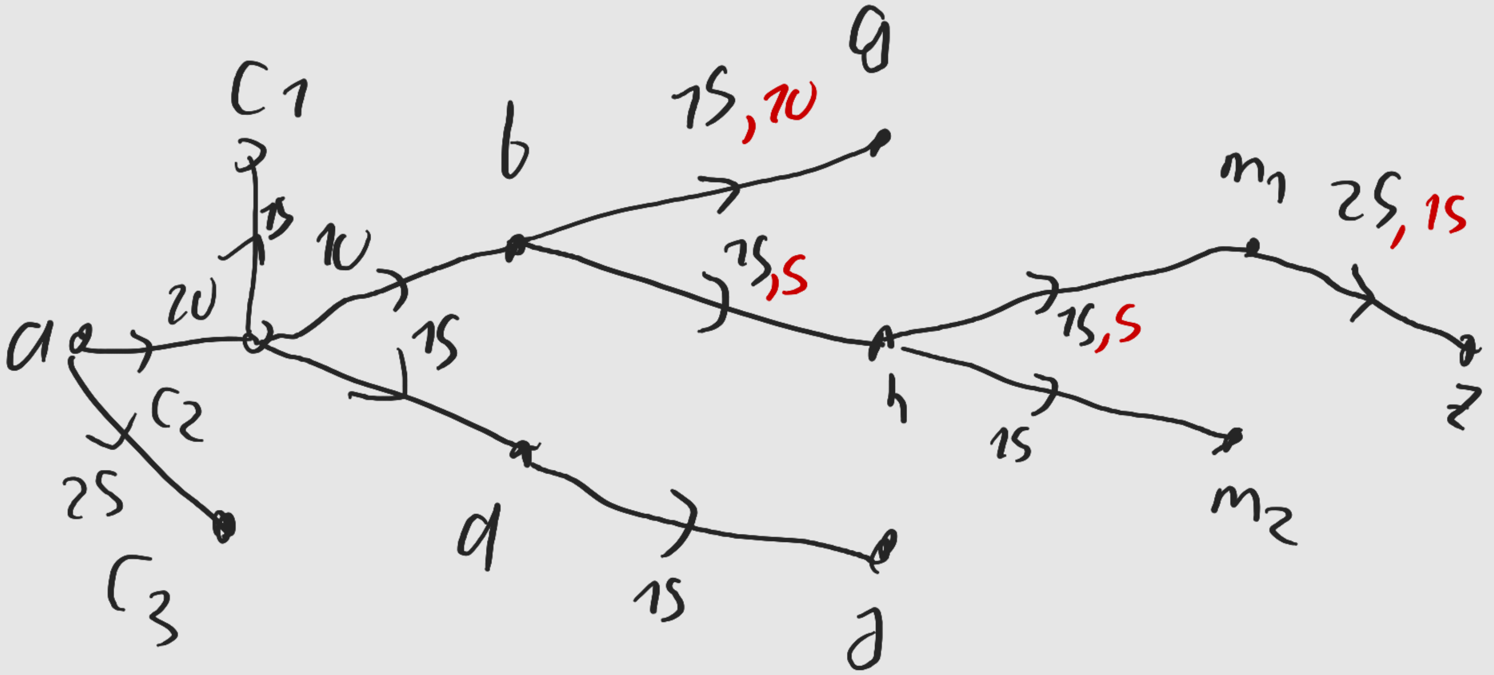


the capacity is now 5 :

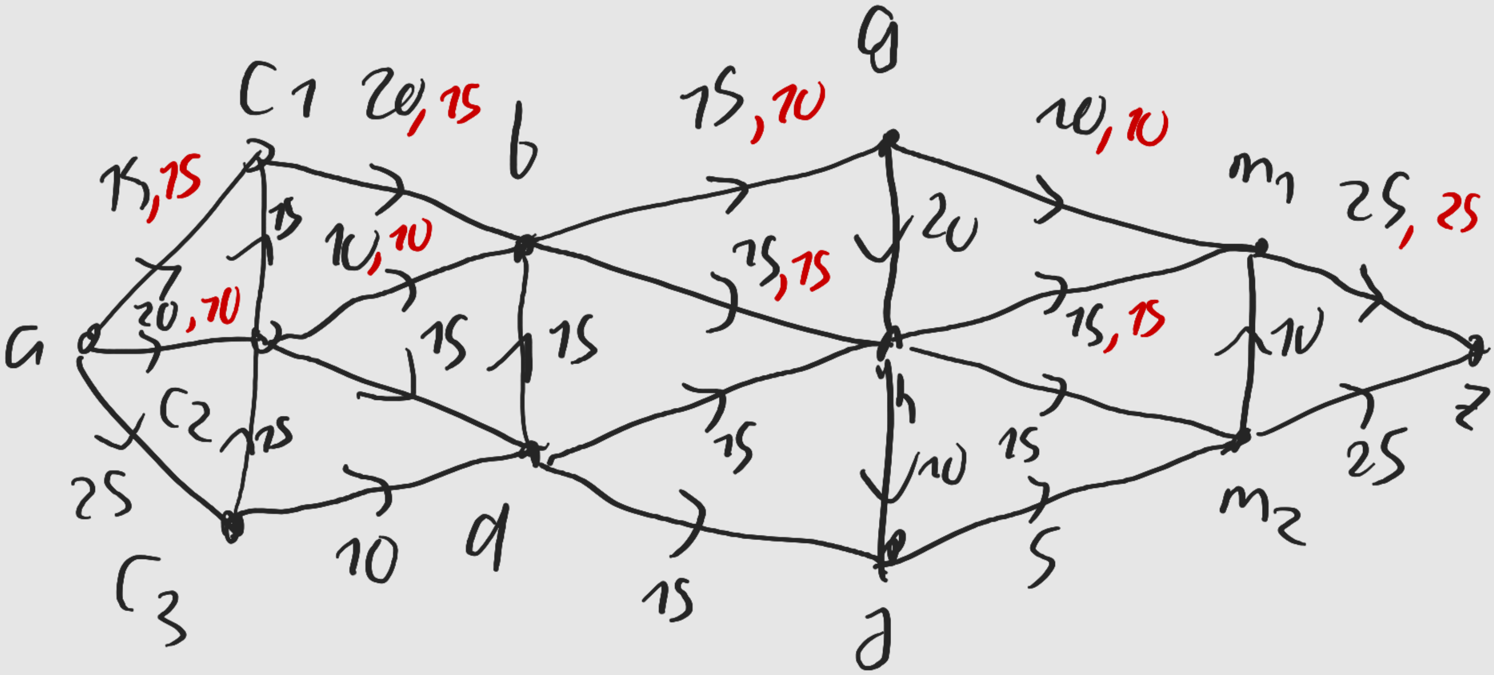


Step 2:

Edmonds-Karp

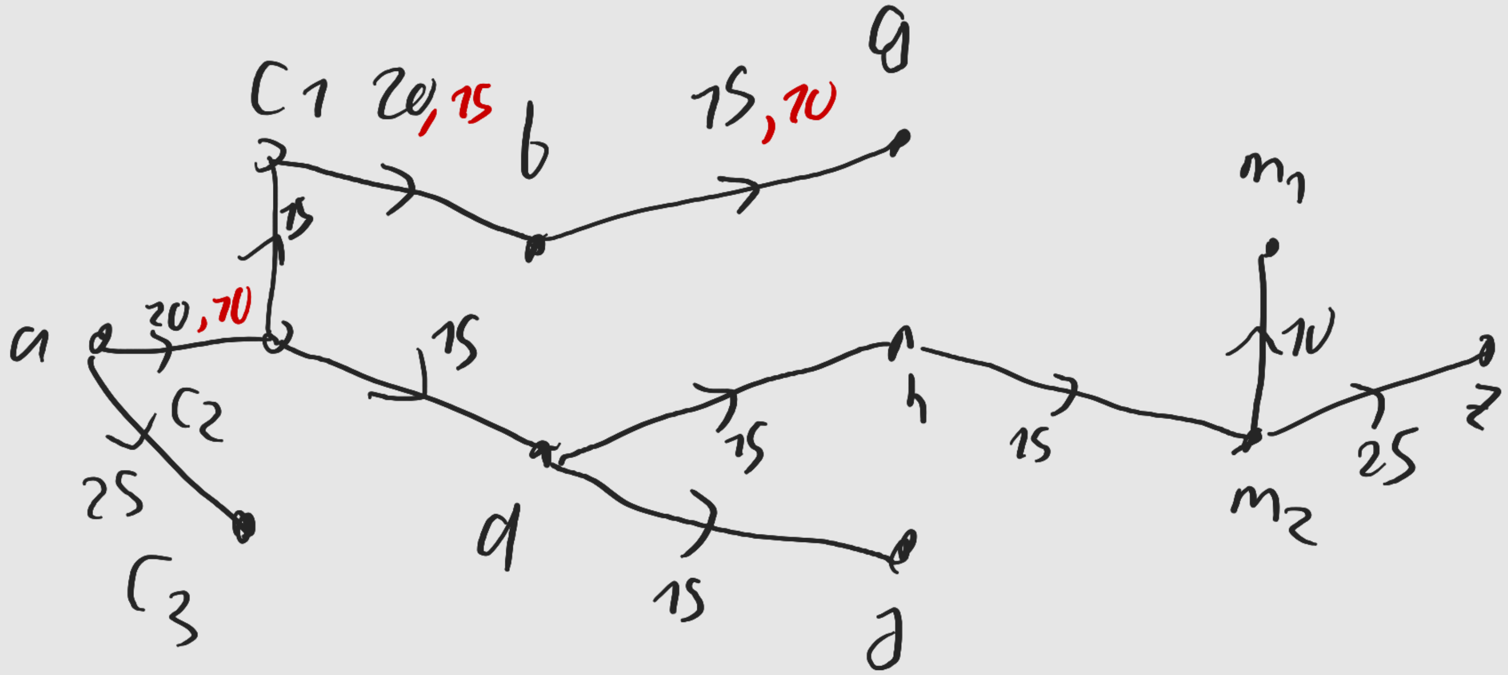


We get a capacity of 10:

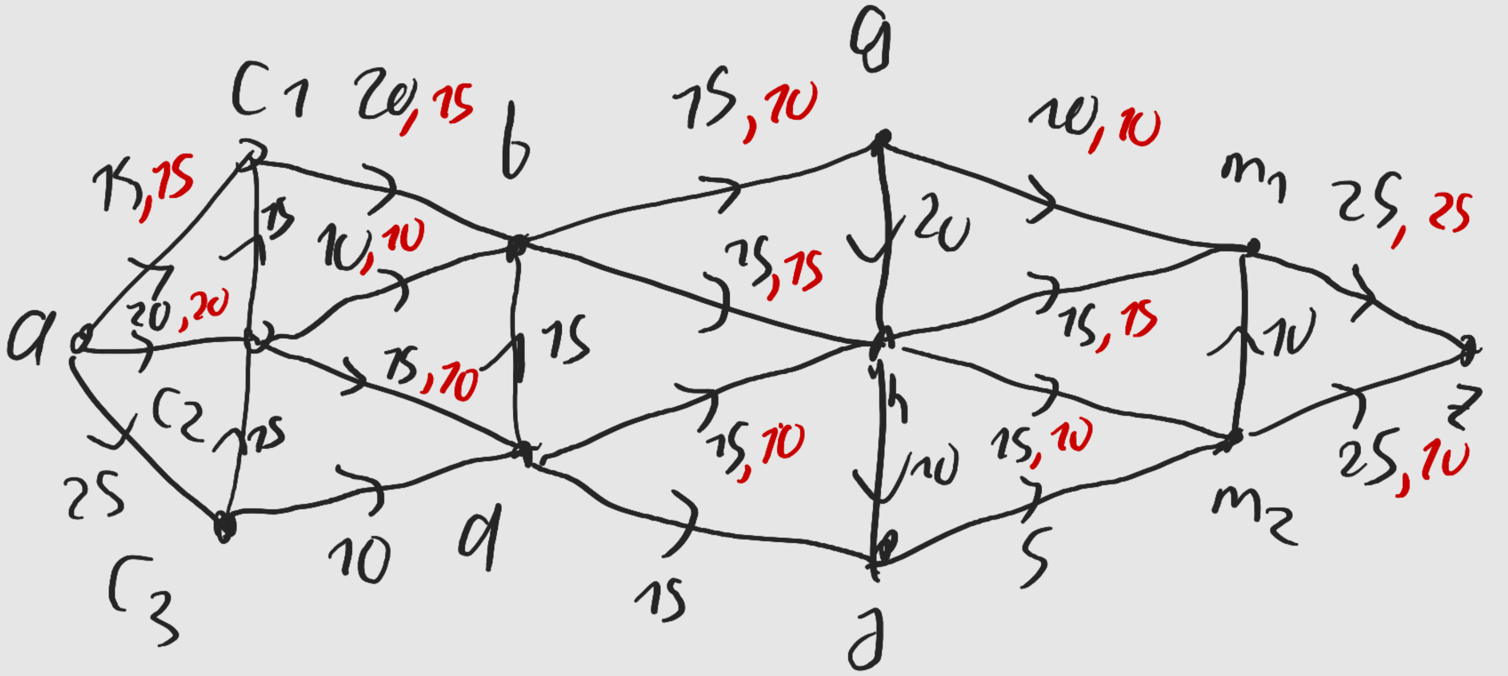


Step 2:

Edmonds-Karp:

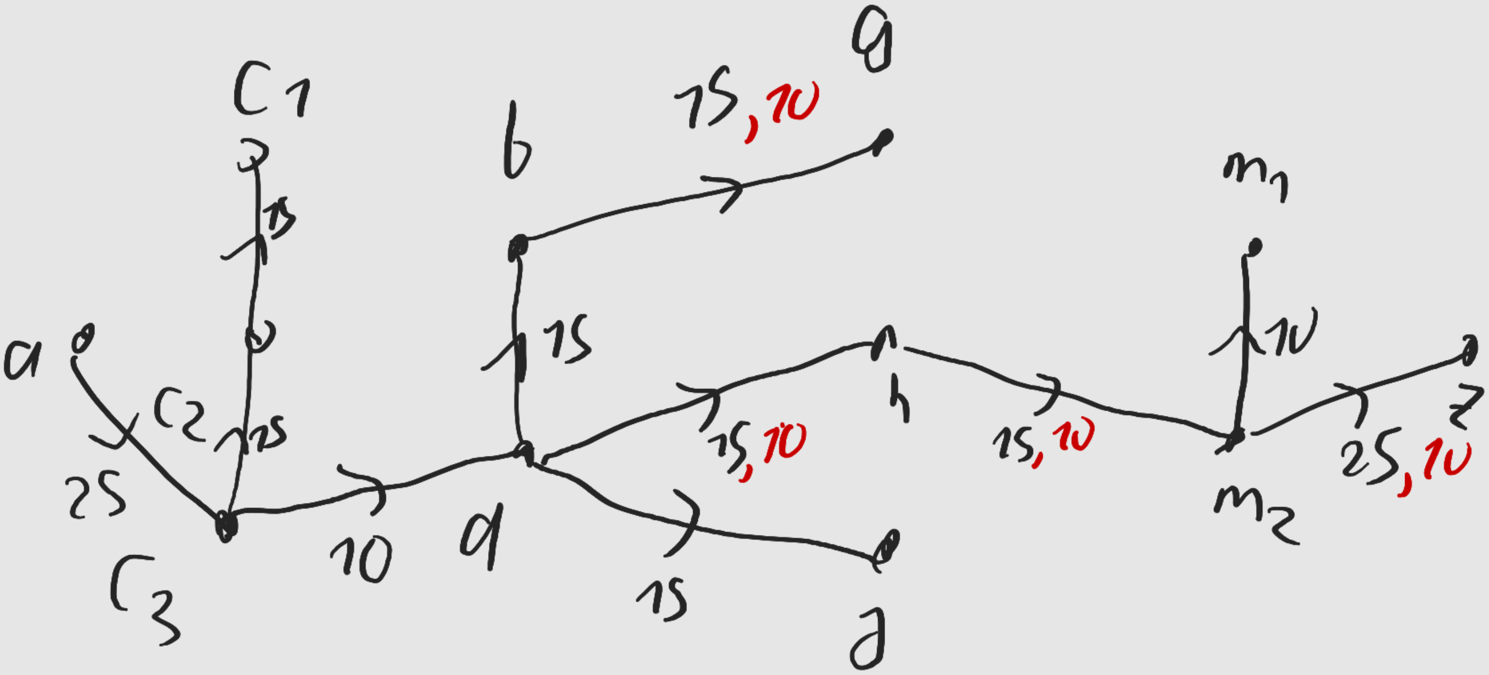


The capacity is 10

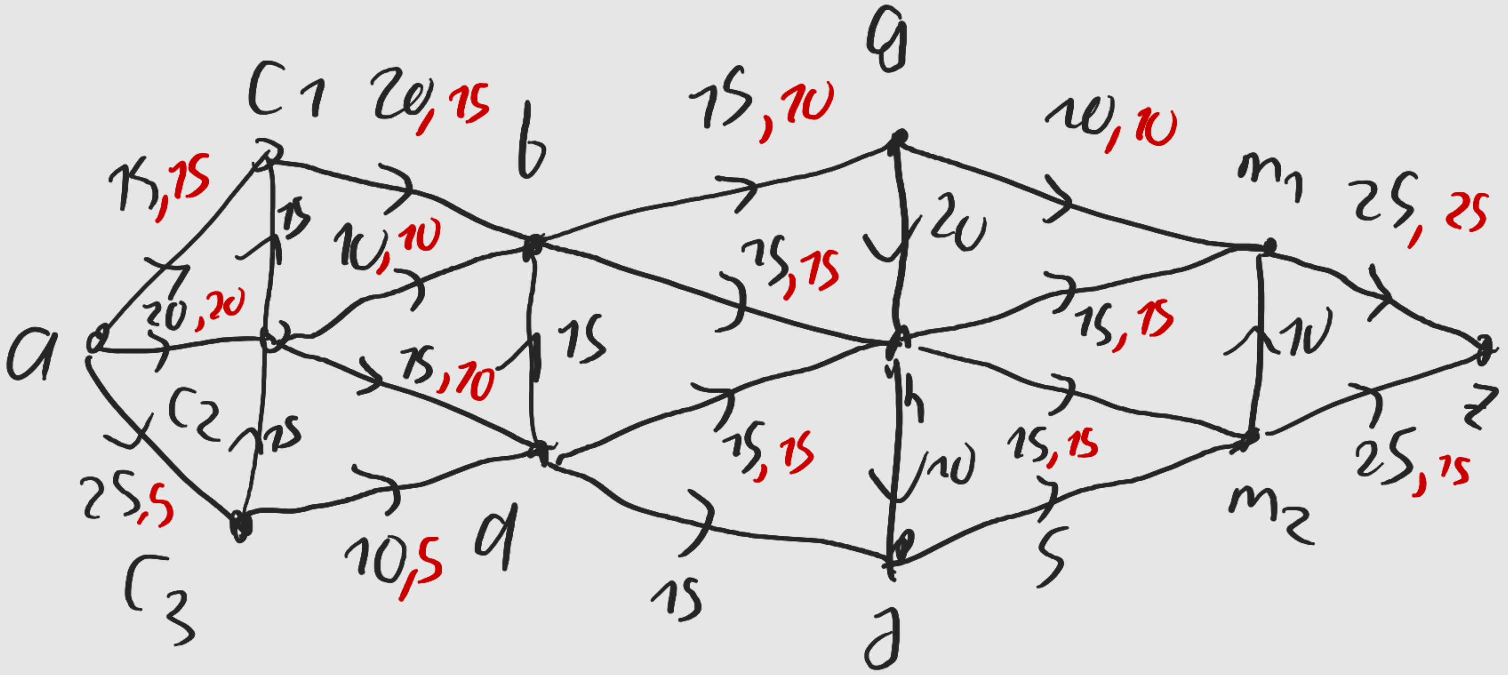


Step 2:

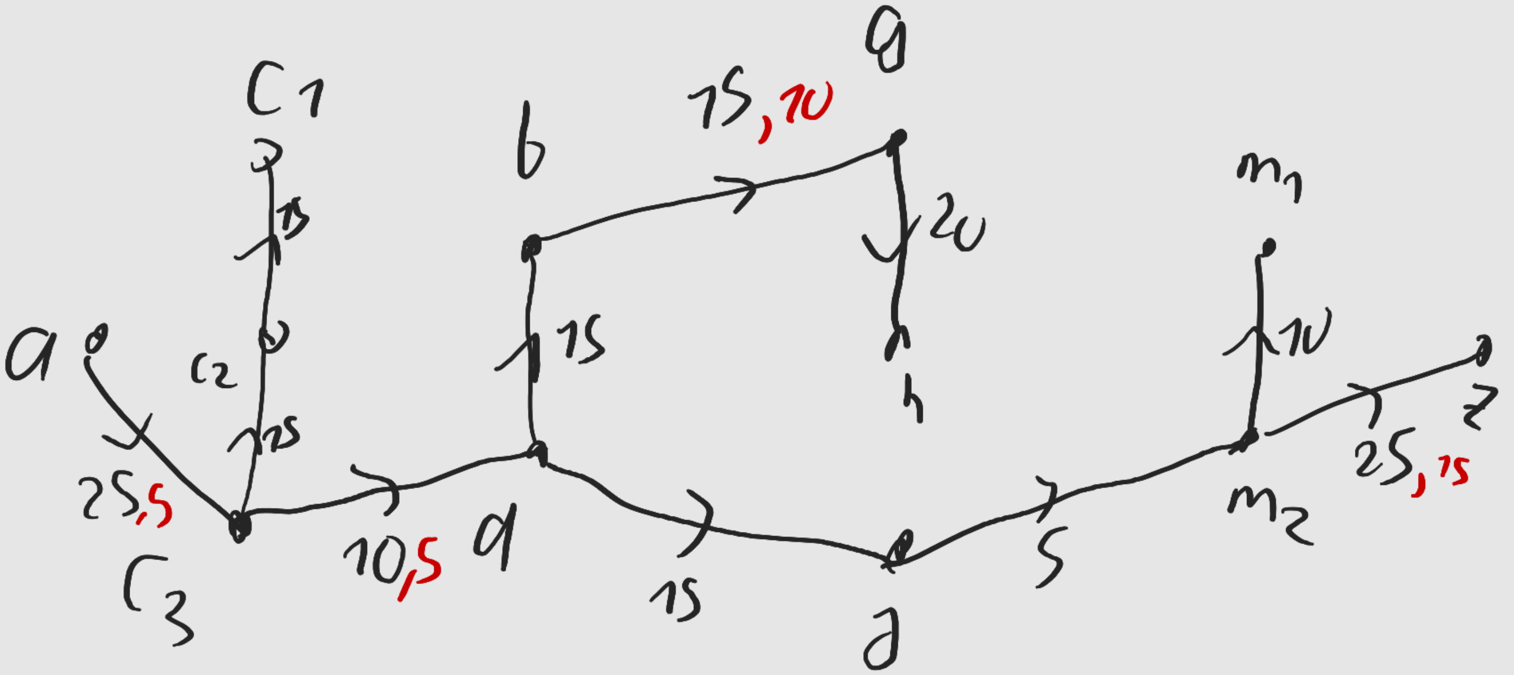
Edmonds-karp :



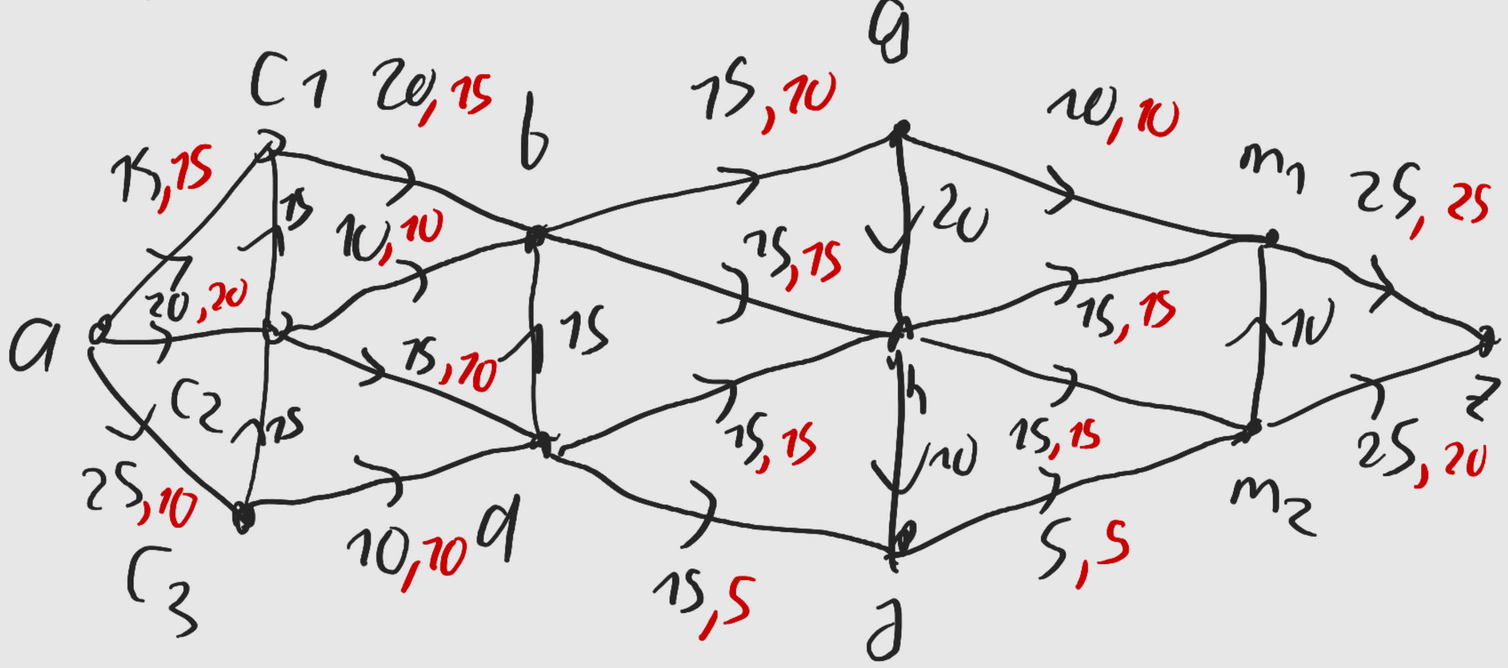
The capacity is 5.



Step 2:

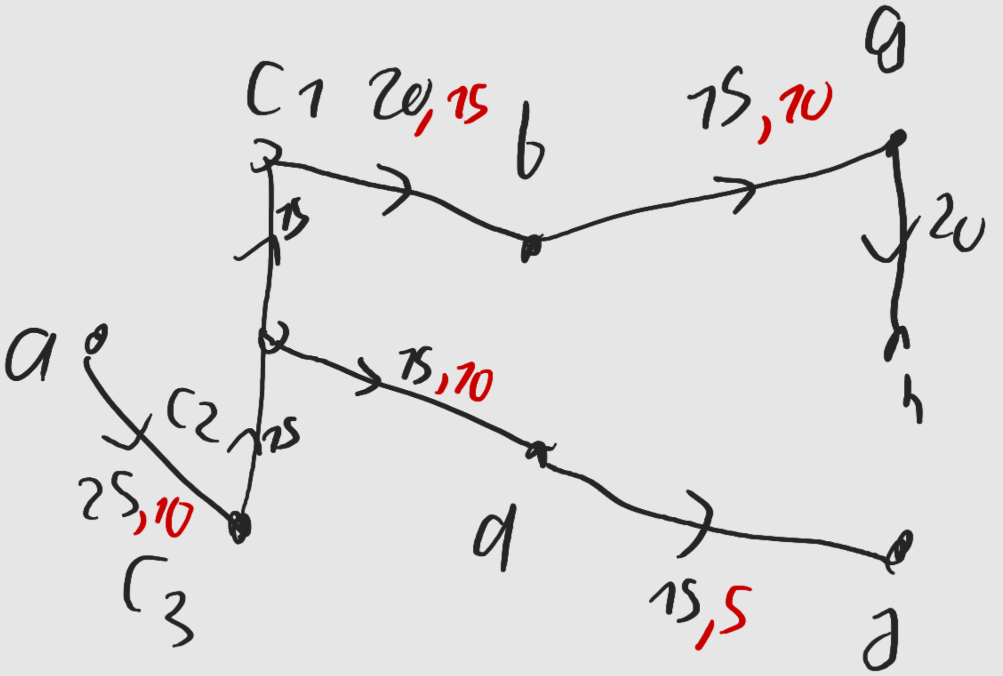


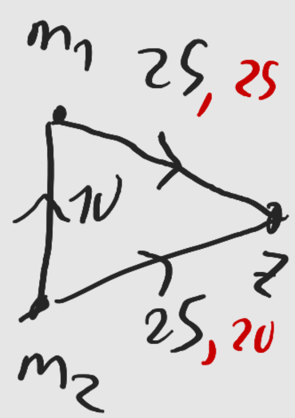
Capacity of S:



<u>Step 2</u>:

Edmonds - Karp:

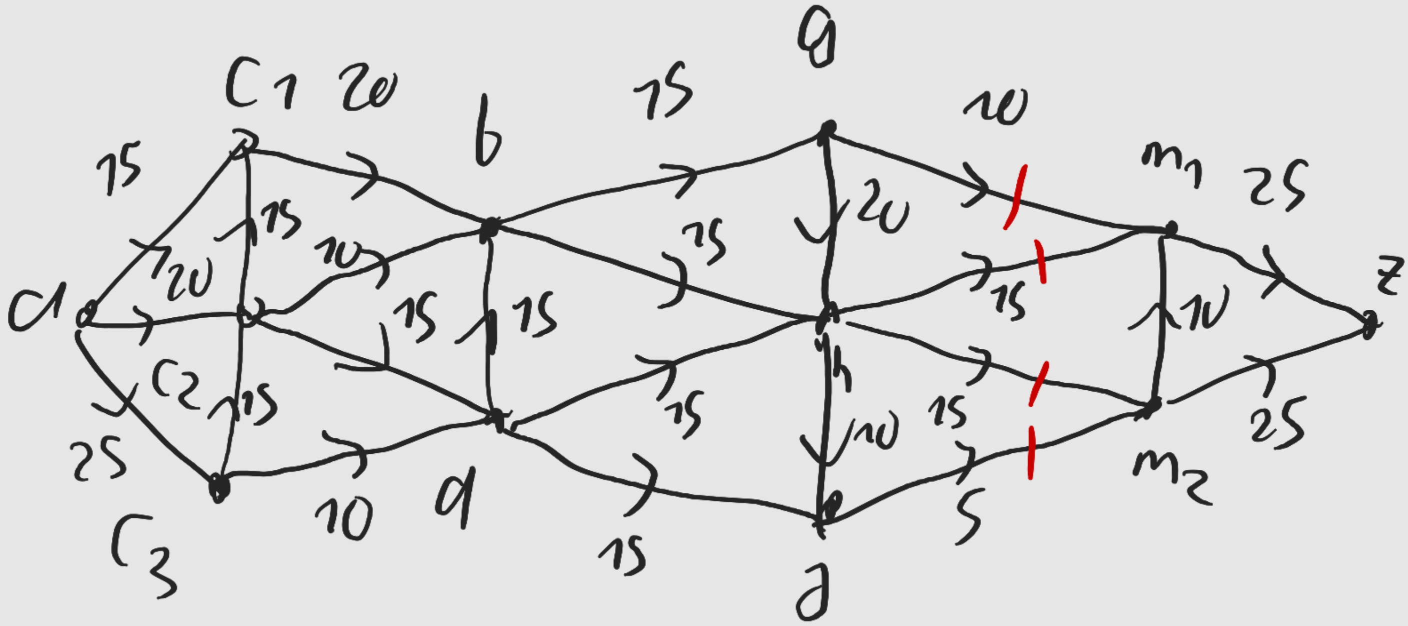




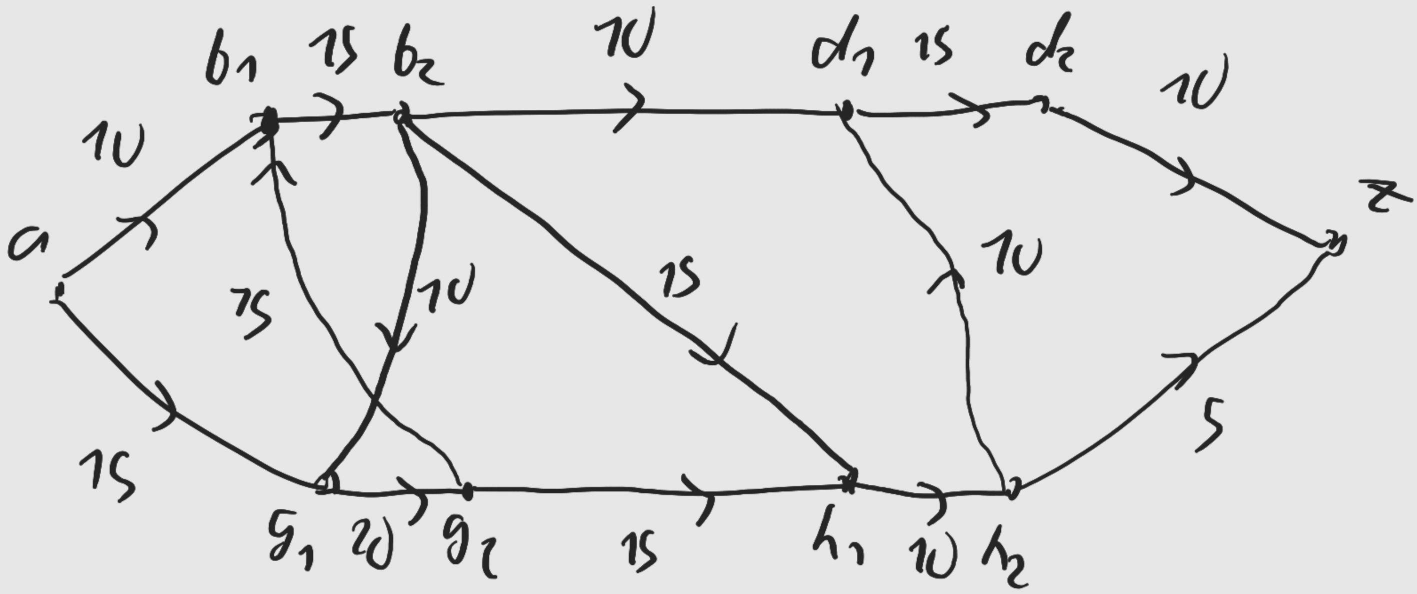
There is no usable edge, so we get a cut

$P = \{a, c_1, c_2, c_3, b, d, g, h, j\}$, $\overline{P} = \{m_1, m_2, z\}$

with capacity 45:

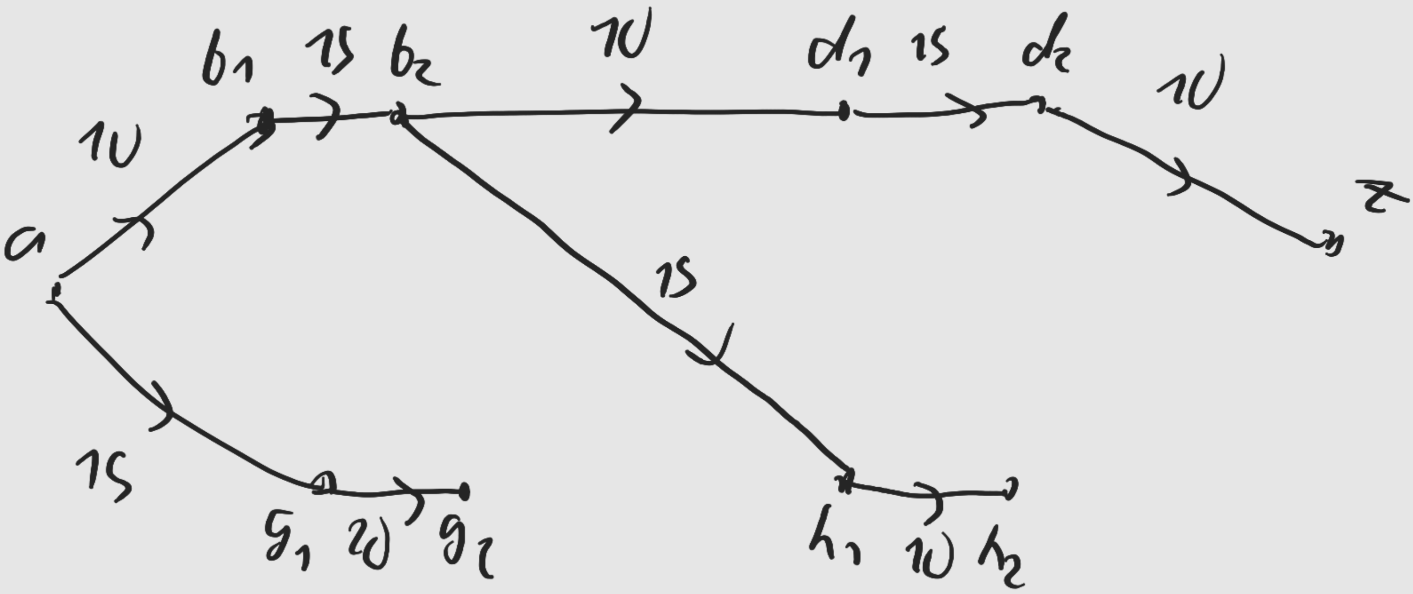


Example 13:

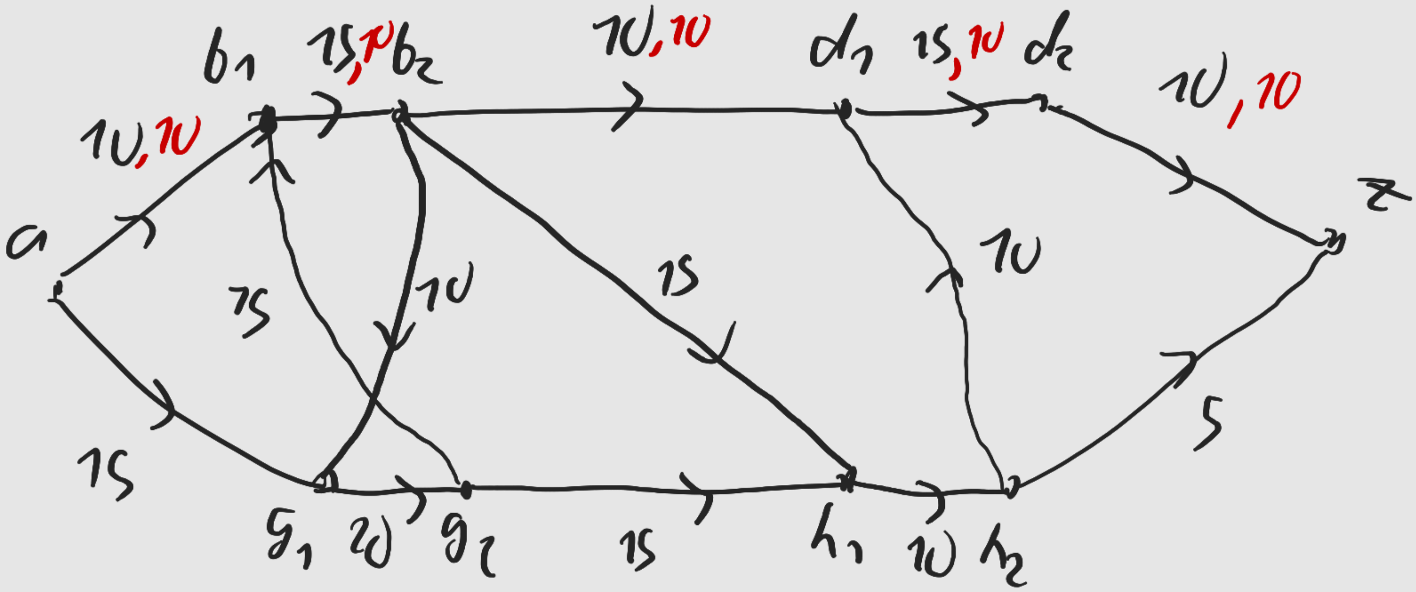


Initial flow is 0.

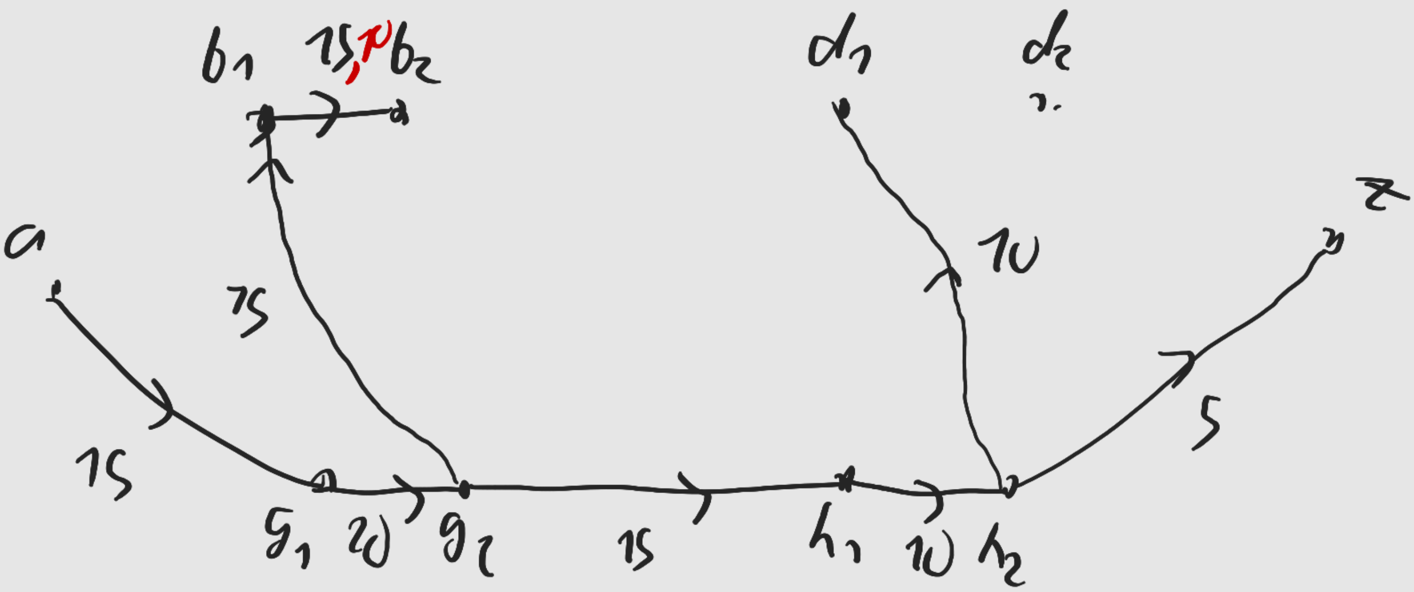
Edmonds-Karp:



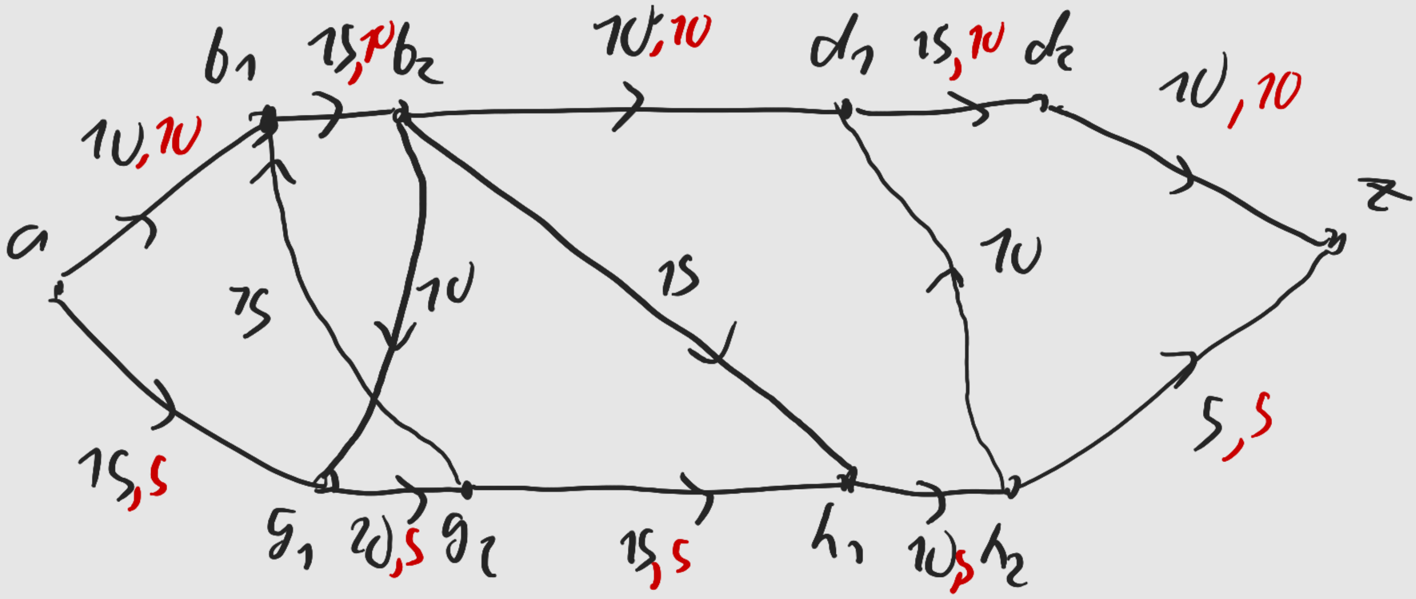
Capacity is 10:



Edmonds - karp



the capacity is 5



We should do Edmonds-Karp once more, but we already see that the cut is

$P = \{a, b_1, b_2, g_1, h_2, d_1, d_2, h_1, h_2\}$, $\bar{P} = \{z\}$