Mm5023 lecture 15

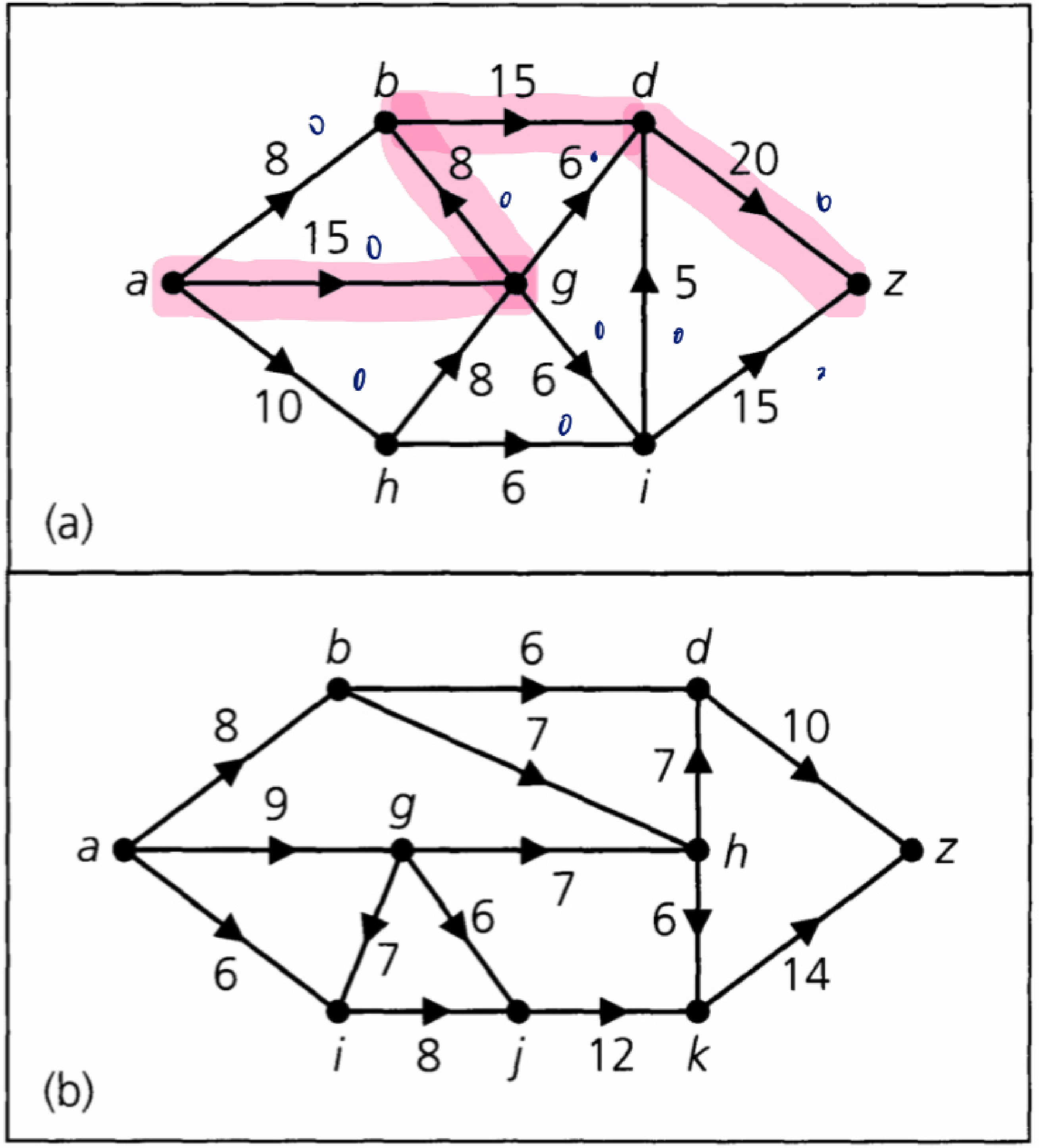
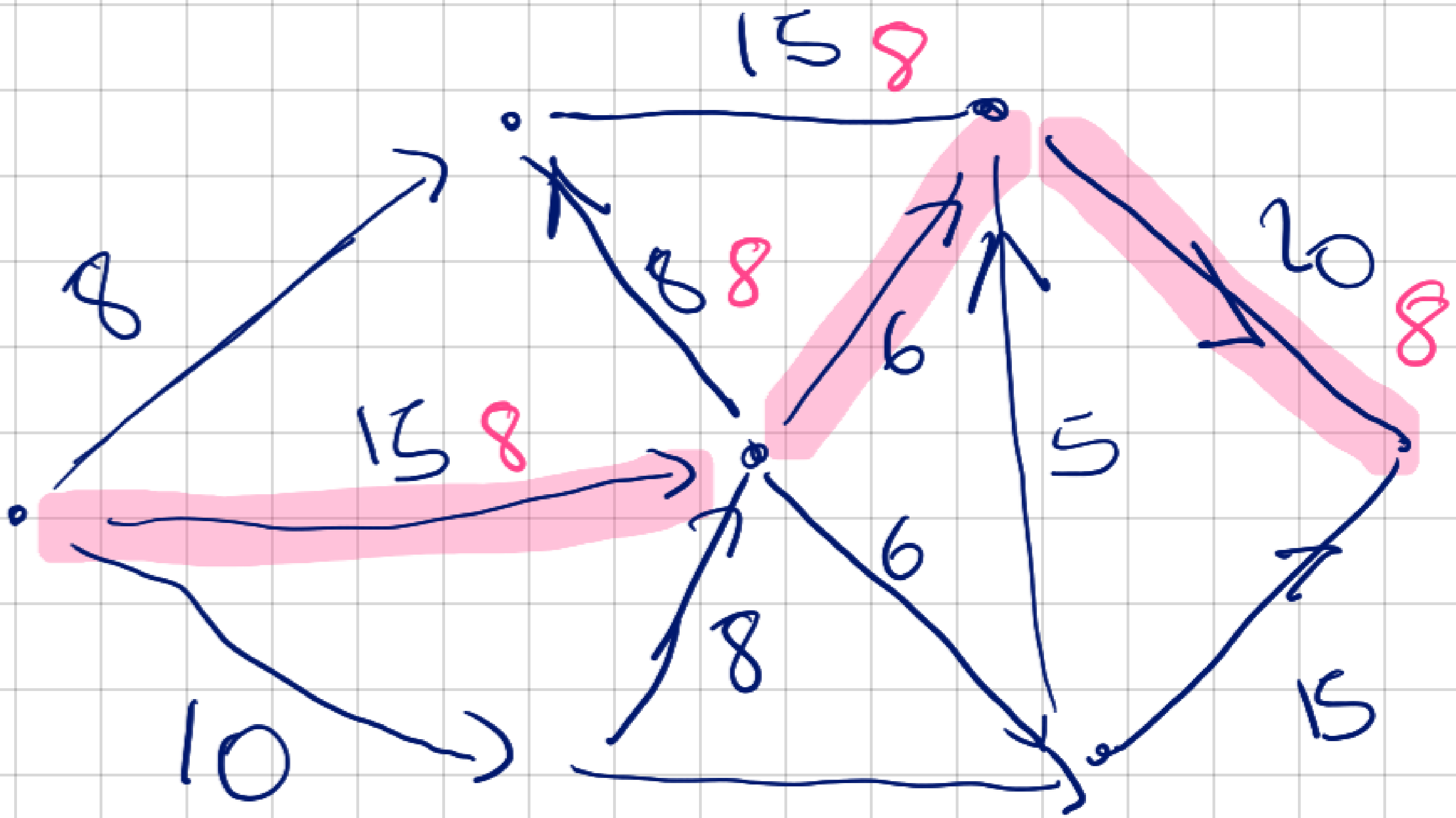


Figure 13.21

Max flow min cut.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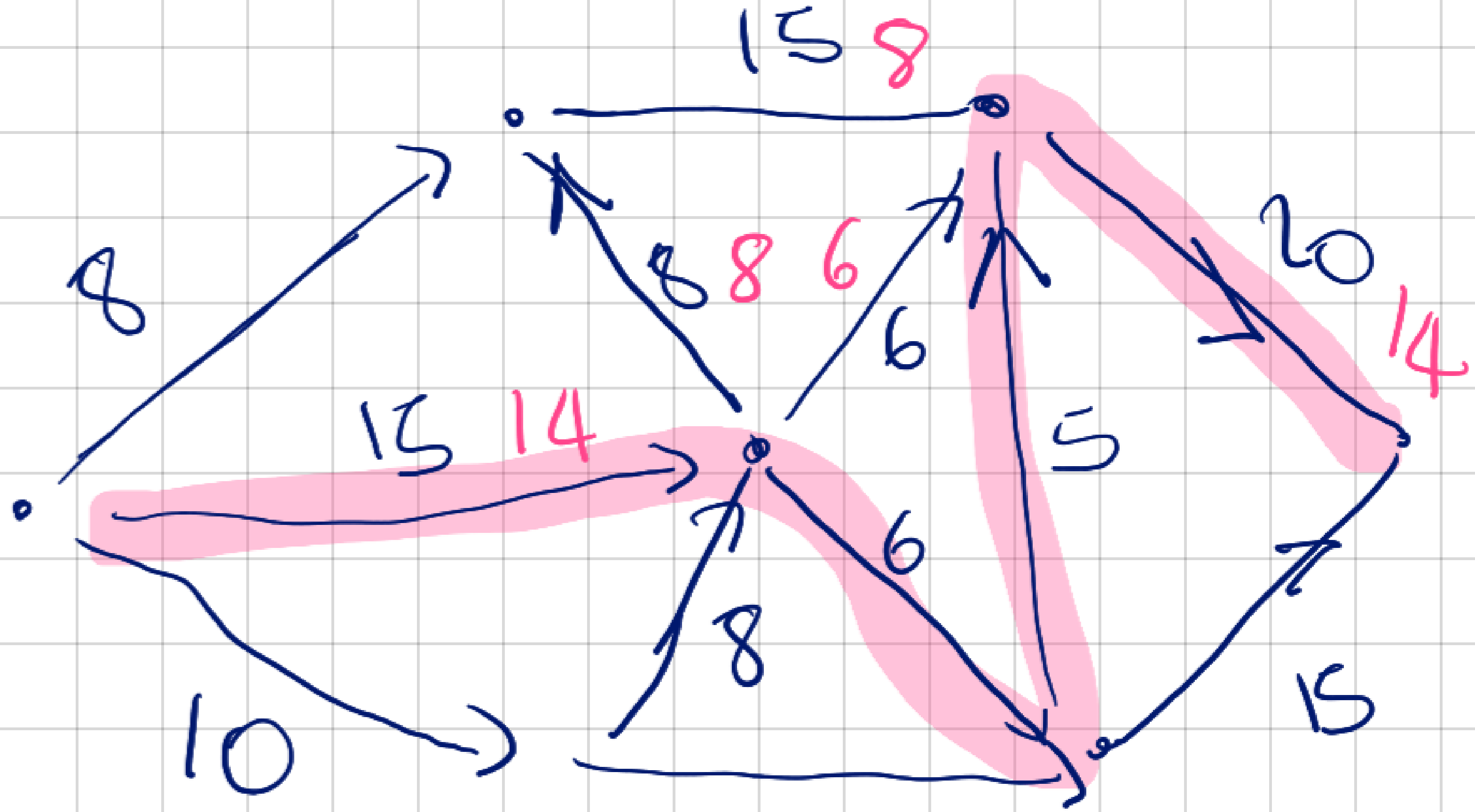
8
8 8
15 8
6
20
8
5
6
8
10
15


15 8
8
8 8 6
6
20
14
15 14
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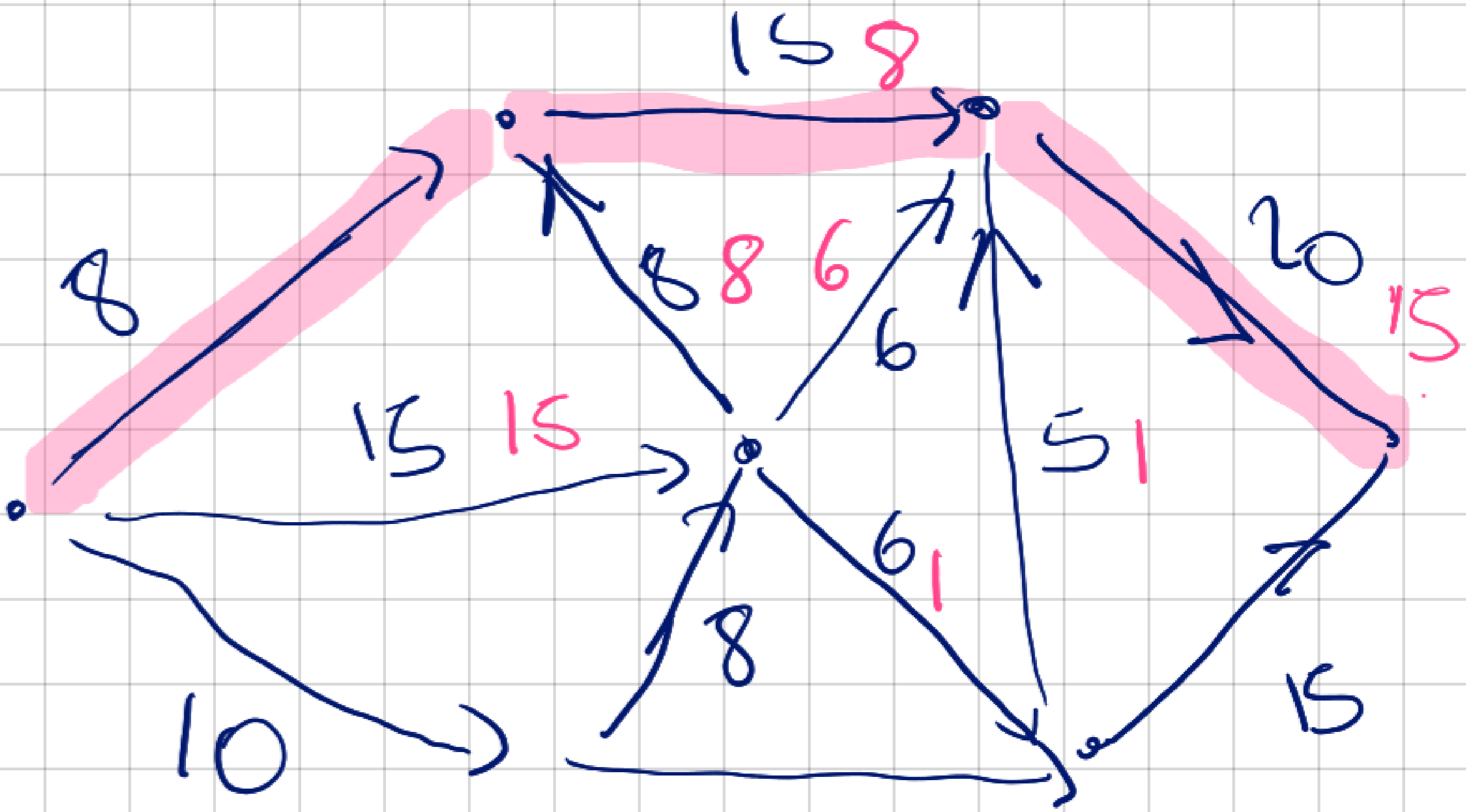
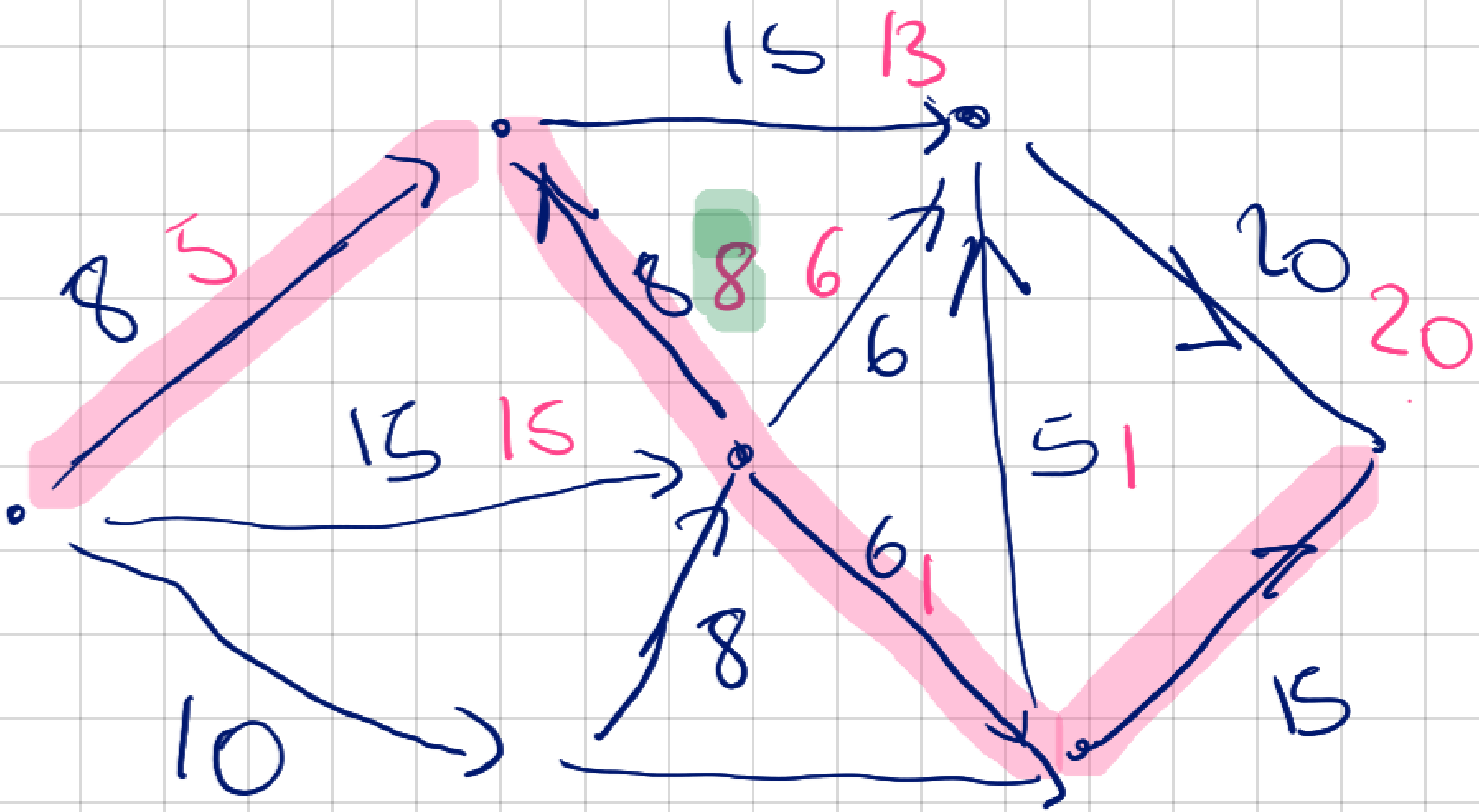
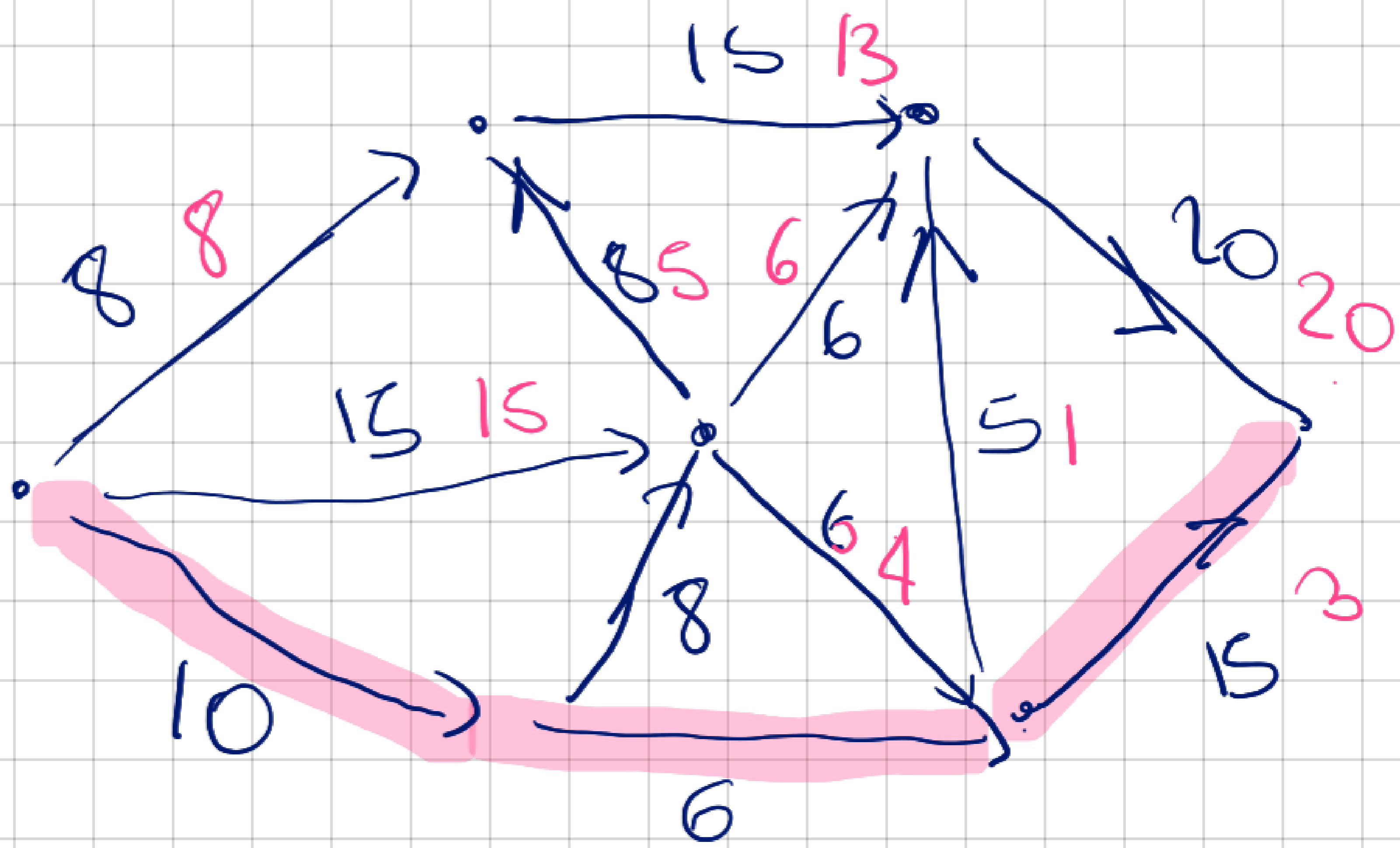
15



15 8
8
8 8 6
6
20
15
15 15
51
61
8
10
15



15 13
8 5
8 8 6
6
20
20
15 15
5 1
6 1
8
10
15


15 13
8 8
85 6
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15 15
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64
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3
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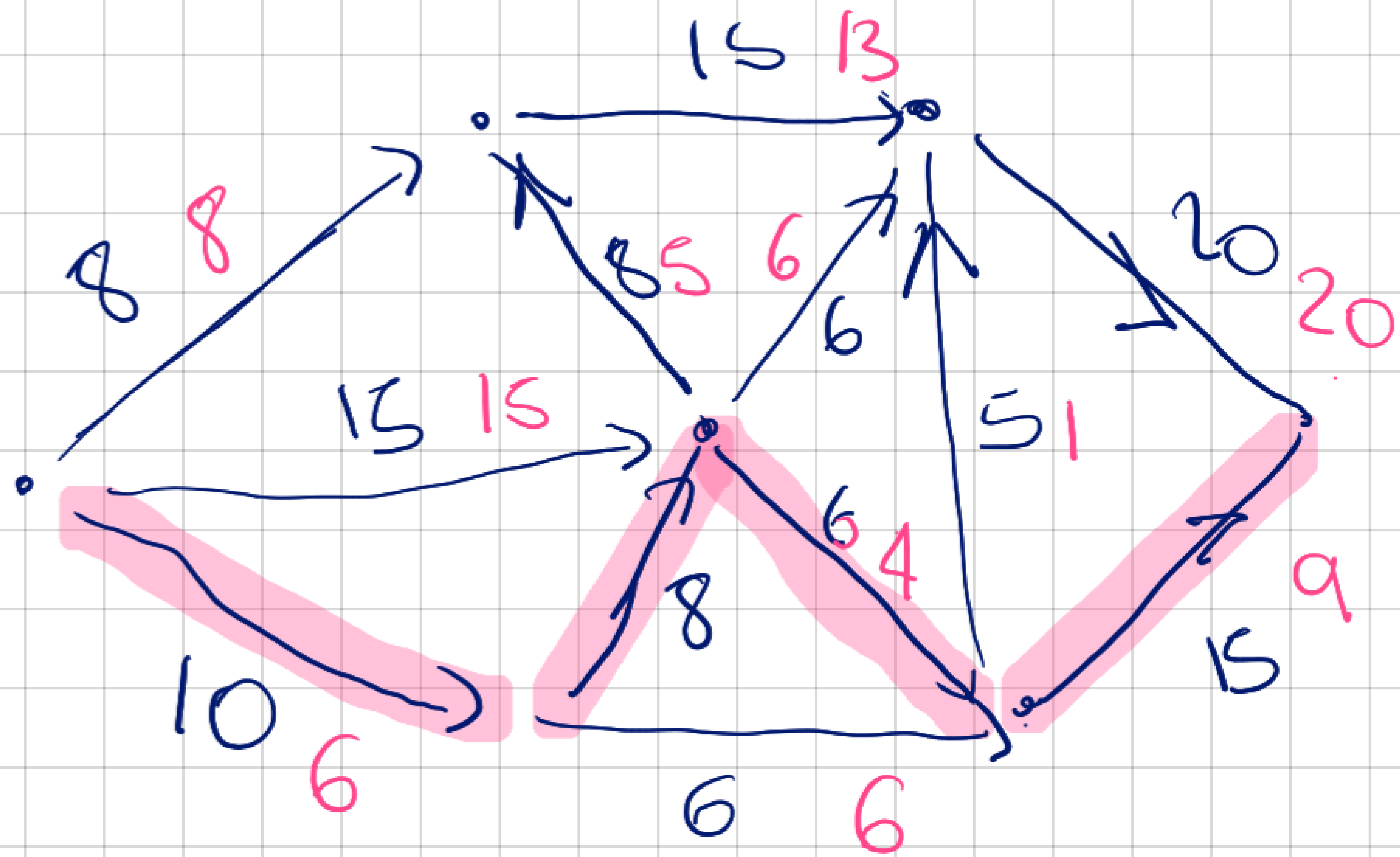
15
6


15 13
8 8
8 5 6
6
20
20
15 15
5 1
6 4
8
9
15
10
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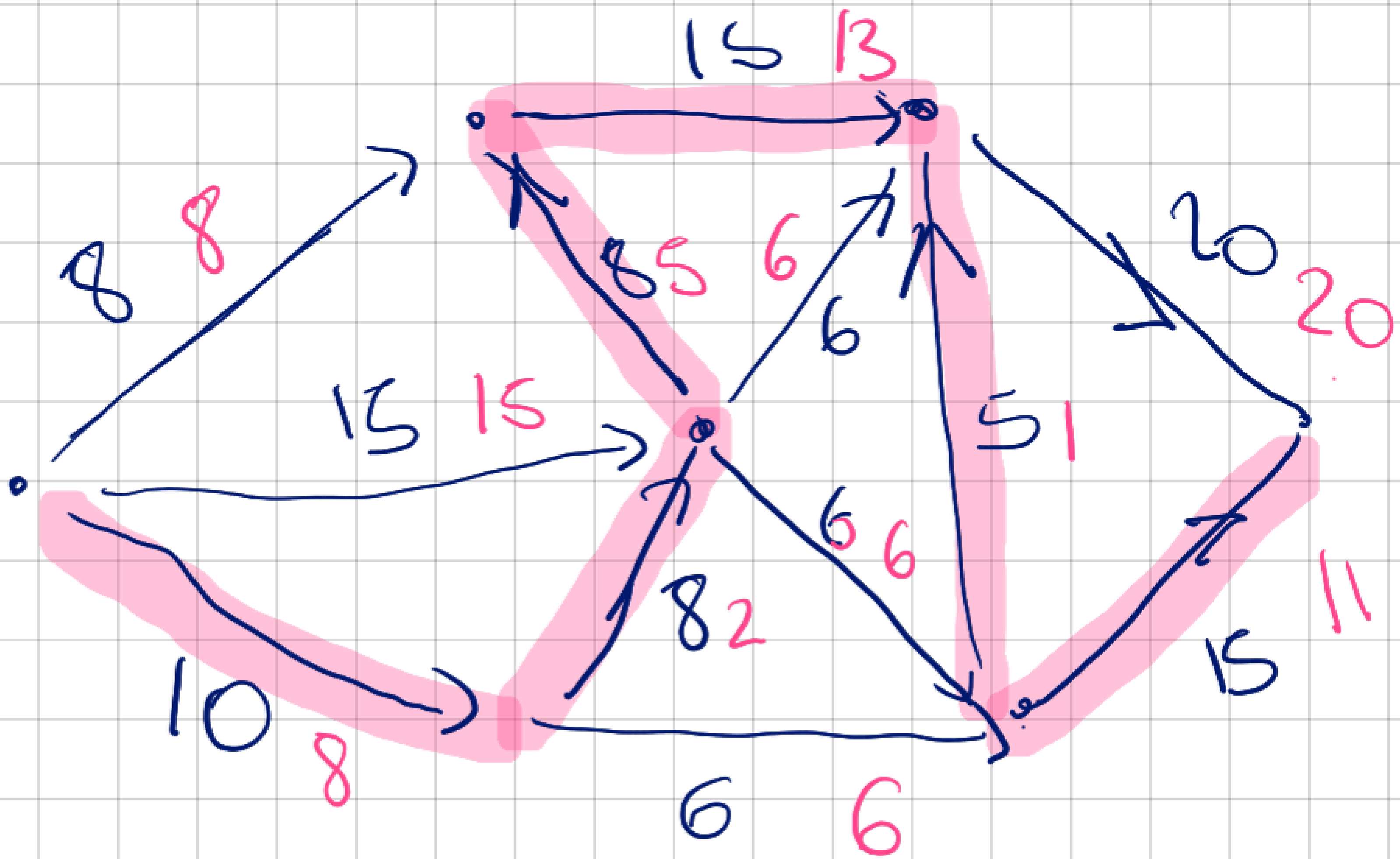
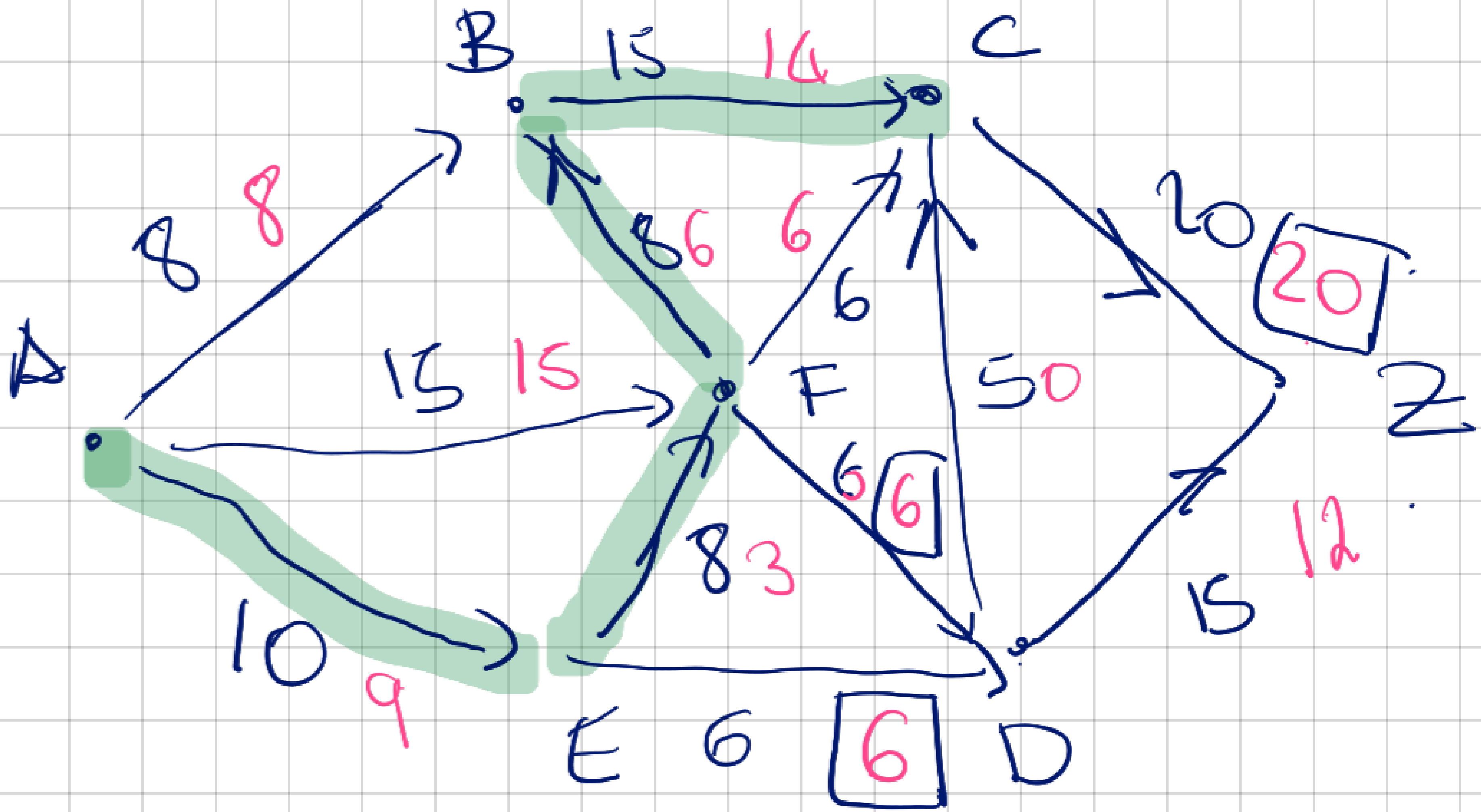
6 6


15 13
8 8
85 6
6
20
20
15 15
51
66
82
15
11
10
8
6 6



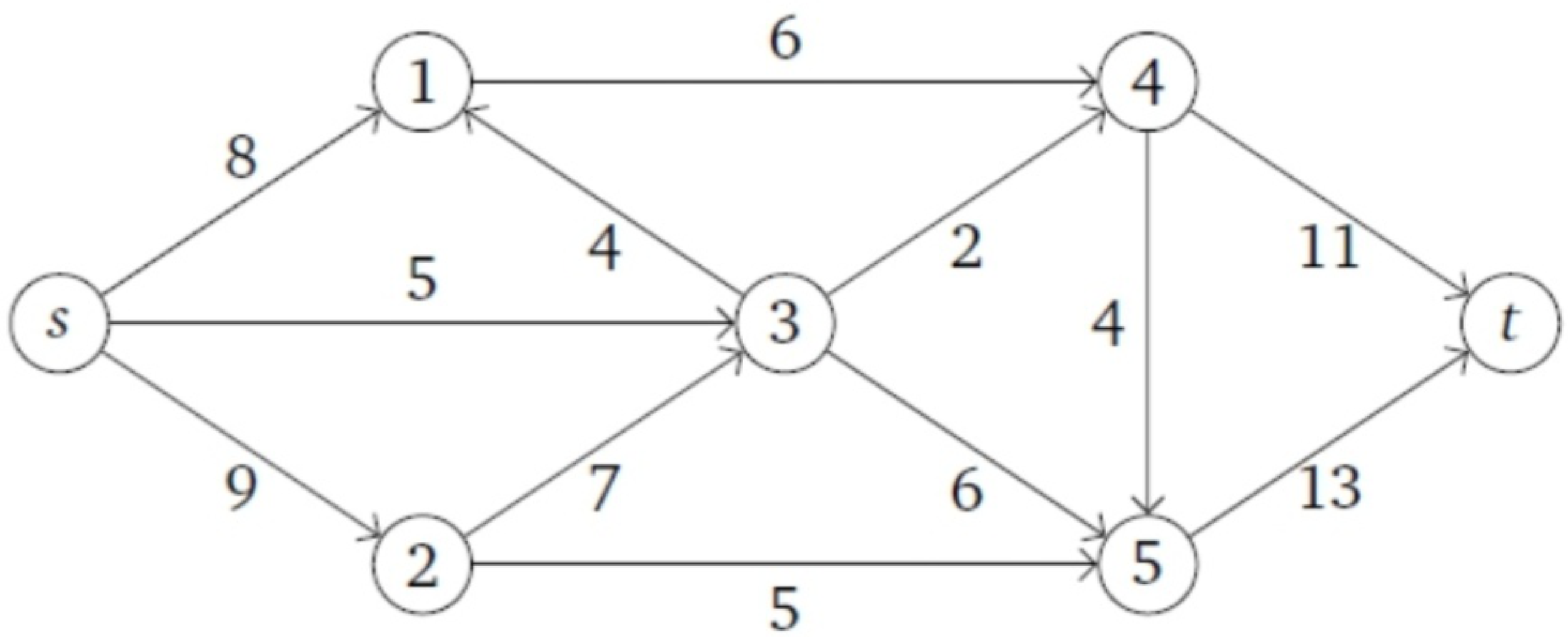
$$\mathrm{Val}(f) = 8 + 15 + 9 = 32$$

$$= 20 + 12$$

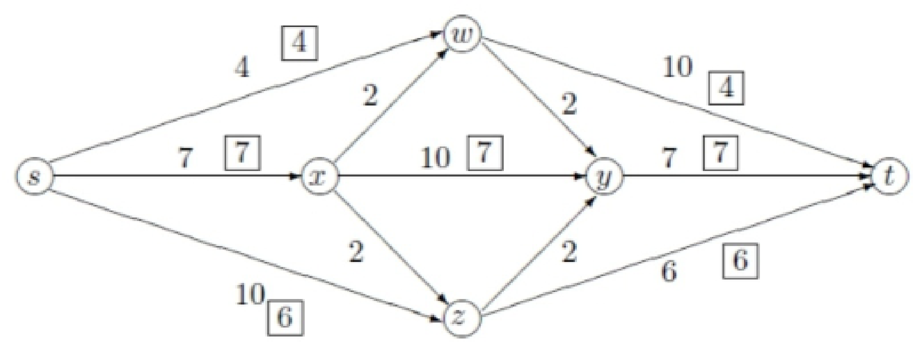
$P = \{A, B, F, E, C\}$ $\quad P^c = \{D, Z\}$

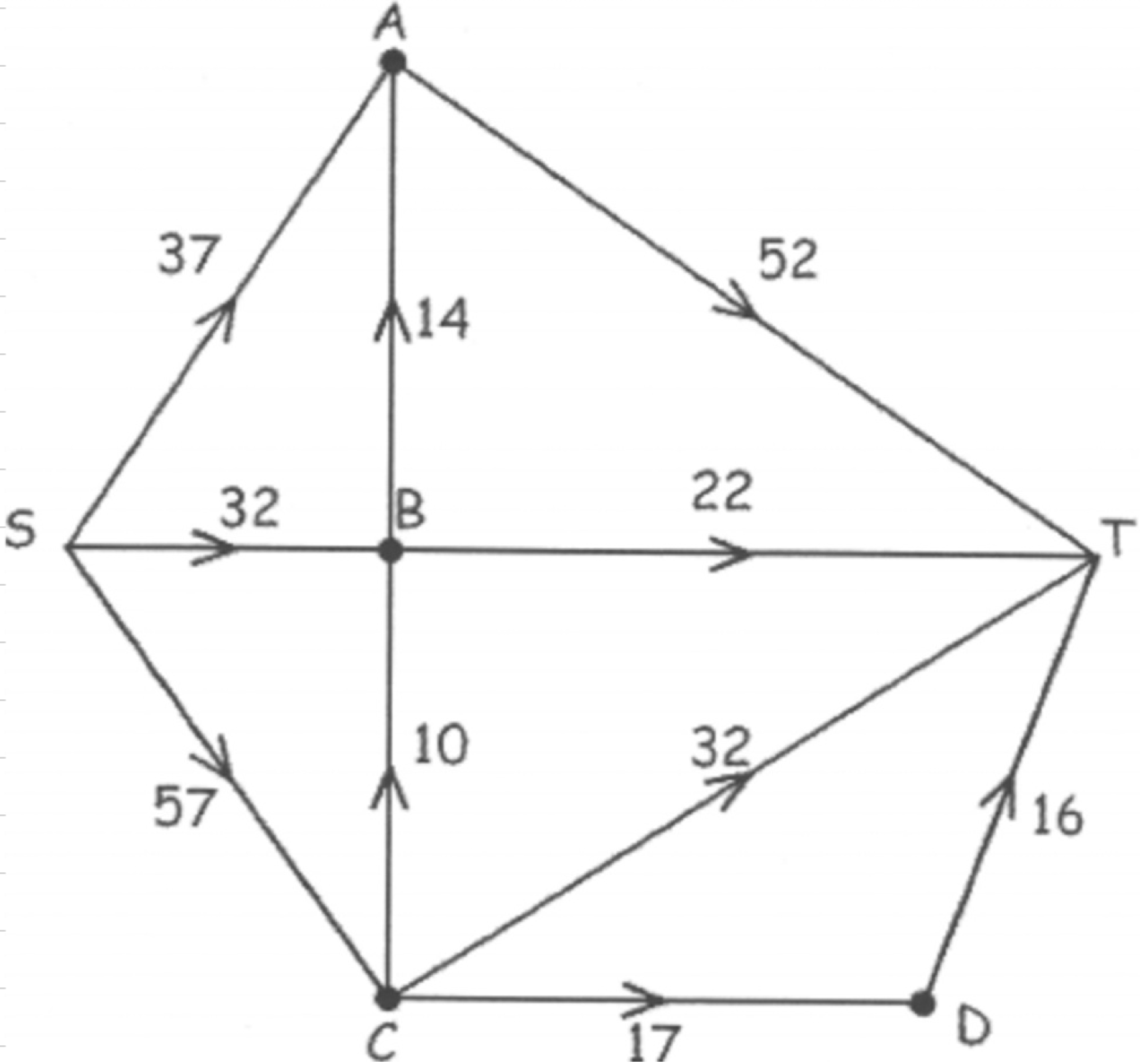
The capacity of the cut

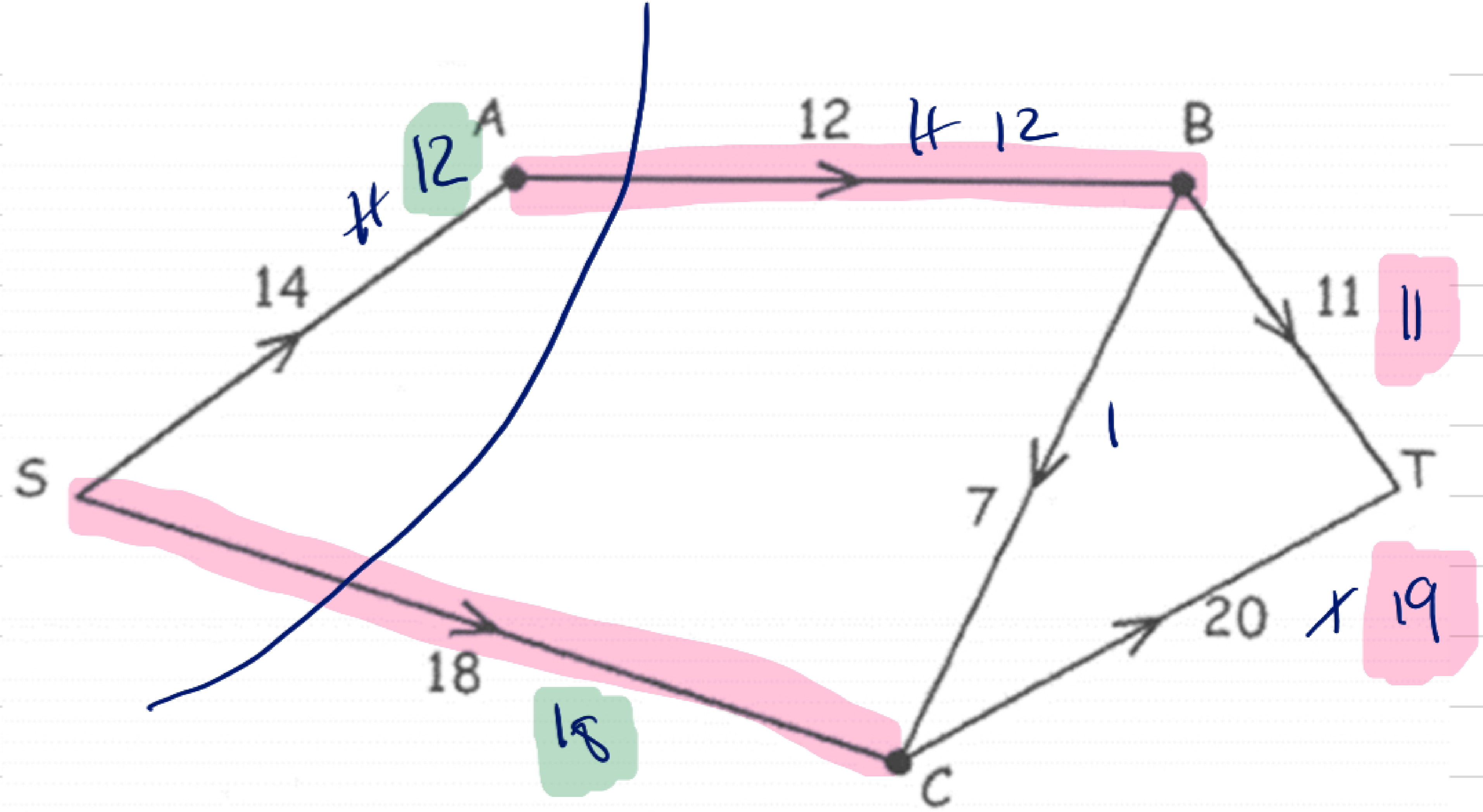
$$C(P, P^c) = 6 + 6 + 20 = 32$$



The figure below shows a flow network on which an st flow is shown. The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge. (Edges without boxed numbers have no flow being sent on them.) What is the value of this flow? Is this a maximum st flow in this graph? If not, find a maximum st flow. Find a minimum st cut. (Specify which vertices belong to the sets of the cut.)




A
B
C
D
S
T
37
52
14
32
22
10
32
57
16
17



$Val(f) = 12 + 18 = 11 + 19 = \boxed{30}$

$C(P, P^c) = 12 + 18 = \boxed{30}$

$\Rightarrow$ the flow is maximal & the cut is minimal.

Chromatic Polynomials

$G = G_1 \cup \ \cup G_n$ connected cp then

- $\chi_G(x) = \prod \chi_{G_i}(x)$

- $\chi_{G-e} - \chi_{G'_e} = \chi_G$

- $G = G_1 \cup G_2 \qquad G_1 \cap G_2 = K_n$

$$\chi_G = \frac{\chi_{G_1} \cdot \chi_{G_2}}{\chi_{K_n}}.$$

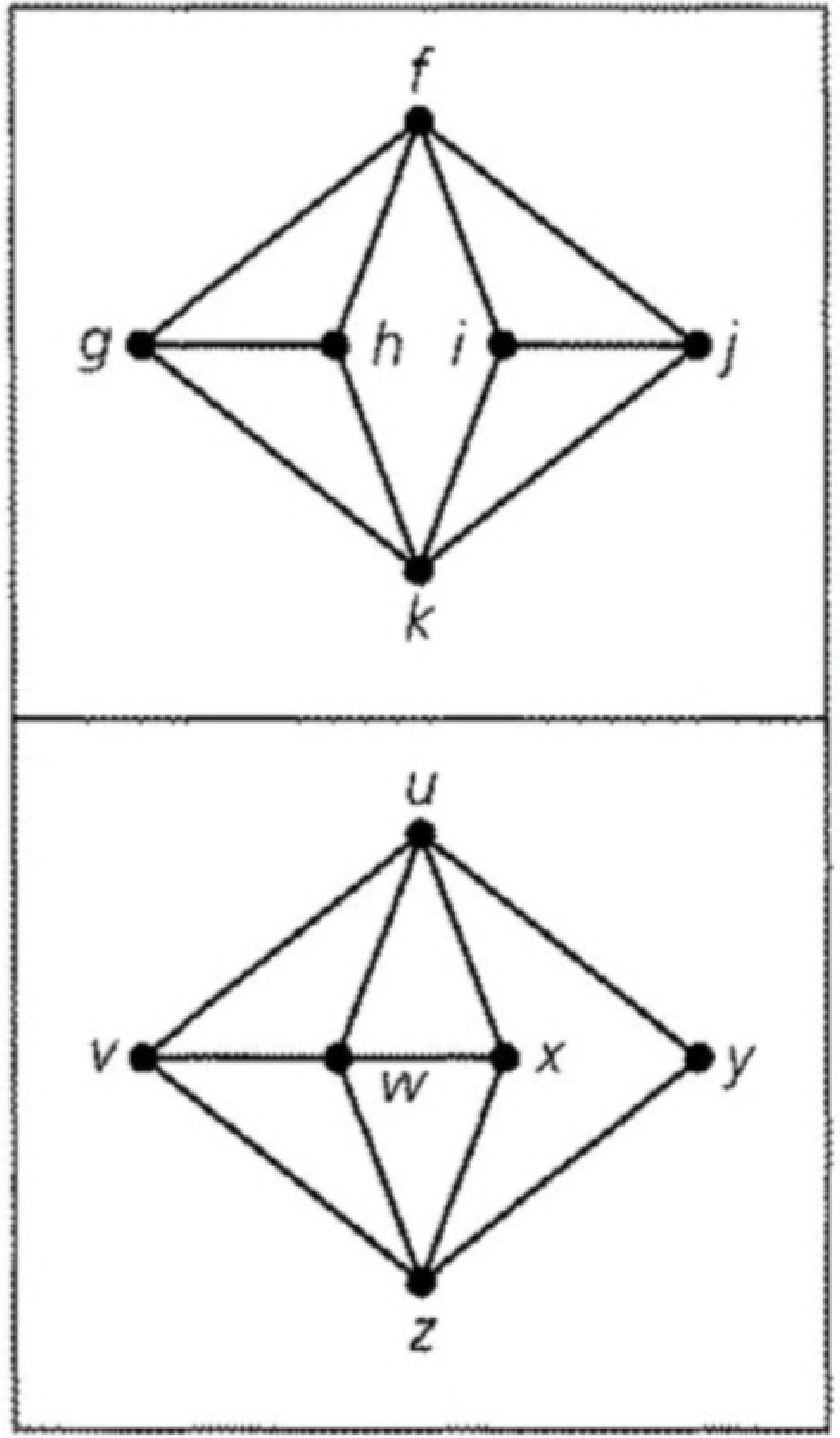
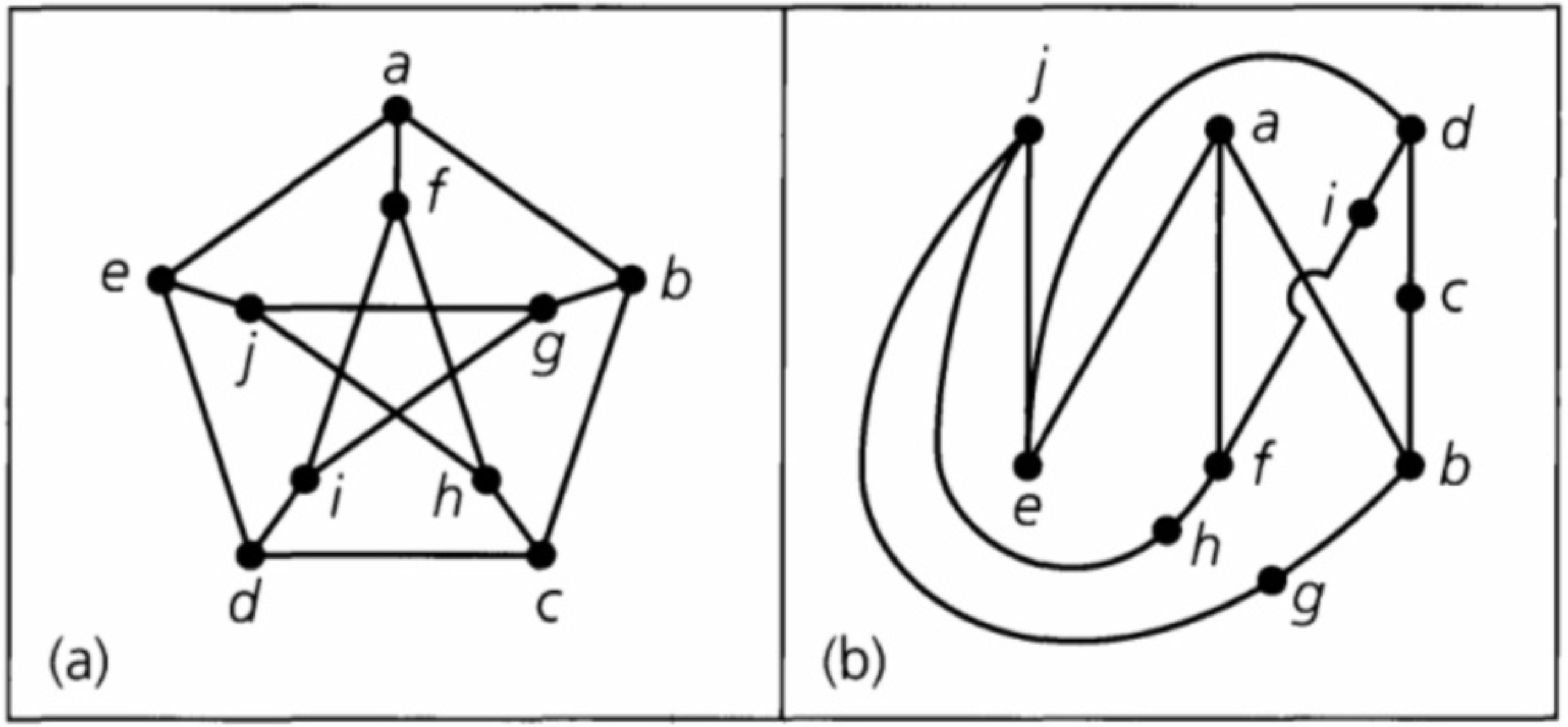


Figure 11.93


a
f
e
b
j
g
i
h
d
c
(a)
j
a
d
i
c
e
f
b
h
g
(b)

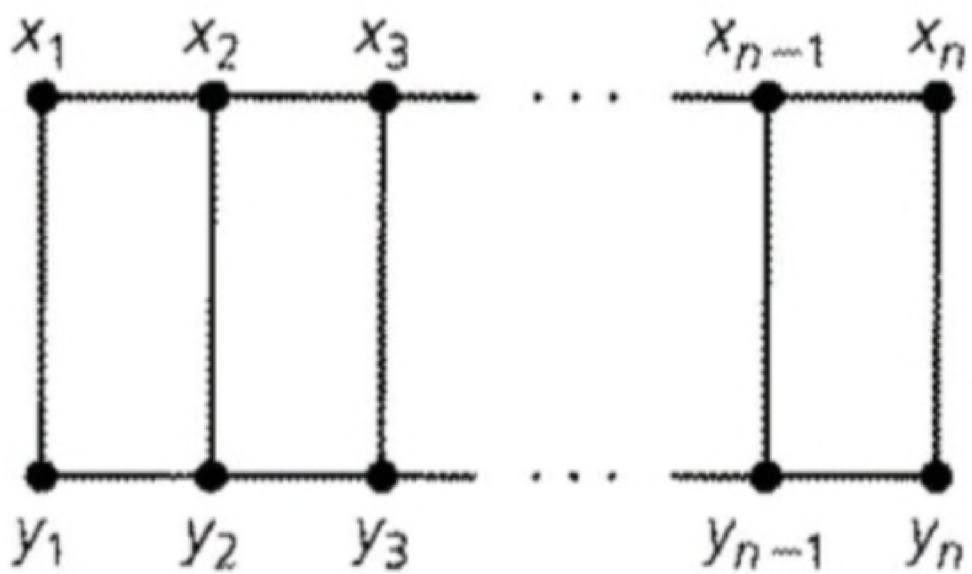
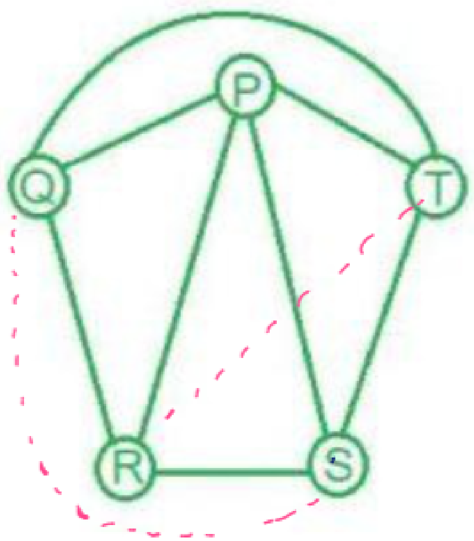


Figure 11.94

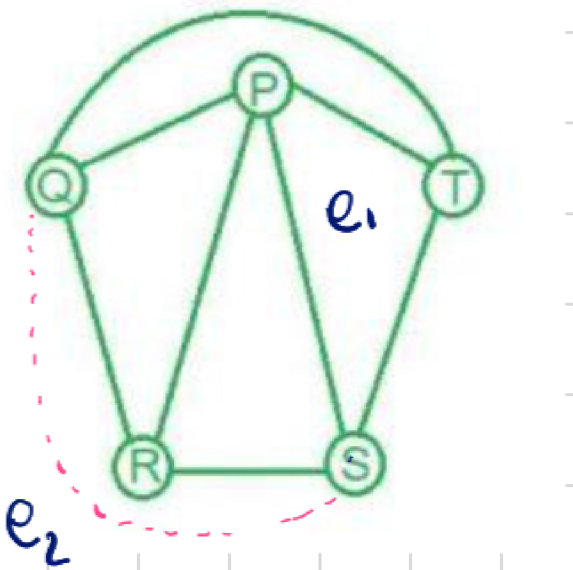


In order to color K_n
with λ color

→ you chose n of λ
color. you multiply by $n!$

$$\binom{\lambda}{n} n! = \frac{\lambda}{(\lambda-n)!}$$

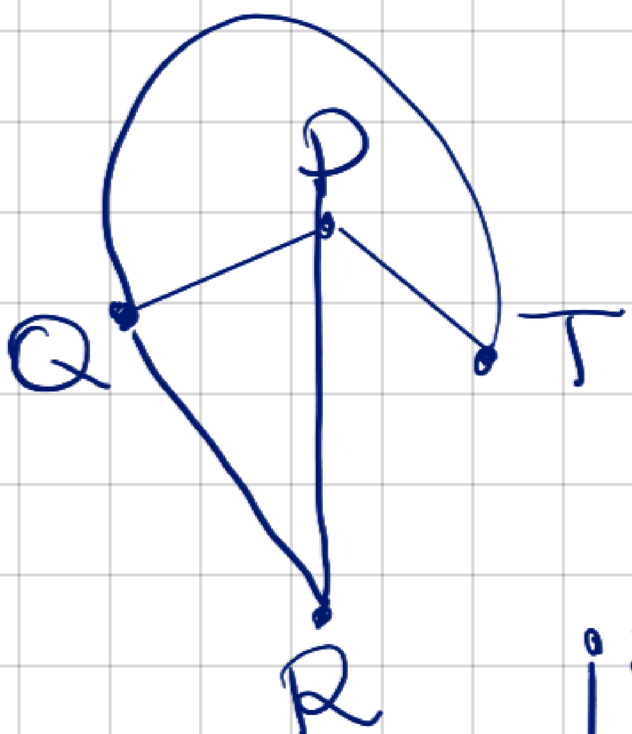
$$= \lambda(\lambda-1)(\lambda-2)\cdots(\lambda-n+1)$$

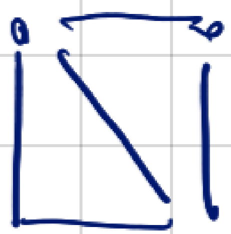


$$\chi_{K_5} = \chi_{K_n - e_1} - \chi_{K_{n\,e_1}}$$

$$= \chi_{K_5 - e_1} - \chi_{K_4}$$

$$= \boxed{\chi_{K_5 - e_1 - e_2}} - \underbrace{\chi_{(K_5 - e_1)_{e_2}}}_{K_4 - e} - \chi_{K_4}$$





$$\chi_{6} = \chi_{K_5} + \chi_{K_4 - e_1} + \chi_{K_4}$$

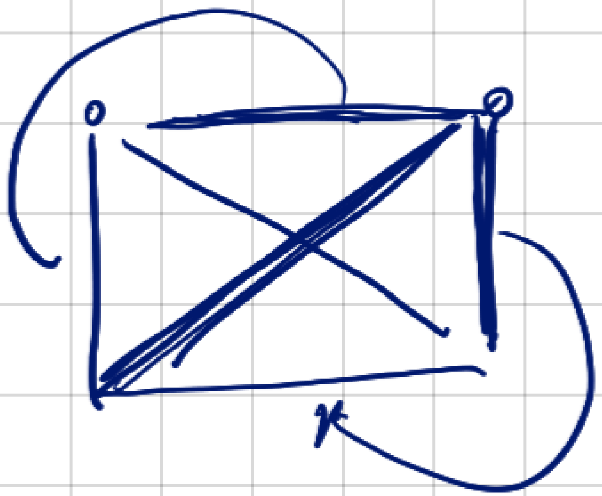
$$= \lambda(\lambda-1)(\lambda-2)(\lambda-3)(\lambda-4)$$

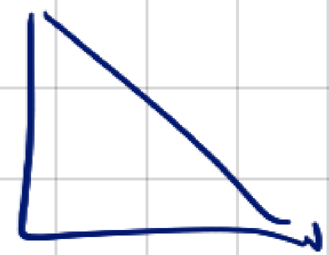
$$+ \lambda(\lambda-1)(\lambda-2)(\lambda-3) + \lambda(\lambda-1)(\lambda-2)$$

$$+ \lambda(\lambda-1)(\lambda-2)(\lambda-3)$$

Remember: this is not a generating function => in how many ways you can color it with k color → plug $k \to \lambda$

$$\chi_{K_4} = \chi_{K_4 - e_1} - \chi_{K_4 e}$$





$$\chi_{K_4} = \chi_{K_4 - e_1} - \chi_{K_3}$$

$$\chi_{K_4 - e_1} = \chi_{K_4} + \chi_{K_3}$$

$$= \lambda(\lambda-1)(\lambda-2)(\lambda-3) + \lambda(\lambda-1)(\lambda-2)$$

Hamilton Cycles

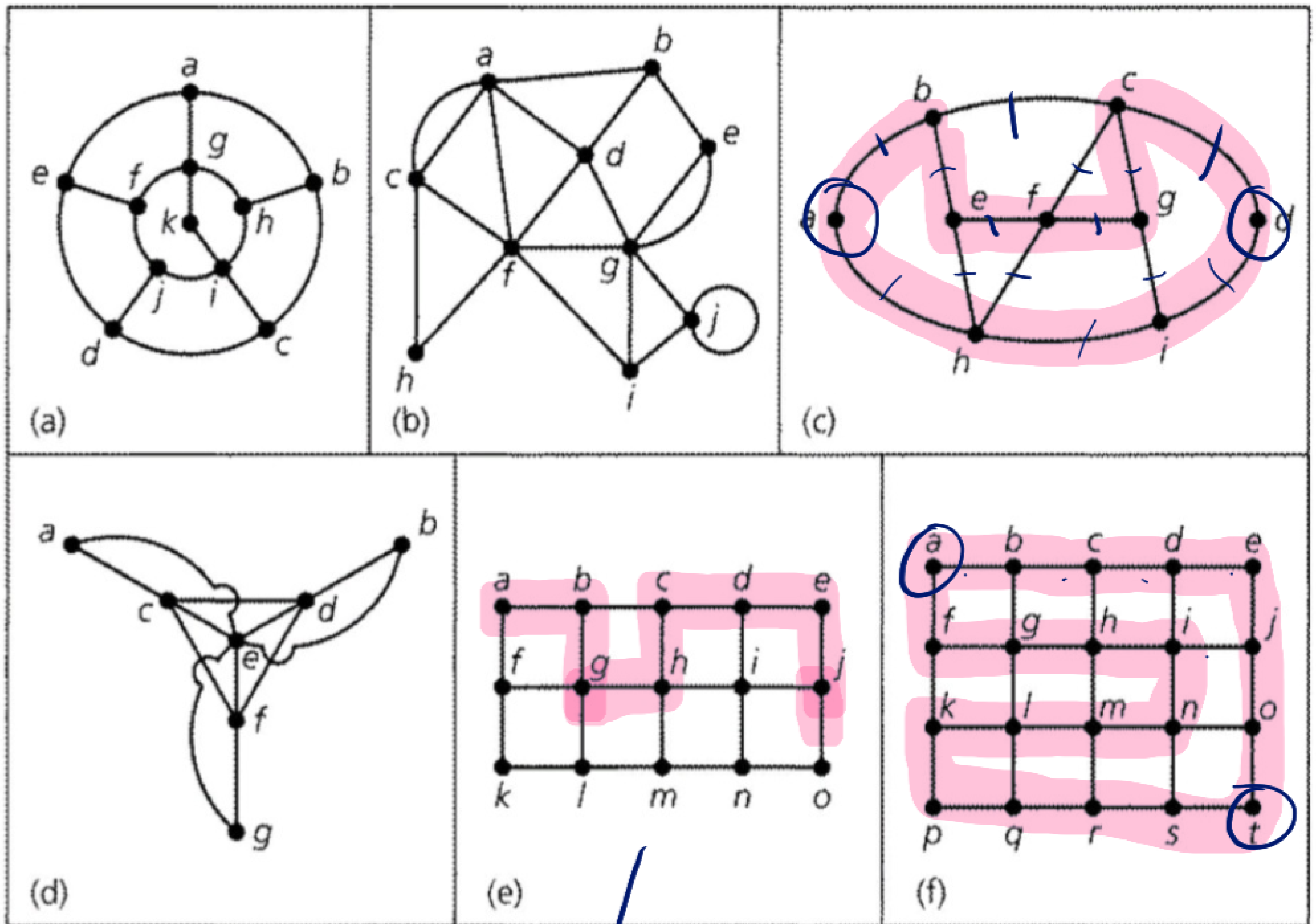


Figure 11.84

Not an HC

Recall

- $\deg(v) + \deg(w) \geq |V|$
- $|E(G)| \geq \binom{|V|-1}{2} + 2$

$\Downarrow$

G has an HC

(f) $|V| = 20$

$\deg(a) + \deg(t) = 4 < 20$

$|E(6)| = 16 + 15 = 31$

$\binom{19}{2} = \frac{19 \cdot \cancel{18}}{\cancel{2}} = 19 \cdot 9$

(C) · has 9 vertices

$$\deg(a) + \deg(d) = 4 < |V|$$

· $E(G) = 14$ edges

$$\binom{9-1}{2} = \frac{8!}{2!\,6!} = \frac{\cancel{8}\cdot 7}{\cancel{2}} = 4\cdot 7$$

$$= 28$$

$$14 < 28 + 2 = 30$$

Planar Graphs

$G=(V,E)$ connected & planar

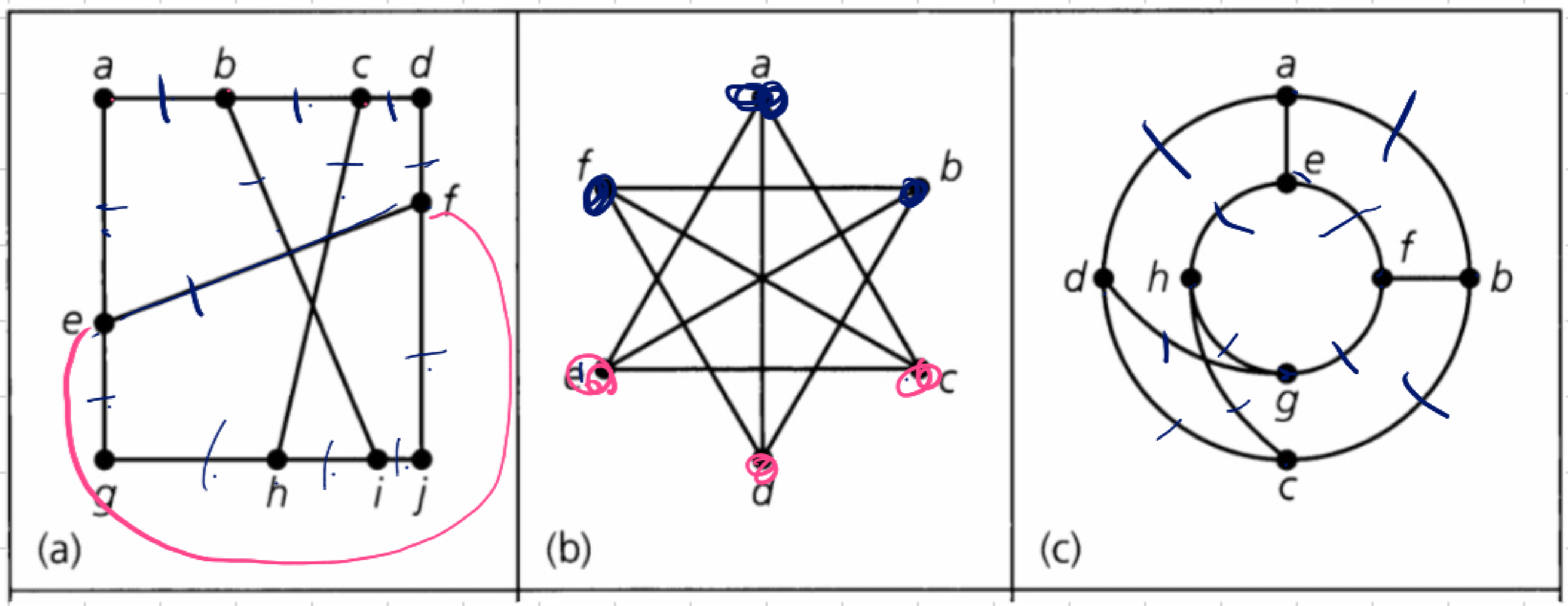
$$v-e+r=2$$

necessary condition.

$e \leq 3v-6$

bip $e \leq 2v-4$

G non planar $\iff$ has a subgraph iso to K_5 or $K_{3,3}$



(b) 6 vertices

9 edges $< 18-4$.

it can be plane.

(c) 8 vertices

10 edges

10 < 24 - 6   22

It could be planar

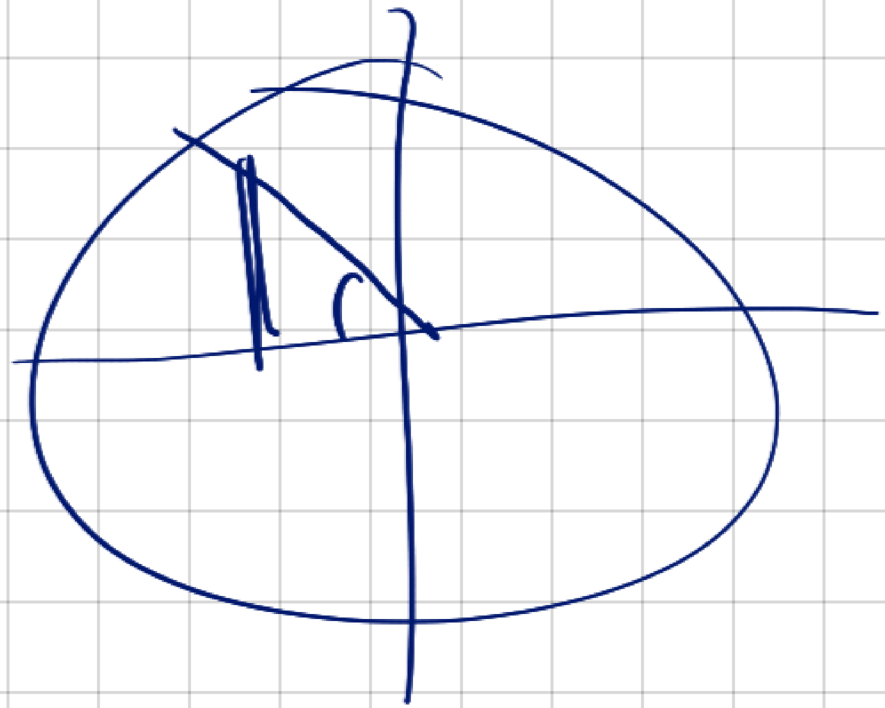
(a) $|V| = 10$

$|E| = 13$

$$3v - 6 = 30 - 6 = 24$$

$13 < 24 \Rightarrow$ it can be planar

$$\frac{-1+i\sqrt{3}}{2} = \cos\left(2\pi/3\right) + i\sin\left(\frac{2\pi}{3}\right)$$



$$1^n\left(A\cos\left(n\frac{2\pi}{3}\right) + B\sin\left(n\frac{2\pi}{3}\right)\right)$$

General solution of hom problem.

<u>Check</u> 1 is not a solution of the char equation

Recursion

$$a_{n+2} + a_{n+1} + a_n = 3 \cdot 1^n \qquad a_0 = 1$$

$$a_1 = 1$$

$$x^2 + x + 1$$

roots $\dfrac{-1 \pm \sqrt{-3}}{2} = \dfrac{-1 \pm i\sqrt{2}}{2}$

Solution of the homogeneous problem.

Educate guess $a_n^{(p)} = A$.

$$3A = 3 \Rightarrow A = 1$$

$$a_{n+2}^{(p)} + a_{n+1}^{(p)} + a_n^{(p)} = B$$

General solution

$$A\cos\left(n\frac{2\pi}{3}\right) + B\sin\left(n\frac{2\pi}{3}\right) + 1$$

$$\cos(2\pi/3) \quad \frac{\sqrt{3}}{2}\sin\left(n\frac{2\pi}{3}\right) + 1$$

$$A\cos(0) + B\cancel{\sin(0)} + 1 = 1$$

$$A = 1$$

$$\cos\left(\frac{2\pi}{3}\right) + B\sin\left(\frac{2\pi}{3}\right) \cancel{+1} = \cancel{1}$$

$$B \, \frac{\sqrt{3}}{2} = -\frac{1}{2}$$

$$B = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

With the method of gf:

$$a_{n+2} + a_{n+1} + a_n = 3$$

$$\sum_{n=0}^{\infty} a_{n+2} x^{n+2} + \sum_{n=0}^{\infty} a_{n+1} x^{n+2} + \sum_{n=0}^{\infty} a_n x^{n+2} = 3$$

$$f(x) - a_0 - a_1 + x\left(f(x) - a_0\right) + x^2\left(f(x)\right) = 3$$

$$f(x)\left(1 + x + x^2\right) - 2 - x = 3$$

$$f(x) = \frac{5-x}{1+x+x^2}$$

?

Mc Laurin expansion

Rook Polynomials

formula

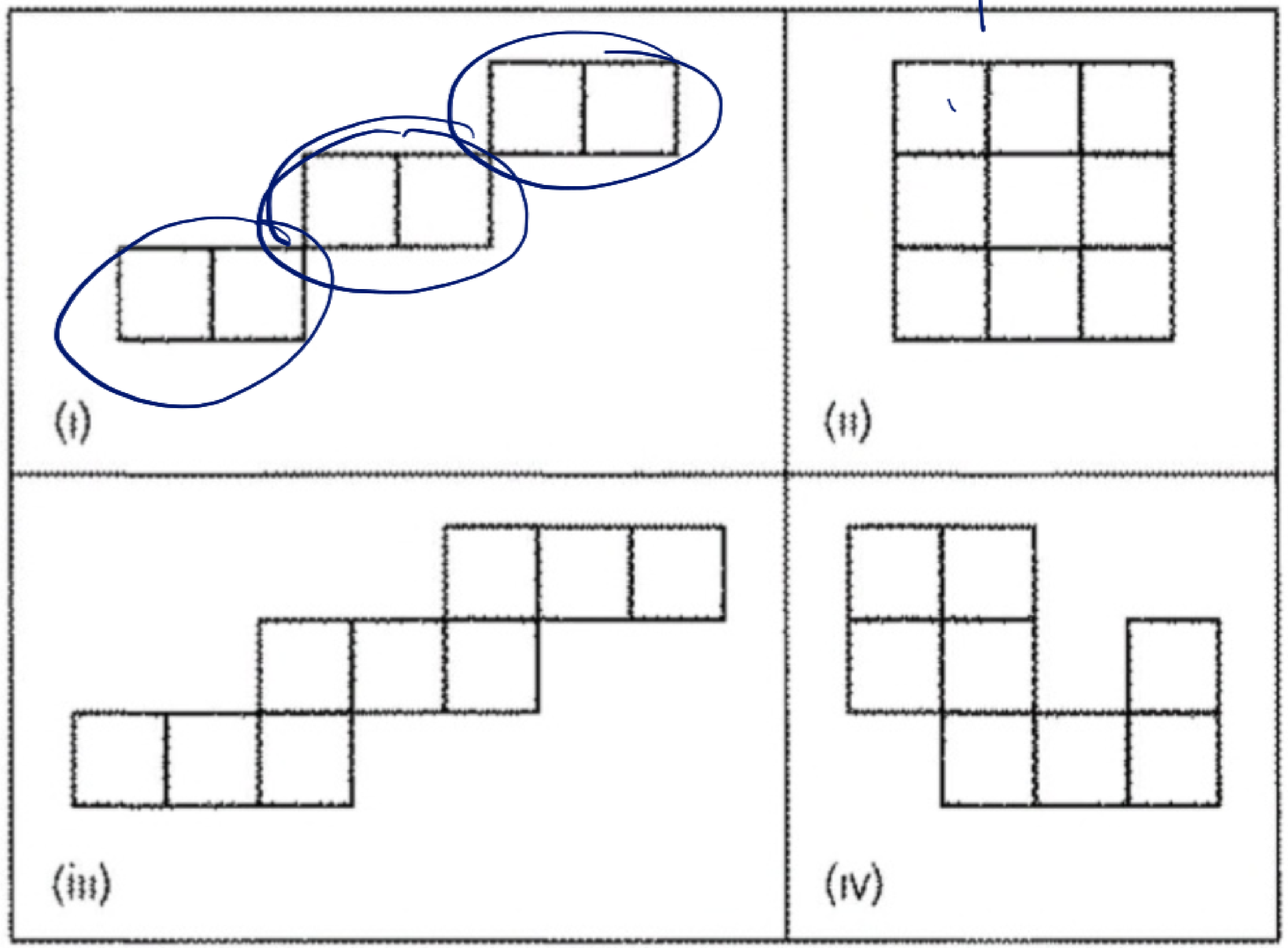
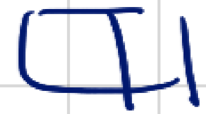


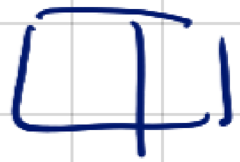
Figure 8.13

- $r(C,x) = r(C',x) + x \cdot r(C'',x)$

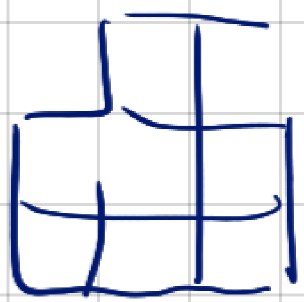
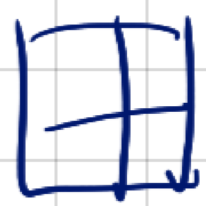
- $C_1 \cup C_2$ disj.

$$r(C,x) = r(C_1,x) \cdot r(C_2,x)$$

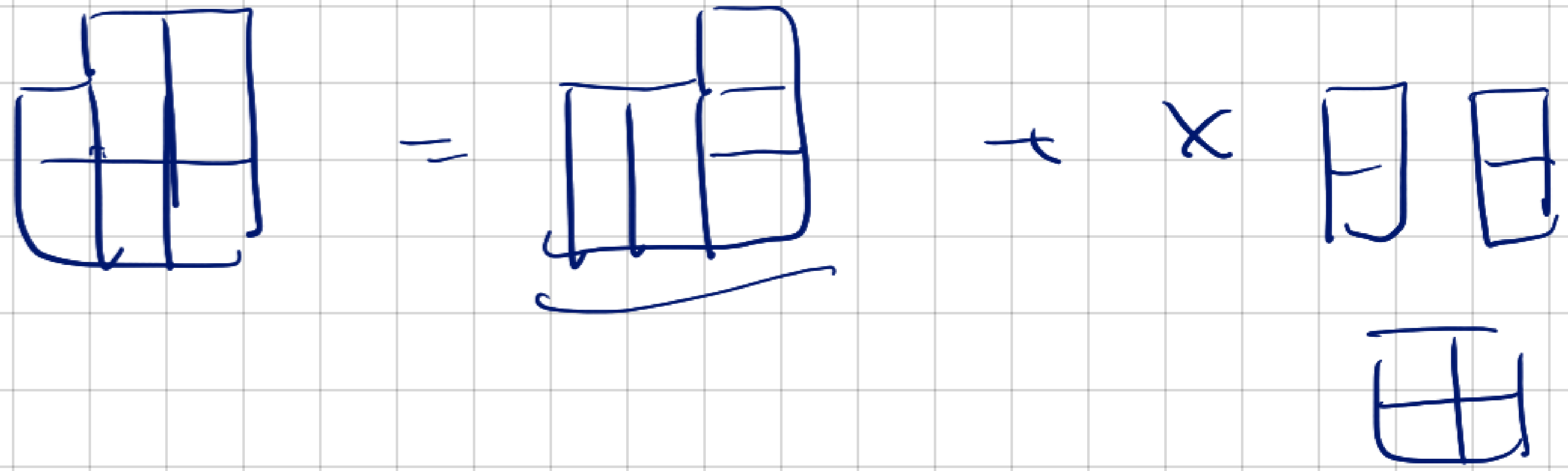
(a) $r(C\,a) = r\Big($  $, x\Big)^3$

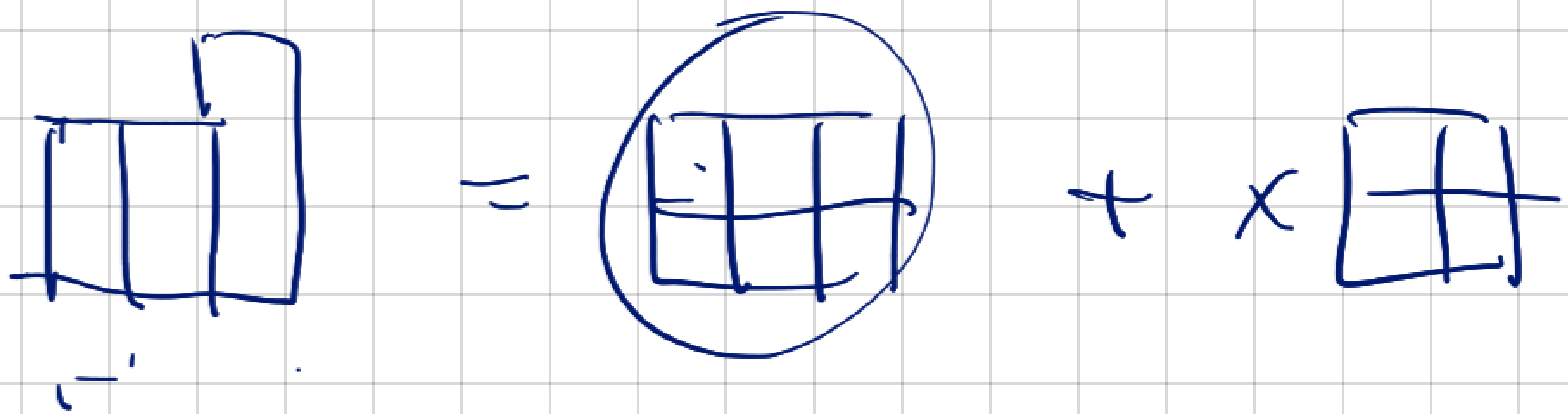
 $1 + 2x$

$= (1+2x)^3$

(b)  $+ x$ 

 $= 1 + 4x + 2x^2$





$$1 + 6x + 3x^2 + x(1+4x+2x^2)$$
$$+ x(1+4x+2x^2)$$
$$+ x(1+4x+2x^2)$$

Generating function

- Compute the generating function of

n^3

- Compute te exponential generating function of $n \cdot 2^{n+1}$

$$\sum \frac{n \cdot 2^{n+1} \cdot x^n}{n!} = 2 \sum \frac{n}{n!} (2x)^n$$

$$2 \sum_{1} \frac{m}{n!} (y)^{n}$$

$$= 2 \left(0 + \sum_{1}^{\infty} \frac{m}{n!} (y)^{n} \right)$$

$$= 2 \left(0 + y \sum_{1}^{\infty} \frac{m}{n!} (y)^{n-1} \right) \quad e^{y}$$

$$= 2 \left(0 + y \frac{d}{dy} \left(\sum_{1}^{\infty} \frac{1}{n!} y^{n} \right) \right)$$

$$2\left(ye^{y}\right)$$

$$= 2(2x)e^{2x}$$