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Department of Mathematics
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Final Exam
Mathematics for Economic and Statistical Analysis
MM1005, Fall term; 7,5 ECTS
Tuesday 30 September 2025

Instructions:
- During the exam you may not use any textbook, class notes, or any other supporting material.
- Non-graphical calculators will be provided for the exam by the department. Other calculators may not be used.
- In all solutions, justify your answers.
- Use natural language, not just mathematical symbols. Write clearly and legibly
- Mark your final answer to each question clearly by putting $\boxed{\text{a box}}$ around it.
- Begin each problem (1–6) on a new page and place the problem number at the top of the page. For problems with multiple parts, you do not need to start each part on a separate page.

Grades: Each solved problem is awarded up to 10 points. At least 30 points guarantee grade E, 36 for D, 42 for C, 48 for B, and 54 for A. Note that the problems are not ordered according to the difficulty!

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1. (a) Determine the value of the parameter a such that

$$\lim_{x\to 0} \frac{e^{ax}-1-3x}{x^2+x} = 5.$$

(b) Find the smallest possible positive real value ω such that, for all natural number N with $N \geq \omega$ it is satisfied that

$$\left|1-\sum_{j=1}^{N} 2^{-j}\right| < 2^{-2024}.$$

Solution (a) Applying L'Hopital's rule gives us that

$$\lim_{x\to 0} \frac{e^{ax}+1-3x}{x^2+x} = \lim_{x\to 0} \frac{(ae^{ax}-3)}{2x+1} = (a-3) = 5 \Leftrightarrow a = 8.$$

(b) The seek ω is

$$\omega = 2025.$$

2. Calculate, argumenting your answer, the integrals

$$\text{(a)} \quad \int x\left(2^x+3x-\frac{1}{x^2}\right) \mathrm{d}x \qquad\qquad \text{(b)} \int_0^{+\infty} \frac{x^3-4}{(x^4-16x+32)^{6/5}} \mathrm{d}x$$

Solution

$$\int x\left(2^x+3x-\frac{1}{x^2}\right) dx = \frac{2^x}{(\ln 2)^2}(x\ln 2-1)+x^3-\ln|x|+C, \qquad C\in\mathbb{R}.$$

$$\int_0^{+\infty} \frac{x^3-4}{(x^4-16x+32)^{6/5}} \mathrm{d}x = \lim_{s\to\infty} -\frac{5}{4\sqrt[5]{x^4-16x+32}}\Bigg|_0^s = \frac{5}{8}.$$

3. For a and b real numbers, consider the function

$$f(x) = e^{-x^2}(x^2-ax+b).$$

(a) Find the values of the parameters a and b such that the function has a critical point at $x = 1$, with $f(1) = 0$.

(b) For the values of $a = 2$ and $b = 1$, determine in which intervals is the function $f(x)$ increasing or decreasing.

Solution (a) The condition $f(1) = 0$ tells us

$$1 - a + b = 0.$$

Then we compute

$$f'(x) = e^{-x^2}(-a + (2 - 2b)x + 2ax^2 - 2x^3).$$

The condition $f'(1) = 0$ gives us

$$a - 2b = 0.$$

We solve the system

$$\begin{cases} 1 - a + b = 0 \\ a - 2b = 0 \end{cases}$$

and get $a = 2$, $b = 1$.

(b) We have

$$f'(x) = -2e^{-x^2}\left(x^3 - 2x^2 + 1\right) = -2e^{-x^2}\left((x-1)(x - \frac{1}{2}(1-\sqrt{5}))(x - \frac{1}{2}(1+\sqrt{5})\right)$$

A study of the sing of the polynomial term gives us that f is increasing at

$$(-\infty, \frac{1}{2}(1-\sqrt{5}) \cup (1, \frac{1}{2}(1+\sqrt{5}))$$

and decreasing

$$(\frac{1}{2}(1-\sqrt{5}, 1) \cup (\frac{1}{2}(1+\sqrt{5}), +\infty).$$

So there are no other stationary points, therefore $f(x)$ decreases in $(-\infty, -1)$ and increases in $(-1, +\infty)$.

4. The expression

$$y^3 + (x-8)y - 4 = 0,$$

defines y as a smooth function of x around the point $x = 6$: $y = y(x)$.

(a) Find all the possible real values of $y(6)$.

(b) Find the Taylor polynomial of order two of y about the point $x = 6$.

Solution Substituting in the expression and solving gives

$$y(6) = 2.$$

Implicit differentiation yields

$$y'(6) = -1/5, \qquad y''(6) = -1/125.$$

We infer that

$$p(x) = 2 - \frac{1}{5}(x-6) - \frac{1}{250}(x-6)^2.$$

Please, turn page $\longrightarrow$

5. Let $F(x,y) = -8x^2 - 2xy + 38x - y^2 + 10y - 28$.

   (a) Find all stationary points for this function and determine whether they are local maximum, minimum, or saddle points. Does the function have a global maximum/minimum for $(x,y) \in \mathbb{R}^2$? Argument your answer.

   (b) Sketch the set
   $$L = \{(x,y) : x \geq 1, y \geq -3, x+y = 0\}.$$
   and find the maximum and minimum value of F on L.

Solution The partial derivatives of f are

$$f'_x(x,y) = 38 - 16x - 2y, \qquad f'_y(x,y) = 10 - 2x - 2y.$$

So the only stationary point is

$$x = (2,3).$$

Calculating the Hessian gives

$$H = \begin{pmatrix} -16 & -2 \\ -2 & -2 \end{pmatrix}$$

which satisfies that for all $(x,y) \in \mathbb{R}^2$

$$\det H = 28 > 0 \qquad \text{and} \qquad H_{11} = -16 < 0,$$

so we know that the stationary point is a global maximum.

Notice that $F(x,0) = -8x^2 + 38x - 28$ and that $\lim_{x\to+\infty} f(x,0) = -\infty$, so the function has no global minimum.

For the second part of the exercise, we need to study

$$f(x) = F(x,-x) = -28 + 28x - 7x^2, \quad x \in [1,3].$$

This function has a critical point at $x = 2$, with $f(2) = F(2,-2) = 0$. Evaluating at the extremes of the interval yields

$$f(1) = -7, \qquad f(3) = -7.$$

so

$$\min_{x\in I} f(x) = -7, \qquad \max_{x\in I} f(x) = 0.$$

6. Write the following system in a matricial form (i.e. write the system as $A \cdot v = b$, where A is a 3×3-matrix, and v, b are two 3×1 matrices):

$$\begin{cases} 4x_1 + 2x_2 + 2x_3 = 14 \\ 7x_1 - 2x_2 + 8x_3 = 27 \\ x_1 + x_3 = 4 \end{cases}$$

Calculate the determinant of A and use the Gaussian elimination method to solve the system.

Solution Defining A as the matrix

$$A = \begin{pmatrix} 4 & 2 & 2 \\ 7 & -2 & 8 \\ 1 & 0 & 1 \end{pmatrix}$$

we can write the system as

$$A. \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ 27 \\ 4 \end{pmatrix}$$

A direct calculation gives that

$$\begin{aligned}\det A &= 4\begin{vmatrix} -2 & 8 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 7 & 8 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 7 & -2 \\ 1 & 0 \end{vmatrix} \\ &= 4(-2) - 2(7-8) + 2(2) = -8+2+4 = -2,\end{aligned}$$

and a Gaussian elimination yields that

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$