STOCKHOLM UNIVERSITY
Department of Mathematics
Examiner: Gregory Arone

Examination for
MM3001: Matematiska metoder för ekonomer
15th March 2021

Time: 8:00-13:00
Instructions:
- During the exam you MAY NOT use textbooks, class notes, or any other supporting material.
- Use of calculators is permitted for performing calculations. The use of graphic or programmable features is NOT permitted.
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear solutions even if the answer is correct.
- Use natural language when appropriate, not just mathematical symbols.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting $\boxed{\text{A BOX}}$ around it.
- The solutions should be uploaded onto the course's webpage no later than 13:30

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

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1. Let k be a fixed number. Consider the following system of linear equations, with unknowns x, y, z, and w.

$$\begin{aligned} 3x + y - 2z + w &= 5 \\ x - y - z + w &= 6 \\ 5x + 3y - 3z + kw &= 4 \end{aligned}$$

   (a) Use Gaussian elimination to find for which value of k the system of equations has at least one solution. (2p)

   (b) For the value of k that you found in part (a), describe the general solution. Your answer should express x and y in terms of z and w. (2p)

   (c) Find the solution with $z = -1$, $w = 2$. (1p)

2. Consider the equation

$$y^2x^2 + \frac{x}{\sqrt{y}} = 6.$$

   This equation defines a curve in the plane. Notice that $(2, 1)$ is a solution

   (a) Use implicit differentiation to find the slope of the tangent line to this curve at the point $(2, 1)$. (3p)

   (b) Find the equation of the tangent line at the point $(2, 1)$. (2p)

3. (a) Compute the integral $\int (t^2 + 1)e^{t^3+3t}\, dt$ (as a function of t). (2p)

(b) Find a number a for which $\int_a^0 \sqrt{1-x}\,dx = \frac{14}{3}$ (3p).

4. Let a be some fixed number. Consider the function $f(x,y) = x^2 + axy + y^2 - 4x - ax - 2y - 2ay$.
   (a) Prove that $(2,1)$ is a critical point of f, for every a. (2p)
   (b) Find the second derivatives f''_{xx}, f''_{xy} and f''_{yy}. You answer may depend on a. (1p)
   (c) Find for which a (if any) the point $(2,1)$ is a local maximum, for which a it is a local minimum, and for which it is neither. [The formula at the end of the test may help.] (2p)

5. Consider the function
$$f(x,y) = 3x^2 - 12x + 3y^2 - 4y.$$
   Let D be the domain defined by the inequalities $0 \le y$ and $x^2 + y^2 \le 10$.
   Find the global maximum and the global minimum of $f(x,y)$ on D. Remember to show clearly all the necessary steps. (5p)

6. Consider the function $f(x) = \sqrt{\ln(x^2 - x - 2)}$.
   (a) Determine the domain of definition of f. (2p)
   (b) Determine the local extreme points of f (if any). (1p)
   (c) Determine where f is increasing and where f is decreasing. (2p)

Formulas

The second derivative criterion for a function of two variables $f(x,y)$ depends on the determinant $\det \begin{bmatrix} f''_{xx} & f''_{xy} \\ f''_{xy} & f''_{yy} \end{bmatrix}$. It says the following: If, at a critical point

- $\det \begin{bmatrix} f''_{xx} & f''_{xy} \\ f''_{xy} & f''_{yy} \end{bmatrix} > 0$ and $f''_{xx} > 0$ then f has a local minimum at this critical point.
- $\det \begin{bmatrix} f''_{xx} & f''_{xy} \\ f''_{xy} & f''_{yy} \end{bmatrix} > 0$ and $f''_{xx} < 0$ then f has a local maximum at this critical point.
- $\det \begin{bmatrix} f''_{xx} & f''_{xy} \\ f''_{xy} & f''_{yy} \end{bmatrix} < 0$ then f has neither a local maximum nor a local minimum at this critical point.

GOOD LUCK!