MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
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Tentamensskrivning i
Mathematics Methods for Economist MM3001
2022-10-23

Instructions:

- In case of ambiguity, one has to refer to the ENGLISH version of this exam.
- During the exam you MAY NOT use textbooks, class notes, or any other supporting material apart from the formula sheet given to you.
- Use of calculators is permitted for performing calculations. The only approvd calculator are those that The use of graphic or programmable features is NOT permitted.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear and wrong argument, even if the final answer is correct.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting A BOX around it.

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

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(1) (5pt) Compute the degree 3 Taylor polynomial of the function $f(x) = x\ln(1-x^2)$, around the point $x_0 = 0$, and use it to give an approximation of $f(0.1)$.

(2) A company has to buy a new office for \$500 000. They are offered the following payment plan. They have to give a down payment of \$200 000 now. The rest will be due in 10 years, with an yearly interest rate of 6% compounded monthly. This can be payed monthly, with the first installment due after one month.
   (a) (1 pts) Let m denote the monthly fee for the payment plan. Find a formula involving m yielding the amount of the debt after the second payment.
   (b) (1 pts) Find the monthly fee for the payment plan (that is how much is due every month to pay the debt in 10 years -120 monthly payments).

   Suppose that the company pays the down payment and the monthly fee from an account with \$700 000 invested with an yearly interest 4%, compounded monthly.
   (c) (1 pt) Compute how much money is left in the account after 3 months and after 1 year.
   (d) (2 pt) Determine if paying in cash is less advantageous than the paying plan.

(3) Consider the function $f(x) = \frac{x^2+8}{x+1}$.
   (a) (2pt) Find all the critical points and determine their type.
   (b) (2pt) Find where the function is increasing or decreasing and concave or convex.
   (c) (1pt) Find the max and min value of the function on the interval $[1, 3]$.

(4) Compute the following integrals:

(a) (3pt) $\int \left(\frac{3}{\sqrt{t}} e^{\sqrt{t}} + \frac{3}{2t+1} \right) dt,$

(b) (2pt) $\int_0^7 (y+2)^2 \ln(y+2) dy.$

(5) Consider the matrix

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 7 & c & 2 \\ c & 6 & 1 \end{pmatrix}$$

(a) (2 pt) Compute the determinant of A, $|A|$ as a function of c.

(b) (1 pt) Find all the values of c for which A is not invertible.

(c) (2 pt) Determine whether the following linear system has 1, 0, or infinitely many solutions. In case there is just one solution, find this.

$$\begin{cases} +2x & & +z & = & 1 \\ +7x & +y & +2z & = & 3 \\ x & +6y & +z & = & 3 \end{cases}$$

(6) Consider the two variables function

$$f(x,y) = x(y^2 - 1)$$

(a) (2pt) Find all the critical points of $f(x,y)$ and determine their type.

(b) (2pt) Consider now D, the circle

$$\{(x,y) | x^2 + y^2 \leq 1\}.$$

Determine the max and min value taken by $f(x,y)$ on the *boundary* of the D (that is when $x^2 + y^2 = 1$).

(c) (1 pt) Determine the minimum and the maximum value of $f(x,y)$ on D.

GOOD LUCK!!!

Svenska Texten

(1) (5pt) Berk̈na grad 3 Taylor polynom till funtionen $f(x) = x\ln(1-x^2)$, kring $x_0 = 0$, och använda det för att approximera $f(0.1)$.

(2) Ett företag vill kópa ett nytt kontor för \$500 000. De kan betalar nlgit den följande planen: de behöver betala \$200 000 nu, och resten i 10 år med årlig ränta på 6% beräknad månadsvis.
   (a) (2 pt) La m vara den månadsavgiften. Hitta en formel som ange skulden efter den adnra betalningen.
   (b) (2 pt) Beräkna lånets månadsavgiften.

   Antar att företaget betalar avgifte från en konto med \$700 000 på och årlig ränta på 4% beräknad månadsvis.
   (c) (1 pts) Beräkna hur mycket finns i kontoen efter 3 månder och efter ett år.
   (d) (2 pt) Bestäm om att betala kontant är mindre fördelaktigt än betalningsplanen.

(3) Betrakar funktionen $f(x) = \frac{x^2+8}{x+1}$.
   (a) (2pt) Hitta alla de kritiska punkterna och bestäm deras typ.
   (b) (2pt) Bestäm var funktionen är vaxande eller evtagende, conkave eller convex.
   (c) (1pt) Bestäm de max och min värderna till funktionen i intervallen $[1, 3]$.

(4) Beräkna följande integraler:
   (a) (3pt) $\int \left(\frac{3}{\sqrt{t}} e^{\sqrt{t}} + \frac{3}{2t+1} \right) dt,$
   (b) (2pt) $\int_0^{+7} (y+1)^2 \ln(y+2) dy.$

(5) Betrakta matrisen

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 7 & c & 2 \\ c & 6 & 1 \end{pmatrix}$$

   (a) (2 pt) Beräkna determinanten till A, $|A|$ som en funktion av c.
   (b) (1 pt) Hitta alla värderna tillc c var A inteär invertibara.
   (c) (2 pt) Best"am om fóljande system har 0, 1 och oendliga många lössningar. Om det finns bara 1, ange det.

$$\begin{cases} +2x & & +z & = & 1 \\ +7x & +y & +2z & = & 3 \\ x & +6y & +z & = & 3 \end{cases}$$

(6) Betrakta följande funktionen i två varibler:

$$f(x, y) = x(y^2 - 1)$$

   (a) (2pt) Hitta alla kritiska punkterna för $f(x, y)$ och bestäm deras typ;
   (b) (2pt) Btrakta nu

$$D = \{(x, y) | x^2 + y^2 \leq 1\}$$

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Hitta den max och min värden som f tar vid grensen av D.

(c) (1 pt) Bestäm de max och min värden till $f(x, y)$ på D.

Lycka Till!!!