MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Sofia Tirabassi

Tentamensskrivning i
Mathematics Methods for Economist MM3001
2024-08

Instructions:

- In case of ambiguity, one has to refer to the ENGLISH version of this exam.
- During the exam you MAY NOT use textbooks, class notes, or any other supporting material apart from the formula sheet given to you.
- Use of calculators is permitted for performing calculations. The only approved calculator are those that The use of graphic or programmable features is NOT permitted.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear and wrong argument, even if the final answer is correct.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting A BOX around it.

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

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(1) (5pt) Compute the degree 3 Taylor polynomial of the function $f(x) = \ln(1+x^3)$, around the point $x_0 = 0$, and use it to give an approximation of $f(0.1)$.

(2) Consider the curve in the plane given by the following equation

$$x^3y + xy^3 = C,$$

where C is an arbitrary constant.
   (a) (1 pts) Find the value of C such that the curves passes through the point $(1, 1)$.
   (b) (2 pts) With $C = 2$ find the slope of the tangent to the curve in the point $(1, 1)$.
   (c) (1 pt) Find the equation of the tangent to the curve in the point $(1, 1)$.

(3) Consider the function $f(x) = \ln(x^5 - 50000x)$.
   (a) (2pts) Find where the function is increasing or decreasing. Find all the critical points, make sure that they are in the domain of f and determine their type.
   (b) (1pt) Find the maximum and minimum values of the function on the interval $[9, 10]$.
   (c) (2 pts) Compute $\lim_{x\to+\infty} f(x)$ and sketch the graph of $f(x)$.

(4) Compute the following integrals:
   (a) (2pt) $\int \left(t^3e^{-t^2} + 4t^{-\frac{7}{5}}\right) dt$,
   (b) (3pt) $\int_0^1 \ln(x^4+1)x^3dx$.

(5) Consider the matrix

$$A = \begin{pmatrix} -8 & c & 2 \\ -4 & 0 & 1 \\ c & 6 & 1 \end{pmatrix}$$

(a) (2 pt) Compute the determinant of A, $|A|$ as a function of c.
(b) (1 pt) Find all the values of c for which A is not invertible.
(c) (2 pt) Determine whether the following linear system has 1, 0, or infinitely many solutions. In case there is just one solution, find this.

$$\begin{cases} -8x & -4y & +2z & = & 2 \\ -4x & & +z & = & 1 \\ -4x & +6y & +z & = & 1 \\ x & +y & +z & = & 1 \end{cases}$$

(6) Consider the two variables function

$$f(x, y) = y^2 - 2x^2y - y + 200$$

(a) (2pt) Find all the critical points of $f(x, y)$ and determine their type.
(b) (2pt) Consider now

$$D = \{(x, y) | -1 \leq x \leq 1, 0 \leq y \leq 1 - x^2\}.$$

Determine the maximum and minimum values taken by $f(x, y)$ on the *boundary* of the D.
(c) (1 pt) Determine the minimum and the maximum value of $f(x, y)$ on D.

GOOD LUCK!!!

Svenska Texten

(1) (5pt) Beräkna det tredje graden Taylors polynom till funktionen $f(x) = \ln(1 + x^3)$, omkring punkten $x_0 = 0$. Använd det för att approximera $f(0.1)$.

(2) Antar kurven i planen med ekvation:

$$x^3y + xy^3 = C,$$

där C är en godtycklig konstant.
   (a) (1 pts) Hitta värdent till C sådan att kurver går genom punkten med koordinater $(1, 1)$.
   (b) (2 pts) Med $C = 2$ beräkna lutningen till kurven i punkten $(1, 1)$.
   (c) (1 pt) Ange ekvationer of den linjen som är tangenten til kurvan i punkten $(1, 1)$.

(3) Betrakta funktionen $f(x) = e^{-x^3+30000x}$.
   (a) (2pts) Hitta var funktionen är växande eller avtagende. Hitta all kritiska punkter och bestem deras typ.
   (b) (1pt) Hitta den största och den minsta och minimum värden som funktionen tar i intervallen $[-101, -99]$.
   (c) (2 pts) Beräkna $\lim_{x\to\pm\infty} f(x)$ och rita grafen till $f(x)$.

(4) Beräkna de följade integralerna
   (a) (2pt) $\int \left(t^3e^{-t^2} + 4t^{-\frac{7}{5}}\right) dt,$
   (b) (3pt) $\int_0^1 \ln(x^4 + 1)x^3 dx.$

(5) Betrekta matrisen

$$A = \begin{pmatrix} -8 & c & 2 \\ -4 & 0 & 1 \\ c & 6 & 1 \end{pmatrix}$$

   (a) (2 pt) Beräkn determinanten till A, $|A|$ som en funktioner av c.
   (b) (1 pt) Hitta alla värden till c sådana att A int är invertibara.
   (c) (2 pt) Besätm hur många lösningen (1, 0 eller odliga många) har den följande system. Om det finns bara ett, räkna det.

$$\begin{cases} -8x & -4 & +2z & = & 2 \\ -4x & & +z & = & 1 \\ -4x & +6y & +z & = & 1 \\ x & +y & +z & = & 1 \end{cases}$$

(6) Betrkata den följnande funktionen i tva variabler

$$f(x, y) = y^2 - 2x^2y - y + 200$$

   (a) (2pt) Hitta alla den kritiska punkter och bestäm deras typ.
   (b) (2pt) La

$$D = \{(x, y)| - 1 \le x \le 1, 0 \le y \le 1 - x^2\}.$$

   Beräkna den mstörsta och den minsta värden till $f(x, y)$ på *gränsen* till D .
   (c) (1 pt) Beräkna den största och minsta värden till $f(x, y)$ på D.

Lycka Till!!!