Exercise 1

$f(x) = \ln(1+x^3)+3$ $\qquad f(0)=3$

$f'(x) = \frac{3x^2}{1+x^3}$ $\qquad f'(0)=0$

$f''(x) = 6x\frac{1}{1+x^3} - \frac{3x^2\,3x^2}{(1+x^3)^2}$

$= 6x\frac{1}{1+x^3} - \frac{9x^4}{(1+x^3)^2}$ $\qquad f''(0)=0$

$f'''(x) = 6\frac{1}{1+x^3} + 6x\left(\text{something}\right)$

(*) $\qquad + 32x^3\left(\text{something}\right)$ $\qquad f'''(0)=6$

$\qquad + 9x^4\left(\text{something}\right)$

$p(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3$

$= 3 + x^3$

$p(0.1) = \frac{3001}{1000} = 3.001$ is an approximation of $f(0.1)$

(*) You need just to compute $f'''(0)$ so I do not need to compute summand that one multiplied by x

Exercise 2

(a) we set $x=1$ $y=-\sqrt{2}$ in

$$x^4+y^2=c$$

we get

$1+2=C$ that is $\boxed{C=3}$

(b) we derive implicitly considering
$y=y(x)$

$$4x^3+2y(x)y'(x)=0$$

we set $x=1$ $y(x)=-\sqrt{2}$

$$4-2\sqrt{2}\,y'(1)=0$$

$$\boxed{y'(1)=\frac{4}{2\sqrt{2}}=\sqrt{2}}$$

(c) The line is of the form
$y=mx+k$ From (b) $m=\sqrt{2}$

$y=\sqrt{2}x+K$ we set $x=1$ $y=\sqrt{2}$

$-\sqrt{2}=\sqrt{2}+K$ $K=-2\sqrt{2}$

$$\boxed{y=\sqrt{2}x-2\sqrt{2}}$$

Exercise 3

(a) $f(x) = 3000 \ln(1-x^2)$ is defined when

$$1-x^2 > 0 \iff x \in (-1,1)$$

(b) $f'(x) = -\dfrac{6000x}{1-x^2}$

Note that $1-x^2 > 0$ on the domain

$f'(x) \geq 0$ if $x \leq 0$

	-1		0		1
$f'(x)$		$+$		$-$	
$f(x)$		$\nearrow$		$\searrow$	

The function is increasing for $x \leq 0$ and decreasing for $x > 0$. There is a critical pt in $x=0$ which is a local max.

(c) $f''(x) = -6000 \left(\dfrac{(1-x^2)+2x^2}{(1-x^2)^2} \right)$

$$= -6000 \left(\frac{1+x^2}{(1-x^2)^2} \right) < 0 \text{ always}$$

f is concave on the whole domain

(d) $f(0) = 0$ MAX

$f(-\frac{1}{2}) = 3000 \ln\left(\frac{3}{4}\right) < 0$ MIN

$f(\frac{1}{3}) = 3000 \ln\left(\frac{8}{9}\right) < 0$

$$\ln\left(\frac{8}{9}\right) > \ln\left(\frac{3}{4}\right)$$

Exercise 4

(a) $$\int \frac{8x^4}{(x^5+2)^{100}} + e^{\frac{2}{11}x}\, dx$$

$$= \int \frac{8x^4}{(x^5+2)^{100}}\, dx + \int e^{2/11 x}\, dx$$

$$u=(x^5+2) \qquad v = \frac{2}{11}x$$

$$du = 5x^4\, dx \qquad dv = \frac{2}{11} dx$$

$$\frac{8}{5}\int \frac{du}{(u)^{100}} + \frac{11}{2}\int e^v dv$$

$$= \frac{8}{5} \frac{1}{-100+1} u^{-99} + C_1 + \frac{11}{2} e^v + C_2$$

$$= -\frac{8}{5} \frac{1}{99} (x^5+2)^{-99} + \frac{11}{2} e^{\frac{2}{11}x} + C$$

(b) $$\int_0^1 x e^x dx = \left[x e^x\right]_0^1 - \int_0^1 e^x dx$$

$$= e - \left[e^x\right]_0^1$$

$$= e - e + 1 = 1$$

Exercise 5

(a) $A \cdot A$ $(1\times3)(1\times3)$ not possible

$B \cdot B^T$ $(3\times3)(3\times3)$ possible

$B \cdot A$ $(3\times3)(1\times3)$ not possible

$A \cdot B$ $(1\times3)(3\times3)$ possible

(b) $$B \cdot A^T = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 0 & 1 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1\cdot1 + 1\cdot2 + 3\cdot3 \\ 1\cdot1 + 0\cdot2 + 1\cdot3 \\ 2\cdot1 - 1\cdot2 + 0\cdot3 \end{pmatrix} = \begin{pmatrix} 12 \\ 4 \\ 0 \end{pmatrix}$$

$$\det B = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 0 \\ 2 & -1 & -2 \end{vmatrix} = (-1)\begin{vmatrix} 1 & 2 \\ -1 & -2 \end{vmatrix} = 0$$

(c) $$\left(\begin{array}{ccc|c} k & 2k & 3 & 4k \\ 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 \end{array}\right) \quad R_1 \leftrightarrow R_2$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ k & 2k & 3 & 4k \\ 2 & 1 & 1 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ k-3 & 2k-3 & 0 & 4k \\ 1 & 0 & 0 & 1 \end{array}\right) \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1) \end{array}$$

$$\underbrace{}_{C}$$

$\det C = \cancel{(k-3) - (2k-3)}\ 2k \quad +3-2k$

$\neq 0$ if $k \neq \frac{3}{2}$

for $k \neq 3/2$ there is only one solution

for $k = \frac{3}{2}$ we have no solution

$k=1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -2 & -1 & 0 & 4 \\ 1 & 0 & 0 & 1 \end{array}\right)$$

$x=1$ from the last row

$-y = 4-2 \Rightarrow y = -6$

$z = -1+6 = 5$

there is only one solution $(1\ -6\ 5)$

<u>Everyone that solves for k= get full points</u>

Exercise 6

(a) $f(x,y) = y e^{1+3x^2+y}$

$\partial_x f(xy) = 6xy e^{1+3x^2+y} = 0$

$\partial_y f(xy) = e^{1+3x^2+y}(1+y) = 0$

$\Leftrightarrow \begin{cases} 6xy = 0 \\ (1+y) = 0 \end{cases} \Rightarrow x = 0 \quad y = -1$

$\partial_{xx} f(x,y) = 6y e^{1+3x^2+y}(6x^2+1)$

$\partial_{yx} f(xy) = 6x e^{1+3x^2+y}(1+y) = \partial_{xy} f(xy)$

$\partial_{yy} f(xy) = (2+y) e^{1+3x^2+y}$

$$H(0\ {-1}) = \begin{vmatrix} \partial_{xx} f(0\ {-1}) & \partial_{yx} f(0\ {-1}) \\ \partial_{yx} f(0\ {-1}) & \partial_{yy}(0,-1) \end{vmatrix} = \begin{vmatrix} -6 & 0 \\ 0 & 1 \end{vmatrix} = -6 < 0$$

SADDLE POINT

(b) A: $y = 0 \qquad x \in [-1,1]$

B: $y = -1 - x^2 \qquad x \in [-1,1]$

A $\quad f(x\ 0) \equiv 0$

B $\quad g(x) = f(x, 1-x^2) = (1-x^2) e^{2+2x^2}$

$g'(x) = -2x e^{2+2x^2} + 4x(1-x^2) e^{2+2x^2}$

$= 2x e^{2+2x^2}(-1 + 2 - 2x^2)$

$= 2x(1-2x^2)\, e^{2+2x^2} = 0$

$x = 0$

$1-2x^2 = 0 \qquad x = \pm \frac{\sqrt{2}}{2}$

$g(0) = e^2 \qquad \underbrace{g\left(\pm \frac{\sqrt{2}}{2}\right) = \frac{1}{2} e^3}_{\text{MAX}} \qquad \underbrace{g(\pm 1) = 0}_{\text{MIN}}$

The max value on ∂D is $\frac{1}{2}e^3$

The min value on ∂D is 0

(c) We have to compare max and min value on ∂D with the value at critical points in D. But the only critical points is not in D

$\Rightarrow$ the max value on D is $\frac{1}{2}e^3$

& the min value is 0