STOCKHOLMS UNIVERSITET
Matematiska institutionen
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Exam / Tentamen
MM3001 7,5 hp
HT 2025
2025-10-27

Instructions:

- This exam is given in two versions, English and Swedish. In case of ambiguity it is the ENGLISH text that holds.
- During the exam you MAY NOT use textbooks, class notes, or any other supporting material apart from the formula sheet given to you.
- Use of calculators is permitted for performing calculations. The use of graphic or programmable features is NOT permitted.
- You can use the formula sheet that come with the exam.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear and wrong argument, even if the final answer is correct.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting A BOX around it.

Note: The problems are not ordered according to difficulty

Written Exam (English)

1 **Elemetary Algebra**

(a) (1 pt) Solve the following equation

$$e^{x+2} = 1.$$

(b) (2 pts) Find all real numbers x such that

$$e^x(x^2 - 7x + 9) < -e^x.$$

2 **Taylor polynomials**

(a) (4 pts) Compute the Taylor polynomial of degree 3 of the function

$$f(x) = \ln(1 + 2x)$$

around the point $x_0 = 0$.

(b) (1 pts) Use the polynomial computed in (a) to approximate $f(0.1)$.

3 **Implicit derivation:** Consider the curve in the plane given by the equation

$$x + xy + y^2 = 2.$$

(a) (1 pt) Show that the point $(2, -2)$ belongs to the curve.

(b) (2 pts) The equation above describes y as a function of x, that is $y = y(x)$. Compute $y'(2)$.

4 **One variable optimization:** Consider the function

$$f(x) = \frac{x+2}{e^x}.$$

(a) (2 pts) Find all the critical points, determine their type, and decide when $f(x)$ is increasing.

(b) (2 pts) Determine where the function is concave or convex.

(c) (1 pts) Find the maximum and minimun values that the functions takes in the interval $[-3, 2]$.

5 (4 pts) **Integrals:** Compute the following integral

$$\int xe^{-3x^2} + \ln(x) + x^3 dx.$$

6 (5 pts) **Linear Algebra:** Consider the following matrices

$$A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix} \quad \text{and} \quad B = (1, 1, 1).$$

(a) (1 pt) Compute, if possible, $A \cdot B$ and $B \cdot A$.

(b) (1 pt) Compute the determinant of A, $\det A$.

(c) (3 pt) Use Gauss-Jordan elimination to find the number of solutions of the following linear system. If there is just one soultion, compute it.

$$\begin{cases} \quad y & -z & = & -1 \\ x & +z & = & 1 \\ 2x & +y & +z & = & 1 \end{cases}$$

Important: in order to get full points, it is not enough to give the correct answer, you have to use Gauss-Jordan elimination, showing your work.

7 **Multivariable optimization:** Consider the function

$$f(x, y) = x(x^2 + y^2).$$

(a) (2 pts) Find all the critical points and determin their type, if possible.

(b) (2 pts) Find the maximun and minimum value of the function when

$$x^2 + y^2 = \frac{1}{4}.$$

(c) (1 pts) Find the maximun and minimum value of the function when

$$x^2 + y^2 \leq \frac{1}{4}.$$

——— End of exam ———

Skriftligt prov (Svenska)

1 **Elementär Algebra**

(a) (1 pt) Lös den följande ekvationen

$$e^{x+2} = 1.$$

(b) (2 pts) Hitta alla reella tal x sådana att

$$e^x(x^2 - 7x + 9) < -e^x.$$

2 **Taylor polynom**

(a) (4 pts) Beräkna det Taylor polynomet av grad 3 till funktionen

$$f(x) = \ln(1 + 2x)$$

kring punkten $x_0 = 0$.

(b) (1 pts) Använd polynomet beräknat i (a) för att approximera $f(0.1)$.

3 **Implicita derivator:** Betrakta kurva i (x, y)-planet med ekvationen

$$x + xy + y^2 = 2.$$

(a) (1 pt) Visa att punkten $(2, -2)$ tillhör till kurvan.

(b) (2 pts) Ekavtionen ovanför ange y som en funktion av x. Beräkna $y'(2)$.

4 **Optimering i en variabel:** Betrakta funktionen

$$f(x) = \frac{x+2}{e^x}.$$

(a) (2 pts) Hitta alla de kritiska punkterna, bestäm deras typ och bestäm var funktionen är stigande.

(b) (2 pts) Bestäm var funktionen är konkav eller konvex.

(c) (1 pts) Hitta de storsta och minsta värden som funktioner antar i intervallen $[-3, 2]$.

5 (4 pts) **Integraler:** Beräkna den följande integralen

$$\int xe^{-3x^2} + \ln(x) + x^3 dx.$$

6 (5 pts) **Linjär Algebra:** Betrakta de följande matriser

$$A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix} \quad \text{och} \quad B = (1, 1, 1).$$

(a) (1 pt) Beräkna, om möjligt, $A \cdot B$ och $B \cdot A$.

(b) (1 pt) Beräkna determinanten till A , $\det A$.

(c) (3 pt) Andvänd Gauss-Jordan elimination för att hitta antal lösningar till den följande linjär system. Om det finns bara en lösning, beräkna den.

$$\begin{cases} \quad y & -z & = & -1 \\ x & +z & = & 1 \\ 2x & +y & +z & = & 1 \end{cases}$$

Viktig: för att få all poängar, behöver ni andvända Gauss–Jordan elimination och visa ditt arbete. Det är i+INTE nok att ange den korrekta svaret.

7 **Optimering i flera variabler:** Betrakta funktionen

$$f(x, y) = x(x^2 + y^2).$$

(a) (2 pts) Hitta alla den kritiska punkter och bestem deras typ om möjligt.

(b) (2 pts) Hitta de storsta och minsta värden till $f(x, y)$ när

$$x^2 + y^2 = \frac{1}{4}.$$

(c) (1 pts) Hitta de storsta och minsta värden till $f(x, y)$ när

$$x^2 + y^2 \leq \frac{1}{4}.$$

——— Slut på provet ———