MM3001 October 27

Exercise 1

(a) We apply $\ln(\ )$ on both side and we got

$$\ln(e^{x+1}) = \ln(1)$$

$$\Leftrightarrow \quad (x+1)\cdot\ln(e) = 0$$

$$\Leftrightarrow \quad x+1 = 0$$

which has solution

$\boxed{x=-1}$

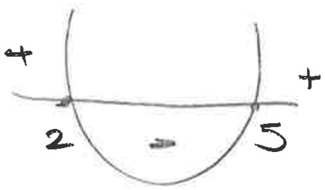
(b) The given inequality is equivalent to

$$e^{x}(x^2-7x+10) < 0$$

since $e^x > 0$ we have to solve

$$x^2-7x+10 < 0$$

$$(x-2)(x-5) < 0$$



we have that $x^2-7x+10 < 0$ when $x \in (2,5)$

Exercise 2

(a) $f(0) = \ln(1) = 0$

$$f'(x) = \frac{2}{1+2x} \qquad f'(0) = 2$$

$$f''(x) = -\frac{4}{(1+2x)^2} \qquad f''(0) = -4$$

$$f'''(x) = \frac{+2\cdot 8}{(1+2x)^3} \qquad f'''(0) = +16$$

$$p(x) = 2x - \frac{4}{2}x^2 + \frac{16}{3}x^3$$

$$= 2x - 2x^2 + \frac{16}{3}x^3$$

(b) Evaluat $p(0.1) = 0.2 - 0.02 + \frac{16}{3} 0.001$

Exercise 3

(a) we set $x = 2$ and $y = -2$

$$VL = 2 + 2(-2) + (-2)^2 = 2 = HL$$

(b) We derive implicitly

$$1 + y + xy' + 2y'y = 0$$

we set $x = 2$ $y = -2$

$$1 - 2 + 2y' - 4y' = 0$$

$$2y' = -1 \qquad y' = -1/2$$

Exercise 4

(a) $$f'(x) = \frac{e^x - (x+2)e^x}{e^{2x}} = \frac{\cancel{e^x}(1-x-2)}{e^{\cancel{2x}}}$$

$$= -\frac{x+1}{e^x} \geq 0$$

$$\iff x+1 \leq 0$$

$$\iff x \leq -1$$

		-1	
$f'(x)$	$+$	0	$-$
$f(x)$	$\nearrow$		$\searrow$

There is only one critical point $x = -1$ which is a <u>local max</u>. The functio

$f(x)$ is increasing when $x \leq -1$

(b) $$f''(x) = -\frac{e^x - (x+1)e^x}{e^{2x}} = +\frac{x}{e^x} \geq 0$$

when $x > 0$

the function is convex when $x > 0$ & concave otherw.

SANITY CHECK $-1 < 0$ so $f''(-1) < 0$

and, by the second derivative test

-1 is a <u>local max</u>

(c) Compare $f(-3)$ $f(2)$ and $f(-1)$

Exercise 5

$$\int x e^{-3x^2} + \ln(x) + x^3 \, dx$$

$$= \int x e^{-3x^2} dx + \int \ln(x) \, dx + \int x^3 dx$$

$$= -\frac{1}{6} \int e^y dy + x\ln(x) - \int \cancel{x} \frac{1}{\cancel{x}} dx + \frac{1}{4} x^4 + C_1$$

$$= -\frac{1}{6} e^y + C_2 + x\ln(x) - x + C_3 + \frac{1}{4} x^4 + C_1$$

$$= -\frac{1}{6} e^{-3x^2} + x\ln(x) - x + \frac{1}{4} x^4 + C$$

Exercise 6

(a) $A \cdot B$ is not possible

$B \cdot A = (3, 2, 4)$

(b) $\begin{vmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 1 & 2 \end{vmatrix} = (-1) \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 0$

(c)

$\left(\begin{array}{ccc|c} 0 & \boxed{1} & -1 & -1 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 1 & 1 \end{array}\right)$ $R_3 \rightsquigarrow R_3 - R_1$

$\left(\begin{array}{ccc|c} 1 & \boxed{1} & -1 & -1 \\ \boxed{1} & 0 & 1 & 1 \\ 2 & 0 & 2 & 2 \end{array}\right)$ $R_3 \rightsquigarrow R_3 - R_2$

$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$

variables - # non 0 row =

$= 3 - 2 > 0$

$\Rightarrow \infty$ many solutions

Exercise 7

(a) $\frac{\partial}{\partial x} f(xy) = 3x^2 + y^2 = 0$

$\frac{\partial}{\partial y} f(xy) = 2yx$

The second equation yields $y=0$ or $x=0$
we set it in the first & get

$y=0 \quad 3x^2=0 \Rightarrow x=0$

$x=0 \quad y^2=0 \Rightarrow y=0$

There is only one critical point $(0,0)$

$$H(xy) = \begin{vmatrix} 6x & 2y \\ 2y & 2x \end{vmatrix} = 12x^2 + 4y^2$$

$H(00) = 0$ the criterion does not give an answer

(b) $f_{180}(xy) = \frac{1x}{4} =: g(x)$

$g'(x) = 1$ no critical point

$g(-\frac{1}{2}) = -\frac{1}{8}$ min $\quad g(\frac{1}{2}) = \frac{1}{8}$ max

(c) $f(0,0)=0$ compare with (b) min value $-\frac{1}{8}$
max value $= 1/8$