

In all your solutions, give explanations to clearly show your reasoning.

Allowed aids: The calculator and formula sheet provided with the exam.

1. (a) Consider the geometric series (3p)

$$S(x) = 3 + 6e^x + 12e^{2x} + \dots$$

Determine for which x the series converges, and solve the equation $S(x) = 6$.

- (b) Compute that Taylor polynomial of degree 2 at the point $x = 1$ for the function $f(x) = e^x \ln(x)$. (2p)

2. Determine the constant a such that the curve (5p)

$$xy^3 + ax^5y = 4$$

passes through the point $(1, 2)$, and then determine the equation of the tangent line at that point.

3. Consider the function $f(x) = \frac{x}{x^2 + 3}$. Determine the local extreme points, the limits as $x \rightarrow \pm\infty$, and where the function is convex or concave. Sketch the graph. (5p)

4. (a) Compute the integral $\int_0^1 xe^{3x} dx$. (2p)

- (b) Show that the integral $\int_0^\infty \frac{x^2}{(1+x^3)^2} dx$ is convergent and compute its value. (3p)

5. (a) Solve the linear system
$$\begin{cases} x + 2z + w = -1 \\ -x + y - 3z = 2 \\ x + 2y + 4w = 2 \end{cases}$$
 (3p)

- (b) Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 3 \\ 1 & 3 & 0 \end{pmatrix}$. Compute the inverse of A . (2p)

6. Consider the function $f(x, y) = xy - x^3 - y^2 + 4$. Find and classify all its stationary points. (5p)