Solutions 2026-08-20

1) a) The montly interest rate is $p = \frac{3,6\,\%}{12} = 0,3\,\%$.

Let $r = 1 + \frac{p}{100} = 1,003$.

There will be 36 deposits:

deposit	future value
1	$500 \cdot r^{35}$
⋮	
35	$500r$
36	500

so total FV is

$$K = 500(1 + r + \ldots + r^{35}) = 500\,\frac{1 - r^{36}}{1 - r}$$

$= 18978$ SEK

b) We get

$$f(x) = e^{2x+2}$$
$$f'(x) = 2e^{2x+2}$$
$$f''(x) = 4e^{2x+2}$$
$$f'''(x) = 8e^{2x+2}$$

so

$$p(x) = f(-1) + f'(-1)(x+1) + $$
$$+ f''(-1)\frac{(x+1)^2}{2!} + f'''(-1)\frac{(x+1)^3}{3!}$$

$$= 1 + 2(x+1) + 2(x+1)^2 + \frac{4}{3}(x+1)^3$$

2) a) $(x,y)=(2,1)$ satisfies the equation since

$$1^3+1=2^3-3\cdot 2$$

so $y(2)=1$.

Implicit derivation gives

$$3y^2\cdot y'+y'=3x^2-3 \qquad (*)$$

so for $x=2$

$$3y'(2)+y'(2)=9$$

so $y'(2)=9/4$, and the tangent is therefore

$$y-1=\frac{9}{4}(x-2) \iff \underline{y=\frac{9x-14}{4}}$$

b) Putting $y'(x)=0$ in $(*)$ gives $0=3x^2-3 \iff$

$$\underline{x=\pm 1}$$

3) The function is continuous on $\mathbb{R}$.

$$\lim_{x \to -\infty} x^3 e^{-x} = -\infty \quad \text{and}$$

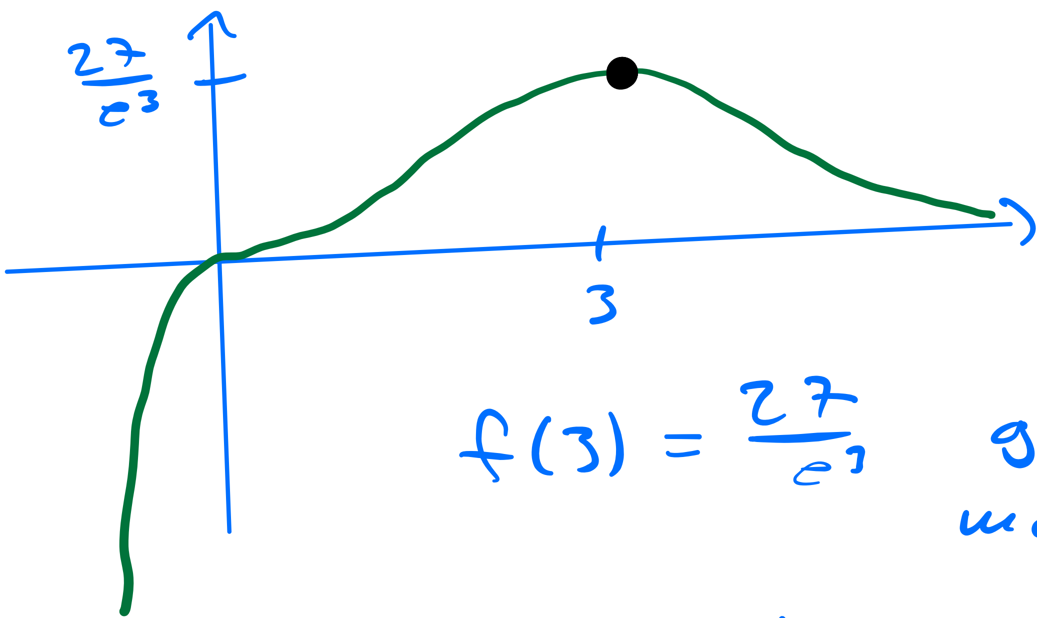
$$\lim_{x \to \infty} \frac{x^3}{e^x} = \begin{bmatrix} \text{l'Hospital} \\ \text{3 times} \end{bmatrix} = 0$$

We get

$$f'(x) = x^2 (3-x) e^{-x} \quad \text{so}$$

$$f'(x) = 0 \iff x = 0 \text{ or } x = 3$$

x	$(-\infty)$		0		3		(∞)
$f'(x)$		$+$	0	$+$	0	$-$	
$f(x)$	$(-\infty)$	$\nearrow$	0	$\nearrow$	$\frac{27}{e^3}$	$\searrow$	(0)



$f(3) = \frac{27}{e^3}$ global max

no global min

4 a) The integral I can be written

$$I = \int (2x+1)^{1/2}\,dx + \int x^2 (x^3+1)^4\,dx$$

$$\int (2x+1)^{1/2}\,dx = \frac{(2x+1)^{3/2}}{2 \cdot 3/2} + C_1 \quad \text{and}$$

$$\int x^2 (x^3+1)^4\,dx = \begin{bmatrix} u = x^3+1 \\ du = 3x^2\,dx \end{bmatrix} =$$

$$= \frac{1}{3} \int u^4\,du = \frac{1}{3} \cdot \frac{u^5}{5} + C_2 =$$

$$= \frac{1}{15}(x^3+1)^5 + C_2 \quad \text{so}$$

$$I = \frac{(2x+1)^{3/2}}{3} + \frac{1}{15}(x^3+1)^5 + C$$

b) $$\int x e^{-x^2}\,dx = \begin{bmatrix} u = -x^2 \\ du = -2x\,dx \end{bmatrix} =$$

$$= -\frac{1}{2} \int e^u\,du = -\frac{1}{2} e^u + C = -\frac{1}{2} e^{-x^2} + C$$

so $$\int_0^\infty e^{-x^2}\,dx = \lim_{N \to \infty} \left[-\frac{1}{2} e^{-x^2} \right]_0^N$$

$$= \lim_{N \to \infty} \left(\frac{1}{2} - \frac{e^{-N^2}}{2} \right) = \frac{1}{2}$$

5 a)

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} \; -1 = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 0 \end{vmatrix} = [\text{col. } 1]$$

$$= 1 \cdot \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

b)

$$(A \mid I) = \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) -1 \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 2 & 1 & 0 & 1 & 0 \end{array}\right) -2$$

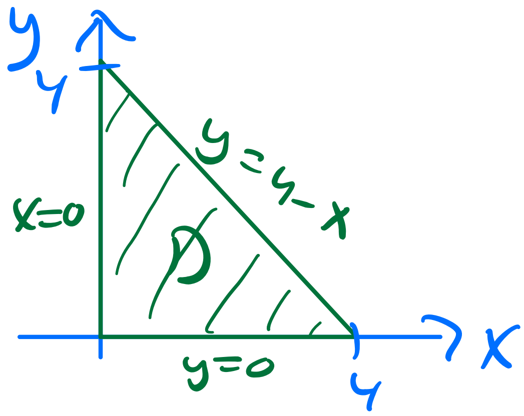
$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 & -2 \end{array}\right) -1 \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -1 & 2 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 & -2 \end{array}\right)$$

$$= (I \mid A^{-1}) \quad \text{so} \quad A^{-1} = \begin{pmatrix} -1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{pmatrix}$$

c)

$$X = A^{-1} B = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

6) The domain is a triangle:



Stationary points:

$$\begin{cases} f'_x = (2xy - x^2y)e^{4-x-y} = 0 \\ f'_y = (x^2 - x^2y)e^{4-x-y} = 0 \end{cases}$$

$\Leftrightarrow \begin{cases} xy(2-x) = 0 \\ x^2(1-y) = 0 \end{cases}$ In the interior of D we have $x, y \neq 0$

so $(x,y) = (2,1)$ the only interior stationary point.

On the boundary $x=0$ and $y=0$ we have $f(x,y)=0$

On the side $y=4-x$ we get

$$h(x) = f(x, 4-x) = x^2(4-x) \qquad 0 \leq x \leq 4$$

$$h'(x) = (8x - 3x^2) = 0 \Leftrightarrow$$

$x = 0$ el. $x = \frac{8}{3}$

The case $x=0$ is already examined, but we get the new candidate $(x,y) = \left(\frac{8}{3}, \frac{4}{3}\right)$

We compare

(x,y)	$(0,y)$	$(x,0)$	$\left(\frac{8}{3},\frac{4}{3}\right)$	$(2,1)$
$f(x,y)$	0	0	$\frac{256}{27}$	$4e$

The maximum value is $4e$