MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Jonathan Rohleder

Tentamensskrivning i
Matematik III Komplex Analys
7.5 hp
17 December 2019

No calculators, books, or other resources allowed. Max score on each problem is 5p; grade of E guaranteed at 15p. Appropriate amount of details required for full marks.

1. Calculate all Laurent series expansions of the function
$$f(z) = \frac{1}{z^2 + (4-3i)z - 12i}$$
centered at $z_0 = 3i$.

2. Find all $a, b, c \in \mathbb{R}$ such that $ax^2 + be^{x-y} + cy^2$ is the real part of an analytic function. Moreover, for each such triple (a, b, c) determine all these analytic functions.

3. Determine the value of the integral
$$\int_{-\infty}^{\infty} \frac{1}{(x^2+a^2)(x^2+b^2)} \,\mathrm{d}x,$$
where $a, b > 0$ are real with $a \neq b$.

4. Let $a \in \mathbb{C}$ with $|a| > e$. Prove that the equation
$$e^z = az^n$$
has precisely n solutions in the open unit disc $|z| < 1$.

5. Show that if f is analytic at z_0 and $f'(z_0) \neq 0$ then there exists an open disk D centered at z_0 such that f is injective on D. (In particular, f is conformal on D.)

6. (a) Show that the function $A \operatorname{Log}|z| + B$, with $A, B \in \mathbb{R}$ constant, is harmonic in each domain that does not contain the origin.
   (b) Find a pair of complex numbers that are symmetric with respect to both the real axis and the circle $|z + 5i| = 4$.
   (c) Determine a harmonic function in $\{z \in \mathbb{C} : \operatorname{Im} z < 0, |z+5i| > 4\}$ that is equal to 0 on the circle $|z + 5i| = 4$ and equals 1 on the real axis.

Exams will be returned on 19 December 2019 at 3 pm in room 414, building 6, and will be stored in the students' office afterwards.