MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Jonathan Rohleder

Tentamensskrivning i
Matematik III Komplex Analys
7.5 hp
20 March 2020

Max score on each problem is 5p; grade of E guaranteed at 15p. Appropriate amount of details required for full marks. Solutions have to be submitted in one pdf file through the submission tool on the course page. No submission later than 5 p.m. will be accepted.

As the exam questions were individualized, we provide here one set of example questions.

0. (Mandatory, but gives no marks.) The PDF document that contains your home exam should start by you writing the following sentence:

   I, the author of this document, hereby guarantee that I have produced these solutions to this home exam without the assistance of any other person. This means that I have for example not discussed the solutions or the home exam with any other person.

1. Find all solutions to the equation $3\tan z + 4i = ie^{2iz}$.

2. Calculate all Laurent series expansions of the function

$$f(z) = \frac{1}{z(z-i)^2}$$

   centered at $z_0 = i$.

3. Use residue calculus to determine the value of the integral

$$\int_0^{2\pi} \frac{1}{2+\sin x} \mathrm{d}x.$$

4. Determine the number of zeroes of $z^5 - 3z^4 - 2$ in the disk $|z| < 2$.

5. (a) Show that, for $A, B \in \mathbb{R}$ constant, the function $A \operatorname{Arg} z + B$ is harmonic in the right half-plane $\operatorname{Re} z > 0$.

   (b) Construct a Möbius transformation of the unit disk $|z| < 1$ onto the right half-plane $\operatorname{Re} z > 0$ such that the upper half-disk is mapped onto the first quadrant $\operatorname{Re} z > 0, \operatorname{Im} z > 0$.

   (c) Find a harmonic function u in the unit disk $|z| < 1$ that satisfies $u = \phi$ on the boundary $|z| = 1$, where

$$\phi = \begin{cases} 2 & \text{on the upper half-circle,} \\ -2 & \text{on the lower half-circle.} \end{cases}$$

6. Compute the integral

$$\iint_{\partial_0 P} \frac{1}{2zw - 3} \,\mathrm{d}z\,\mathrm{d}w,$$

   where $\partial_0 P = \{(z,w) : |z| = |w| = 1\}$ is the distinguished boundary of the unit polydisk centered at the origin, taken with the usual orientation.