MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Jonathan Rohleder

Tentamensskrivning i
Matematik III Komplex Analys
7.5 hp
22 April 2020

Max score on each problem is 5p; grade of E guaranteed at 15p. Appropriate amount of details required for full marks. Solutions have to be submitted in one pdf file through the submission tool on the course page. No submission later than 5 p.m. will be accepted.

1. Find all solutions to the equation $\tan z = \frac{1}{2}(1 + \sqrt{3}i)$.

2. Calculate all Laurent series expansions of the function
$$f(z) = \frac{1}{(z-1)(z-i)}$$
centered at $z_0 = 0$.

3. Use residue calculus to determine the value of the integral
$$\int_0^\infty \frac{x^2}{(x^2+1)(x^2+9)} \mathrm{d}x.$$

4. Assume that $f(z)$ is analytic in the disk $|z| < 2$ and continuous in the closed disk $|z| \leq 2$ with $|f(z)| \leq 48$ for all z with $|z| = 2$. Moreover, assume that $f(z)/z^3$ is analytic in $|z| < 2$ as well. Find an upper bound for $|f(i/6)|$ and show that it is optimal.

5. Prove that all zeroes of the polynomial $z^6 - 5z^2 + 10$ lie in the annulus $1 < |z| < 2$.

6. (a) Show that, for $A, B \in \mathbb{R}$ constant, the function $A \operatorname{Log}|z| + B$ is harmonic in each domain that does not contain the origin.
   (b) Find a pair of complex numbers that are symmetric with respect to both circles $|z| = 1$ and $|z - \frac{3}{10}| = \frac{3}{10}$.
   (c) Determine a harmonic function in the domain enclosed by the two circles in (b) that is constantly equal to zero on $|z - \frac{3}{10}| = \frac{3}{10}$ and constantly equal to 1 on $|z| = 1$.