MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
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Tentamensskrivning i
Topologi, MM8002/SF2721
7.5 hp
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(1) Show the following statements
   (a) A subspace of a discrete space is always discrete;
   (b) A product of two discrete spaces is always discrete.
   (c) The quotient of discrete spaces is discrete.

   In addition determine whether the above statement are still true if "discrete" is substituted with "trivial", by either giving proofs or providing counterexamples. [30 points]

(2) Consider the space $X := [-1, 1]^2$ endowed with the topology with basis
$$\mathcal{B} = \{X \cap (a, b) \times (c, d) \mid a, c < 0, \text{ and } b, d > 0\}.$$
   (a) Determine whether $[-1, -\frac{1}{2}] \times [\frac{1}{2}, 1]$ is closed in X.
   (b) Determine whether X is Hausdorff (T_2)
   (c) Determine whether X is connected.
   (d) Let $\mathbb{I} := [-1, 1]$ endowed with the Euclidean topology. Determine if the identity map $X \to \mathbb{I}^2$ is continuous. What about the identity map $\mathbb{I}^2 \to X$?
   (e) Determine whether X is compact. [25 points]

(3) Which of the following maps are covering? Justify your answers.
   (a) $\rho : [0, 1) \to \mathbb{S}^1$ defined by $t \mapsto (\cos t, \sin t)$.
   (b) $\rho : \mathbb{S}^1 \to X$, where X is the Möbius band and ρ is the inclusion of the boundary.
   (c) The quotient map $\rho : \mathbb{S}^2 \to \mathbb{P}^2(\mathbb{R})$ which identifies antipodal points. [15 points]

(4) Consider the square $[0, 1] \times [0, 1]$ with cyclically identified edges. That is
$$(x, 0) \sim (1, x) \sim (1 - x, 1) \sim (0, 1 - x).$$
   (a) Draw a cell complex representing X and determine the n-skeleton of X for $n = 0, 1, 2$.
   (b) Use Van Kampen theorem to compute the fundamental group of X. [10 points]

(5) State the classification theorem for topological compact surfaces and give an outline of the proof. [20 points]