

Exercises

Prove whether each of the following polynomials is irreducible on the given polynomial ring. If they are not, factor them.

1. $f(X) = X^3 + 4X^2 + X - 6$ in $\mathbb{Q}[X]$.

2. $f(X) = X^4 + X^2 + 1$ in $\mathbb{Z}/2\mathbb{Z}[X]$.

3. $f(X) = X^4 + 1$ in $\mathbb{Z}[X]$

4. $f(X) = X^5 + 3X^4 + 30X^2 - 9X + 12$ in $\mathbb{Q}[X]$.

5. $f(X) = X^5 + 4X^3 - X + iX + 3 + 3i$ in $\mathbb{Z}[i][X]$.

6. $f(X) = X^3 + 6$ in $\mathbb{Z}/7\mathbb{Z}[X]$.

7. $f(X, Y) = X^3 + X^2Y + 3XY^2 + 5XY + 2Y$ in $\mathbb{Z}[X, Y]$.

8. $f(X) = X^6 + X^5 + X^4 + X^3 + X^2 + X + 1$ in $\mathbb{Z}[X]$.