

A first step towards a Swedish validation of the Early Trauma Inventory

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Kandidatuppsats i matematisk statistik Bachelor Thesis in Mathematical Statistics

Kandidatuppsats 2016:1 Matematisk statistik Juni 2016

www.math.su.se

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Mathematical Statistics Stockholm University Bachelor Thesis **2016:1** http://www.math.su.se

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The Early Trauma Inventory Self-Report (ETI-SR) is shown to discriminate well between post-traumatic stress disorder (PTSD) and healthy subjects in a Swedish young adult sample. This result increases the validity of ETI-SR scores as a valid measure of childhood trauma within a Swedish population.

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Acknowledgments

I would like to thank my supervisors Gudrun Brattström and Jan-Olov Persson for all their help and support. I also want to thank Mia Ramklint for allowing me to use real research data.

Introduction

Risk factors of mental illness are often searched for in genetic and/or environmental factors. Childhood trauma has been associated with a range of adverse psychiatric outcomes including depression, borderline personality disorder and post-traumatic stress disorder (PTSD) (J. Douglas Bremner, Vermetten, and Mazure 2000). The Early Trauma Inventory Self-Report (ETI-SR) is a psychometric instrument (J. Douglas Bremner, Vermetten, and Mazure 2000; J Douglas Bremner, Bolus, and Mayer 2007) meant to measure the experience of childhood trauma. ETI-SR measures the presence of 28 life events that are considered to be indicators of four distinct trauma categories: general (T), physical (P), emotional (E) and sexual (S) trauma. To evaluate the quality of a psychometric instrument, its reliability and validity are assessed. Reliability refers to the internal correlational structure of the instrument and can be assessed through inter-item, inter-rater and test-retest correlational analysis. Validity refers to whether or not the instrument measures what it is intended to measure. If both reliability and validity are confirmed, an instrument is said to be valid. When a psychometric instrument is introduced in a new country, it is standard practice to reinvestigate both reliability and validity. ETI-SR is considered a valid instrument in several countries (Osorio et al. 2013: J.-R. Jeon et al. 2012: Plaza et al. 2011) and is now on its way to being validated in Sweden. This paper will focus on the validity of ETI-SR within a swedish context by studying if ETI-SR scores can discriminate between healthy subjects and subjects diagnosed with post-traumatic stress disorder (PTSD). The ability to discriminate between PTSD subjects and healthy subjects increases the validity of ETI-SR within a swedish context, given PTSD's strong association with trauma.

Early Trauma Inventory Self-Report (ETI-SR)

Early Trauma Inventory Self-Report (ETI-SR) is a psychometric instrument measuring the presence of traumatic childhood events (J Douglas Bremner, Bolus, and Mayer 2007). ETI-SR measures the exposure to 28 childhood events that are considered to be potentially traumatic. ETI-SR events are divided into the following four categories: twelve general traumatic events (T), five physical traumatic events (P), five emotional traumatic events (E) and six sexual traumatic events (S). A complete list of measured items is presented in table 1. For all 28 events, frequency of exposure is measured using the following response choices.

For general traumatic events, except event T17 and T18, frequency of exposure is measured as:

0 = never, 1 = 1 time, 2 = 2-3 times, 3 = 4-5 times, 4 = 6-10 times and 5 = more than 10 times

For event T17 and T18, frequency of exposure is measured as:

0 = never, 1 = 1 year, 2 = 2-3 years, 3 = 4-5 years, 4 = 6-10 years and 5 = more than 10 years

For physical, emotional and sexual traumatic events, exposure is measured as:

0 = never, $1 = single \ occasion$, 2 = 1 time per year, 3 = 2-11 times per year, 4 = 1-3 times per month, 5 = 1-6 times per week, 6 = 1 time per day and 7 = more than 1 time per day.

For physical, emotional and sexual traumatic events, ETI-SR also measures age of onset and perpetrator using the following response choices.

For age of onset, the following choices are offered:

0 = 0.5 years, 1 = 1 year, 2 = 2.3 years, 3 = 4.5 years, 4 = 6.10 years and 5 = more than 10 years.

and for perpetrator:

1 = Mother or equivalent, 2 = Father or equivalent, 3 = Other male adult, 4 = Brother, 5 = Unknown man, 6 = Unknown woman, 7 = Other female adult, 8 = Sister, 9 = Other.

ETI-SR also measures the respondent's opinion as to how each trauma category affects daily life on an emotional, functional and social level using the following seven-point Likert scale.

1 = extremely negative, 1 = negative, 2 = to some extent negative, 3 = no effect, 4 = to some extent positive, 5 = positive, 6=extremely positive.

This gives ETI-SR a total of 28 frequency items, 12 daily-life-affect (three for each category) items, 16 age-of-onset and 16 perpetrator items. In total, there are 72 items.

Post-traumatic stress disorder (PTSD)

Post-traumatic stress disorder (PTSD) is a mental illness diagnosed by the assessment of two criteria. The first criterion is the exposure to actual or threatened death, serious injury or sexual violence. The exposure can be both direct and indirect (i.e., witnessed, heard). The second criterion is that the exposure has to cause mental distress. Intrusion symptoms (i.e., distressing memories/dreams, flashbacks, etc.) associated with the event as well as avoidance of situations similar to the event have to be present. Furthermore, a negative impact on cognition and mood has to be observed as well as a marked alteration in arousal (irritable behavior, angry outbursts, problems with concentration and sleep, etc.) (American Psychiatric Association 2013).

Overview

The discriminative capacity of ETI-SR will be studied using logistic regression, and section one gives a theoretical background to logistic regression. Section two presents the sample and its characteristics, and part three explains the analysis. Part four summarizes the results, and part five contains a more thorough discussion of the results as well as some suggestions for future studies.

Logistic regression, theory and notation

The following presentation is due to Perigborn (Pregibon 1981). Consider a $I \times 2$ contingency table, where row *i* is assumed to represent an outcome from an independent binomial response

 $y_i \sim \text{Bin}(n_i, p_i)$. Here n_i denotes the total number of observations in row *i*. This model is called the unstructured binomial model and letting $\theta_i = \text{logit}(p_i) = \log(p_i/(1-p_i))$, the probability function of y_i can be written as

$$f(y_i, \theta_i) = \exp(y_i \theta_i - a(\theta_i) + b(y_i)) \text{ where } a(\theta_i) = n_i \log(1 + e^{\theta_i}) \text{ and } b(y) = \log\binom{n_i}{y_i}.$$

The loglikelihood of $f(y_i, \theta_i)$ becomes

$$l(\theta_i, y_i) = y_i \theta_i - a(\theta_i) + b(y_i)$$

Differentianting $l(\theta_i, y_i)$ with respect to θ_i gives the score function

$$s(\theta_i, y_i) = l(\theta_i, y_i) = y_i - \dot{a}(\theta_i) = y_i - n_i p_i.$$

Negating the score function and differentiating with respect to θ_i gives the information function

$$v(\theta_i, y_i) = -\dot{s}(\theta_i, y_i) = -\dot{l}(\theta_i, y_i) = \ddot{a}(\theta_i) = n_i p_i (1 - p_i).$$

Here $\dot{a}(\theta_i)$ and $\ddot{a}(\theta_i)$ are the first and second order derivate with respect to θ_i . Standard results now yield that $E[y_i] = \dot{a}(\theta_i)$ and $\operatorname{Var}[y_i] = \ddot{a}(\theta_i)$. The maximum likelihood estimate of θ_i becomes

$$MLE(\theta_i) = \dot{a}^{-1}(y_i) = \text{logit}(y_i/n_i).$$

For a sample of size I of independent binomial responses, the loglikelihood becomes

$$l(\boldsymbol{\theta}, \boldsymbol{y}) = \sum_{i=1}^{I} l(\theta_i, y_i) = \sum_{i=1}^{I} y_i \theta_i - a(\theta_i) + b(y_i).$$

The above model is overspecified since it has as many parameters as observations. The logistic regression model introduces m covariates and specifies the following relationship between the parameter vector $\boldsymbol{\theta}$ and the m covariates by

$$oldsymbol{ heta} = ext{logit}(oldsymbol{p}) = ext{ln}\left(rac{oldsymbol{p}}{oldsymbol{1}-oldsymbol{p}}
ight) = oldsymbol{X}oldsymbol{eta}.$$

where X is an $I \times m$ design matrix specifying the observed covariate patterns for the $I \times 1$ observation vector y and β is an $m \times 1$ dimensional vector of parameters. This offers a new loglikelihood function

$$l(\boldsymbol{X}\boldsymbol{\beta},\boldsymbol{y}) = \sum_{i=1}^{I} l(\boldsymbol{x}_{i}\boldsymbol{\beta}, y_{i}) = \sum_{i=1}^{I} y_{i}\boldsymbol{x}_{i}\boldsymbol{\beta} - a(\boldsymbol{x}_{i}\boldsymbol{\beta}) + b(y_{i}).$$

The MLE estimate $\hat{\theta} = X\hat{\beta}$ maximizes the above likelihood. By differentiating the likelihood function with respect to β , $\hat{\beta}$ becomes the solution to the following system of equations.

$$\sum_{i=1}^{n} x_{ij}(y_i - \dot{a}(\boldsymbol{x}_i \boldsymbol{\beta})) = 0 \ j = 1, \dots, m$$

The above system of equation can be rewritten in matrix form as

$$X^{\mathrm{T}}e = 0$$

where $\boldsymbol{e} = \boldsymbol{y} - \dot{a}(\boldsymbol{X}\hat{\boldsymbol{\beta}}) = \boldsymbol{y} - \boldsymbol{n}\hat{\boldsymbol{p}} = \boldsymbol{y} - \hat{\boldsymbol{y}}$. Here \boldsymbol{n} is the *I* dimensional vector of row counts.

Iterated reweighted least square (IRLS)

The above system of equations is nonlinear in $\hat{\beta}$ and demands an iterative process to be solved. Since the second derivatives of the loglikelihood function are easy to compute, an iterated rewighted least square algorithm (IRLS) is often employed. The IRLS approach can be described by the following iterative scheme

$$\boldsymbol{\beta}^{t+1} = (\boldsymbol{X}^{\mathrm{T}} \boldsymbol{V}^{t} \boldsymbol{X})^{-1} \boldsymbol{X}^{\mathrm{T}} \boldsymbol{V}^{t} \boldsymbol{z}^{t}$$

where $z^t = X\beta^t + (V^t)^{-1}e^t$ is called the pseudo-observation vector. When the IRLS algorithm has converged, the pseudo-observation vector becomes

$$oldsymbol{z} = X oldsymbol{\hat{eta}} + V^{-1} oldsymbol{e}$$

and $MLE(\boldsymbol{\beta}) = \hat{\boldsymbol{\beta}}$ can now be written as

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}^{\mathrm{T}} \boldsymbol{V} \boldsymbol{X})^{-1} \boldsymbol{X}^{\mathrm{T}} \boldsymbol{V} \boldsymbol{z}$$

this matrix formulation offers a way to describe the projection matrix in a logistic regression model as

$$H = V^{1/2} (X^{\mathrm{T}} V X)^{-1} X^{\mathrm{T}} V^{1/2}$$

Unusual observations

The following presentation is due to Williams (Williams 1987). An observation can be considered unusual if it satisfies any of the following three characteristics:

- 1. The observation is an outlier, i.e., its response value y_i is far away from the mean \bar{y}
- 2. The observation is a leverage point, i.e., its covariate value x_i is far away from the center of the covariate space.
- 3. The observation is an influence point, i.e., removing the observations drastically changes the value of the estimated parameters.

To estimate if an observation is an outlier, Williams proposed the following studentized residual:

$$st_i = \sqrt{(1 - h_{ii})d_i^2 + h_{ii}\chi_i^2}$$

where

$$\chi_i = (y_i - n_i \hat{p}_i) / \sqrt{n_i \hat{p}_i (1 - \hat{p}_i)}$$

is the pearson residual and

$$d_i = \pm \sqrt{2(l(\hat{\theta}_i, y_i) - l(\boldsymbol{x}_i \hat{\boldsymbol{\beta}}, y_i))}$$

is the deviance residual, h_{ii} is the diagonal element of the projection matrix H. The studentized residual approximates the change in deviance

$$D^2 = 2(l(\hat{\boldsymbol{\theta}}, \boldsymbol{y}) - l(\boldsymbol{X}\hat{\boldsymbol{\beta}}, \boldsymbol{y}))$$

when observation i is excluded from the fitting process. It is important to note that, in general, st_i does not have an easily accessible distribution and should primarily be considered as a ranking statistic.

To estimate if observation i is a leverage point, the hat-value h_{ii} is used. Hat-values range from 0 to 1 and large hat-values indicate a potential leverage point.

A specific observation exerts influence on the model parameters if the parameters change drastically when the observation is removed. Influential observations can be seen as both outliers and leverage points, and a standard measure of influence is the cook's distance:

$$C_i = \frac{\chi_i^2}{p+1} \times \frac{h_{ii}}{(1-h_{ii})}$$

An influence plot shows studentized residuals st_i versus hat-values h_{ii} as well as circles whose area are proportional to the corresponding cook's distance. Influence plots will be used in this paper to investigate influential observations.

Assessment of model fit

Several statistics exist that assert the need for respecification of the model with respect to its linear predictor and/or link function. All these statistics are asymptotically χ^2 or normally distributed under the null hypothesis that the specified model is correct. Which statistic to choose depends largely upon the following factors:

- 1. The number of observed covariate groups and the number of possible covariate groups
- 2. How large the sample is and how it is distributed among the observed covariate groups.

Here, covariate groups refer to the index I, i.e., the number of rows in the contingency table that define the unstructured binomial model. If the specified model only permits a finite number of covariate groups and if all cell counts are reasonably large, both the Pearson statistic

$$X^2 = (\boldsymbol{y} - \boldsymbol{\hat{y}})^T \boldsymbol{V}^{-1} (\boldsymbol{y} - \boldsymbol{\hat{y}})$$

and the deviance statistic

$$D^2 = 2(l(\hat{\theta}, y) - l(X\hat{\beta}, y))$$

are adequate goodness-of-fit statistics that are both asymptotically chi-squared. If the number of covariate groups are large and data is sparse, both X^2 and D^2 will no longer be adequate goodness-of-fit statistics (D. W. Hosmer et al. 1997).

In a simulation study, Hosmer and Lemeshow (D. W. Hosmer et al. 1997) studied several goodnessof-fit statistics and their ability to detect any of the following misspecifications of the predictor:

- 1. An incorrect link function
- 2. A missing linear term in the predictor
- 3. A missing quadratic term in the predictor
- 4. A missing interaction term between continuous and categorical predictors

They showed that the following three tests:

- 1. Hosmer and Lemeshow's decile of risk test (\hat{C})
- 2. Osius-Rojek's standardized chi-squared test (Z_{χ^2})
- 3. Stukel's score test (χ_S^2)

were optimal for detecting any of the above predictor misspecifications. Stukel's score test was included since it is specifically designed to test misspecifications of the link function. Misspecifications of the linear predictor were best detected by Z_{χ^2} and \hat{C} .

It is important to note that all statistics proposed by Hosmer and Lemeshov have low to moderate power to detect the need for respecification with a sample size of n = 100. For a more detailed description of the simulation results, please see Hosmer and Lemeshov (D. W. Hosmer et al. 1997).

Hosmer and Lemeshow's decile of risk test (\hat{C})

Hosmer-Lemeshow's decile of risk test is based on grouping all observations based on percentiles of expected cell frequencies. Given g groups and I covariate patterns, the first percentile consists of all I/g covariate patterns with the lowest expected cell frequencies and so forth. The statistic can be written as

$$\hat{C} = \sum_{k=1}^{g} \frac{(o_{1k} - n'_k \bar{\pi}_k)^2}{n'_k \bar{\pi}_k (1 - \bar{\pi}_k)}$$

where, o_{1k} is the number of postive outcomes in the k:th percentile, n'_k is the number of observations in the k:th percentile and $\bar{\pi}_k$ is the average expected cell frequency in the k:th percentile. The statistic can also be computed without grouping by covariate pattern first and this gives, in general, different results. In this paper, all statistics are computed based on covariate patterns.

Hosmer and Lemeshov showed, by using simulations, that \hat{C} is approximately $\chi^2(g-2)$ for large samples where I = n (D. W. Hosmer and Lemeshow 1980).

Osius-Rojek's standardized chi-squared test (Z_{χ^2})

To better deal with situations where the model has a large number of possible covariate groups, Osius and Rojek (Osius and Rojek 1992) showed that the statistic

$$Z_{\chi^2} = \frac{X^2 - (n - m - 1)}{\sqrt{2(i - \sum_{i=1}^{I} \frac{1}{m_i}) + \text{RSS}}}$$

is asymptotically standard normal if I is unbounded. Here, RSS is the residual sum of square from a weighted linear regression of $V^{-1}(1-2\hat{p})$ onto X with weights V.

Stukel's score test (χ_S^2)

Stukel (Stukel 1988) proposed a new link function defined by:

$$oldsymbol{ heta} = ext{logit}(oldsymbol{p}) = ext{ln}\left(rac{oldsymbol{p}}{oldsymbol{1}-oldsymbol{p}}
ight) = h_lpha(oldsymbol{X}oldsymbol{eta})$$

where the new part of the link function $h_{\alpha}(\boldsymbol{x}_{i}\boldsymbol{\beta})$ is defined as follows. If $\boldsymbol{x}_{i}\boldsymbol{\beta} \geq 0$ then:

$$h_{\alpha}(\boldsymbol{x}_{i}\boldsymbol{\beta}) = \begin{cases} \alpha_{1}^{-1}(\exp(\alpha_{1}|\boldsymbol{x}_{i}\boldsymbol{\beta}|) - 1), & \text{if } \alpha_{1} > 0\\ \boldsymbol{x}_{i}\boldsymbol{\beta}, & \text{if } \alpha_{1} = 0\\ -\alpha_{1}^{-1}\ln(1 - \alpha_{1}|\boldsymbol{x}_{i}\boldsymbol{\beta}|), & \text{if } \alpha_{1} < 0 \end{cases}$$

and if $x_i \beta \leq 0$ then:

$$h_{\alpha}(\boldsymbol{x}_{i}\boldsymbol{\beta}) = \begin{cases} -\alpha_{2}^{-1}(\exp(\alpha_{2}|\boldsymbol{x}_{i}\boldsymbol{\beta}|) - 1), & \text{if } \alpha_{2} > 0\\ \eta_{i}, & \text{if } \alpha_{2} = 0\\ \alpha_{2}^{-1}\ln(1 - \alpha_{2}|\boldsymbol{x}_{i}\boldsymbol{\beta}|), & \text{if } \alpha_{2} < 0 \end{cases}$$

This new link function creates a family of generalized logistic models parameterized by the two new parameters α_1 and α_2 . Standard binary link functions like the probit, log-log as well as the complimentary log-log can be approximated up to the fourth moment by members of the generalized logistic family. When $\alpha_1 = \alpha_2 = 0$, $h_{\alpha}(\mathbf{x}_i \boldsymbol{\beta}) = \mathbf{x}_i \boldsymbol{\beta}$ and the generalized logistic model becomes the standard logistic model making it possible to test the logistic assumption by a score test (χ_S^2) .

Assessment of discriminative power

Even a well specified model can have a low discriminative power, i.e., not being able to separate postive from negative outcomes. Several measures of discriminative power exist and what follows is a description of the measures used in this paper.

ROC-curve

The performance of a logistic regression model can be assessed by the ratio of predicted positive outcomes and observed positive outcomes, also called sensitivity. Equivalently, the ratio of predicted negative outcomes and observed negative outcomes, also called specificit, can be used to evaluate the performance of a logistic regression model. To calculate sensitivity as well as specificity, a cut-off probability has to be chosen, i.e., all outcomes with a predicted probability lower than the cut-off probability are classified as negative outcomes. The ROC-curve plots sensitivity versus 1-specificity for different cut-off-probabilities. When the cut-off-probability is zero, all observations are predicted as positive outcomes; hence, sensitivity becomes one and specificity becomes zero. The opposite being a cut-off-probability of one leading to a sensitivity of zero and a specificity of one.

Area under the curve (AUC)

AUC is the area under the ROC curve and $0.5 \le AUC \le 1$ (if the area < 0.5, negative and positive outcomes are reversed). The AUC value is computed using leave-one-out validation.

Leave-one-out validation

Leave-one-out validation is used to reduce model bias towards fitted data when evaluating discrimantive power. The model is refitted to a sample excluding the observation whose value is to be predicted. The excluded observation is then predicted by the fitted model. This is done for all observations in the sample. The procedure gives a set of predicted values with a lower bias towards observed data, and hence, gives a more realistic picture of discriminative power.

Coefficient of Discrimination (D)

The coefficient of discrimination proposed by Tjur (Tjur 2009) measures the model's explanatory capability with respect to fitted data. It is defined as the difference between expected probability for a positive outcome minus the expected probability for a negative outcome, i.e.,

$$D = \hat{\bar{\pi}}_1 - \hat{\bar{\pi}}_0$$

D is related to the classical coefficients of determination

$$R_{\rm mod}^2 = \frac{\sum_1^n (\hat{\pi}_i - \bar{y}_i)^2}{\sum_1^n (y_i - \bar{y}_i)^2} \text{ and } R_{\rm cor}^2 = \left(\frac{\sum_1^n (\hat{\pi}_i - \bar{y}_i)(y_i - \bar{y}_i)}{\sqrt{\sum_1^n (\hat{\pi}_i - \bar{y}_i)^2 \sum_1^n (y_i - \bar{y}_i)^2}}\right)^2$$

by being their geometric mean

$$D = \sqrt{R_{\rm mod}^2 R_{\rm cor}^2}.$$

The relationship between R_{mod}^2 , R_{cor}^2 and D also gives us that D is always between zero and one. D reaches zero if, and only if, all predicted probabilities are equal. D reaches one if, and only if, all predicted probabilities are equal to their observed value, i.e., zero or one.

A way to visualize the idea behind the D coefficient is to plot two histograms, one over predicted probabilities given positive outcome and one over predicted probabilities given negative outcome.

Sample Characteristics

The sample consisted of 131 subjects who were all recruited from either Uppsala University's psychiatric clinic for young adults or from the nursing institution at Uppsala University. All subjects completed the ETI questionnaire. There were two subjects who had more than 20% missing values and were subsequently removed. The final sample consisted of 129 subjects and had a total of 0.87% missing values. Subjects were measured on 72 ETI items, sex, age and PTSD diagnosis. A total of 59 subjects were patients at the psychiatric clinic at Uppsala University with a confirmed PTSD diagnosis and 70 subjects were nursing students/teachers at Uppsala University and were all confirmed healthy by mental health proffesionals. The PTSD group consisted of 86% women, with an age-range of 18 to 27 and a median of 22. The control group consisted of 73% women, with an age-range of 19 to 66 and a median of 23. The control group had 11 members older than 30 years of age. Table 1 shows the frequency of endorsement of the ETI items, i.e., the number of subjects who had experienced the event at some point in their lives.

Table 1: Frequency of endorsement of ETI items

			FREQ. OF E	NDORSME	NT	
ITEMS		PTSD			CONTROL	
	ơ(%)	Q(%)	TOTAL	്(%)	Q(%)	TOTAL
General trauma item						
T1. Natural disaster	2/8(25)	3/51(6)	5/59(8)	0/19(0)	1/51(2)	1/70(1)
T2. Serious accident	3/8(38)	8/51(16)	11/59(19)	1/19(5)	3/51(6)	4/70(6)
T3. Serious personal injury	2/8(25)	8/51(16)	10/59(17)	1/19(5)	2/51(4)	3/70(4)
T6. Serious injury/illness of parent	4/8(50)	19/51(37)	23/59(39)	3/19(16)	17/51(33)	20/70 (29)
T7. Separation of parents	3/8(38)	29/51(57)	32/59(54)	5/19(26)	15/51 (29)	20/70 (29)
T10. Serious illness/injury of sibling	1/8(12)	13/51 (25)	14/59(24)	3/19(16)	5/51(10)	8/70 (11)
T12. Serious injury of friend	6/8 (75)	19/51(37)	25/59(42)	2/19(11)	6/51(12)	8/70 (11)
T13. Observe death/serious injury of others	2/8(25)	14/51(27)	16/59(27)	3/19 (16)	7/51 (14)	10/70 (14)
T15. Witnessing violence	3/8 (38)	25/51(49)	28/59(47)	5/19(26)	11/50(22)	16/69 (23)
T16. Family mental illness	5/8(62)	35/51(69)	40/59(68)	6/19(32)	14/51(27)	20/70 (29)
T17. Alcoholic parents	3/8(38)	13/51(25)	16/59(27)	1/19 (5)	6/51(12)	7/70 (10)
T23. See someone murdered	0/8(0)	0/51(0)	0/59(0)	0/19(0)	0/51(0)	0/70(0)
Physical abuse item	, , ,	, , ,	, , ,	, ()	, , ,	, , ,
P2. Slapped in the face	2/8(25)	23/51 (45)	25/59(42)	3/19(16)	5/51(10)	8/70 (11)
P3. Burned with cigarette	1/8(12)	2/51(4)	3/59(5)	0/19(0)	0/51(0)	0/70 (0)
P4. Punched or kicked	4/8(50)	23/51(45)	27/59(46)	4/19(21)	8/51 (16)	12/70 (17)
P6. Hit with thrown object	3/8(38)	20/51(39)	23/59(39)	1/19(5)	6/51(12)	7/70 (10)
P8. Pushed or shoved	4/8(50)	23/51(45)	27/59(46)	4/19(21)	10/51(20)	14/70 (20)
Emotional abuse item	/ - (/	-/- (-)	.,	, - ()	-/- (-)	/ (
E1. Often put down or ridiculed	5/8(62)	27/51 (53)	32/59(54)	3/19(16)	4/51(8)	7/70 (10)
E2. Often ignored or made to feel you didn't count	5/8(62)	31/51(61)	36/59(61)	3/19(16)	4/51(8)	7/70 (10)
E3. Often told you are no good	6/8(75)	27/51 (53)	33/59 (56)	2/19(11)	5/51(10)	7/70 (10)
E5. Most of the time treated in cold or uncaring way	5/8(62)	27/51(53)	32/59(54)	1/19(5)	5/51(10)	6/70(9)
E7. Parents fail to understand your needs	5/8(62)	37/51 (73)	42/59(71)	5/19(26)	14/51(27)	19/70 (27)
Sexual abuse item	0/0 (0=)	0.702 (10)		0,-0 (-0)	/ (/)	
S5. Touched in intimate parts in way that was uncomfortable	4/8(50)	31/51(61)	35/59(59)	0/19(0)	5/51(10)	5/70(7)
S6. Someone rubbing genitals against you	4/8(50)	19/51(37)	23/59(39)	0/19(0)	3/51(6)	3/70(4)
S7. Forced to touch intimate parts	3/8(38)	18/51(35)	21/59(36)	0/19(0)	0/51(0)	0/70(0)
S8. Someone had genital sex against your will	4/7(57)	21/50 (42)	25/57(44)	0/19(0)	3/51(6)	3/70(4)
S9. Forced to perform oral sex	3/8(38)	17/50(34)	20/58(34)	0/19(0) 0/19(0)	0/51(0)	0/70(0)
S15. Forced to kiss someone in sexual way	2/8 (25)	18/50(36)	20/58(34)	0/19(0) 0/19(0)	0/51(0)	0/70(0)

Table 2 shows the frequency of exposure to the ETI items. For general trauma items, only five frequency levels exists, i.e., level six and seven are set to zero.

Table 2: Frequency of exposure to ETI items: For general trauma items, only five frequency	levels
exist, i.e., level six and seven are set to zero.	

			EXPOSURE		
ITEMS	PT		CON		TOTAL
	$\sigma(0 1 2 3 4 5 6 7)$	Q(0 1 2 3 4 5 6 7)	$\sigma(0 1 2 3 4 5 6 7)$	Q(0 1 2 3 4 5 6 7)	(0 1 2 3 4 5 6 7)
General trauma item					
T1. Natural disaster	6 1 1 0 0 0 0 0	48 3 0 0 0 0 0 0	19 0 0 0 0 0 0 0	50 1 0 0 0 0 0 0	123 5 1 0 0 0 0 0
T2. Serious accident	5 1 2 0 0 0 0 0	43 4 2 1 1 0 0 0	18 0 1 0 0 0 0 0	48 3 0 0 0 0 0 0	114 8 5 1 1 0 0 0
T3. Serious personal injury	6 1 1 0 0 0 0 0	43 3 4 1 0 0 0 0	18 0 1 0 0 0 0 0	49 1 1 0 0 0 0 0	116 5 7 1 0 0 0 0
T6. Serious injury/illness of parent	4 2 1 1 0 0 0 0	32 11 6 0 2 0 0 0	16 1 1 0 1 0 0 0	34 14 3 0 0 0 0 0	86 28 11 1 3 0 0 0
T7. Separation of parents	5 3 0 0 0 0 0 0	22 26 2 1 0 0 0 0	14 5 0 0 0 0 0 0	36 14 1 0 0 0 0 0	77 48 3 1 0 0 0 0
T10. Serious illness/injury of sibling	7 0 1 0 0 0 0 0	38 10 2 0 0 1 0 0	16 3 0 0 0 0 0 0	46 5 0 0 0 0 0 0	107 18 3 0 0 1 0 0
T12. Serious injury of friend	2 1 4 1 0 0 0 0	32 11 5 3 0 0 0 0	17 2 0 0 0 0 0 0	45 6 0 0 0 0 0 0	96 20 9 4 0 0 0 0
T13. Observe death/serious injury of others	6 0 2 0 0 0 0 0	37 8 6 0 0 0 0 0	16 1 1 1 0 0 0 0	44 6 1 0 0 0 0 0	103 15 10 1 0 0 0 0
T15. Witnessing violence	5 0 1 0 0 2 0 0	26 5 5 5 1 9 0 0	14 2 1 0 0 2 0 0	39 4 6 0 0 1 0 0	84 11 13 5 1 14 0 0
T16. Family mental illness	3 2 1 2 0 0 0 0	16 9 8 4 0 14 0 0	13 3 3 0 0 0 0 0	37 9 5 0 0 0 0 0	69 23 17 6 0 14 0 0
T17. Alcoholic parents	5 0 0 0 1 2 0 0	38 1 1 3 7 0 0	18 1 0 0 0 0 0 0	45 1 2 0 1 2 0 0	106 3 3 1 5 11 0 0
T23. See someone murdered	8 0 0 0 0 0 0 0	51 0 0 0 0 0 0 0	19 0 0 0 0 0 0 0	51 0 0 0 0 0 0 0	129 0 0 0 0 0 0 0
Physical abuse item					
P2. Slapped in the face	6 0 0 1 0 0 0 1	28 10 3 3 1 4 0 1	16 2 0 1 0 0 0 0	46 4 1 0 0 0 0 0	96 16 4 5 1 4 0 2
P3. Burned with cigarette	7 0 0 0 1 0 0 0	49 0 1 0 0 0 0 0	19 0 0 0 0 0 0 0	51 0 0 0 0 0 0	126 0 1 0 1 0 0 0
P4. Punched or kicked	4 1 1 0 0 2 0 0	28 7 3 3 1 7 0 1	15 2 0 2 0 0 0 0	43 6 0 0 2 0 0 0	90 16 4 5 3 9 0 1
P6. Hit with thrown object	5 1 0 1 0 1 0 0	31 7 3 3 3 4 0 0	18 0 0 1 0 0 0 0	45 5 0 1 0 0 0 0	99 13 3 6 3 5 0 0
P8. Pushed or shoved	4 0 0 1 0 1 1 1	28 2 4 7 0 6 0 2	15 2 1 0 0 1 0 0	41 6 0 2 1 1 0 0	88 10 5 10 1 9 1 3
Emotional abuse item					
E1. Often put down or ridiculed	3 0 1 0 0 2 0 2	24 1 4 4 9 0 5	16 0 0 1 1 1 0 0	47 2 0 1 1 0 0 0	90 3 5 6 6 12 0 7
E2. Often ignored or made to feel you didn't count	3 0 0 0 3 0 2	20 1 3 2 8 9 0 6	16 1 0 0 0 1 0 1	47 1 0 1 1 0 0 1	86 3 3 3 9 13 0 10
E3. Often told you are no good	2 0 0 0 2 1 1 2	24 2 3 6 3 7 0 5	17 0 0 1 0 1 0 0	46 3 0 1 1 0 0 0	89 5 3 8 6 9 1 7
E5. Most of the time treated in cold or uncaring way	3 0 1 0 1 1 1 1	24 1 1 3 5 7 1 8	18 0 0 1 0 0 0 0	46 1 0 2 2 0 0 0	91 2 2 6 8 8 2 9
E7. Parents fail to understand your needs	3 0 0 0 2 1 0 2	14 1 2 5 5 13 1 8	14 0 1 2 0 2 0 0	37 1 1 6 3 2 0 0	68 2 4 13 10 18 1 10
Sexual abuse item					
S5. Touched in intimate parts in way that was uncomfortable	4 0 0 2 0 1 1 0	20 12 4 4 7 2 0 2	19 0 0 0 0 0 0 0 0	46 4 0 0 0 0 0 0 0	89 16 4 6 7 3 1 2
S6. Someone rubbing genitals against you	4 0 0 2 0 2 0 0	32 7 4 1 4 1 0 1	19 0 0 0 0 0 0 0	48 2 0 1 0 0 0 0	103 9 4 4 3 0 1
S7. Forced to touch intimate parts	5 0 0 1 0 2 0 0	33 5 3 2 3 2 0 2	19 0 0 0 0 0 0 0	51 0 0 0 0 0 0 0 0	108 5 3 3 3 4 0 2
S8. Someone had genital sex against your will	3 0 1 1 0 2 0 0	29 10 2 2 2 3 0 0	19 0 0 0 0 0 0 0 0	48 2 1 0 0 0 0 0	99 12 4 3 2 5 0 0
S9. Forced to perform oral sex	5 0 0 1 0 2 0 0	33 5 0 4 1 4 0 1	19 0 0 0 0 0 0 0 0	51 0 0 0 0 0 0 0	108 5 0 5 1 6 0 1
S15. Forced to kiss someone in sexual way	6 0 0 0 2 0 0	32 5 1 3 5 2 0 1	19 0 0 0 0 0 0 0	51 0 0 0 0 0 0 0	108 5 1 3 5 4 0 1

Statistical analysis

To remove the final 0.87% of non-response, column mean imputation was used. Column mean imputation imputes the mean of each column wherever a missing value is recorded. This imputation technique is bias toward column independence. However, given the low amount of missing values, a more advanced technique was not considered necessary.

Since 74 covariates were registerd on 129 subjects, it was necessary to reduce the dimension of the parameter space to handle sparsity as well as overfitting. The control variable age was dropped since it was highly centered at 22 with a few outliers. High correlation between items was also expected since data stemmed from a psychometric measurement instrument. Earlier work (J Douglas Bremner, Bolus, and Mayer 2007) suggests constructing general (T), physical (P), emotional (E) and sexual (S) trauma scores by counting the number of endorsed items in each category. This construction reduced the ETI parameters to $2^4 = 16$ parameters, including all interaction terms. When sex was included as a parameter, the total number of model parameters became $2^5 = 32$.

Table 3 shows some descriptive characteristics of the new composite variables. Only the maximum is shown in the table since the minimum for all variables was zero.

Considering a logistic regression with T,P,E,S and Sex as predictors, all five, four and three-way interactions were non-significant. The only two-way interaction that came close to significance (0.1 level) was the interaction between P and E. Since the E and P interaction potentially could have

some effect, it was included in the model. All other two-way interactions were dropped. Sex was not significant in any of the models and was, therefore, excluded as a model parameter. To investigate potential coorelation between predictors, the Spearman correlation coefficients were calculated. The correlation coefficients revealed a correlation of 0.6 between E and P as well as a correlation of 0.51 between T and E.

Table 4 shows goodness-of-fit statistics for all considered models, and table 5 shows coefficient D, AUC and odds ratios.

Table 3: Median of composite ETI scores: Note that only the maximum is shown since the minimum was zero for all scores.

			MEDIAN COMF	OSITE SCO	RES		
COMPOSITE ITEMS		PTSI)	CONTROL			
	$\sigma(MAX))$	q(MAX)	TOTAL (MAX)	$\sigma(MAX))$	q(MAX)	TOTAL (MAX)	
General trauma (T)	4.5(7)	4 (8)	4 (5)	2(2)	1(7)	1.5(7)	
Physical trauma (P)	1(5)	2(4)	2(3)	0(1)	0(3)	0(4)	
Emotional trauma (E)	4 (5)	3(5)	3(5)	0(1)	0(3)	0(5)	
Sexual trauma (S)	1.5(6)	3(6)	3 (4)	0 (0)	0(3)	0 (3)	

Table 4: Goodness-of-fit tests: number of CVPs (|CVP|), number of CVPs with cell frequencies > 5 (|CVP|>5), Pearson's chi-squared test (χ^2), Hosmer and Lemeshow's test (\hat{C}) with nine groups, Osius and Rojek's test (Z_{OR}) and Stukel's score test (χ^2_S).

MODEL	$ \mathrm{CVP} $	$ \mathrm{CVP} > 5$	χ^2	\hat{C}	Z_{OR}	χ^2_S
T+P+E+S+P:E	87	3	45.77 (1)	4.93(0.67)	-0.03(0.98)	0.33(0.85)
T+P+E+P:E	69	4	85.55(0.04)	9.1(0.25)	1.01(0.31)	0.53(0.77)
S+P+E+P:E	60	3	25.31 (1)	5.1(0.65)	-0.04(0.97)	2.17(0.34)
P+E+P:E	25	6	22.43(0.38)	4.46(0.73)	0.17(0.87)	2.08(0.35)
T+E+S	72	3	39.86(1)	9.99(0.19)	-0.12(0.9)	3.06(0.22)
T+P+S	68	5	43.34(0.98)	5.29(0.62)	-0.12(0.9)	0.86(0.65)
T+E	40	4	37.35(0.45)	5.83(0.56)	0.03(0.98)	0.8(0.67)
T+P	35	6	32.59(0.44)	10.44(0.17)	0.05 (0.96)	0.23(0.89)
E+S	32	5	10.4(1)	3.77(0.81)	-0.21(0.83)	3.29(0.19)
P+S	29	3	12.74(0.99)	6.99(0.43)	-0.16(0.87)	5.79(0.06)
T+S	40	4	28.13(0.85)	5.56(0.59)	-0.05(0.96)	0.28(0.87)
Т	9	8	9.92(0.19)	8.17(0.32)	0.46(0.64)	2.59(0.27)
E	6	6	6.45(0.17)	6.35(0.5)	0.72(0.47)	0.93(0.63)
Р	6	5	1.71(0.79)	1.65(0.98)	-0.47(0.64)	1.47(0.48)
S	7	6	2.11(0.83)	0.47(1)	-0.2(0.84)	0.46(0.79)

Based on the results depicted in table 4 and table 5, influential observations were investigated for models PTSD~T+E+S, PTSD~E+S and PTSD~T+S. Studentized residuals (st_i) , hat-values (h_i) and cook's distance were computed for all observations. Table 6 depicts the three most influential observations for each of the remaining models as well as the observation's covariate pattern (CVP).

Given the normal CVP values in table 6 and the high AUC values of table 5, no observations were excluded from the sample.

MODEL	PREI	DICTION	ODDS RATIOS
	D	AUC	
T+P+E+S+P:E	0.66	0.92	$T=1.54* P=0.3\dagger S=4.33*** P:E=1.48\dagger$
T+P+E+P:E	0.44	0.84	$T=1.54^{**} E=1.85^{**}$
S+P+E+P:E	0.63	0.9	$S=4.39^{***} P=0.33^{\dagger} E=1.75^{*} P:E=1.44^{\dagger}$
P+E+P:E	0.37	0.77	E=2.03***
T+E+S	0.64	0.92	$T=1.47^* E=1.92^{***} S=3.39^{***}$
T+P+S	0.57	0.9	$T=1.75^{***} S=3.42^{***}$
T+E	0.44	0.86	$T=1.54^{**} E=2.08^{***}$
T+P	0.32	0.78	$T=1.7^{***} P=1.71^{**}$
E+S	0.61	0.89	$E=2.24^{***} S=3.65^{***} $
P+S	0.49	0.8	$P=1.83^{**} S=3.39^{***}$
T+S	0.56	0.89	$T=1.91^{***} S=3.75^{***}$
Т	0.27	0.74	T=1.92***
Ε	0.37	0.76	E=2.36***
Р	0.2	0.6	P=2.16***
S	0.42	0.68	S=3.92***

Table 5: Prediction of PTSD: coefficient of discrimination (D), area under the ROC-curve (AUC) and estimated odds ratios. Significance level 0.1 is denoted with \dagger .

Table 6: Influential observations: observation number (i), studentized residuals (st_i) , hat-value (h_i) and Cook's distance (C_i)

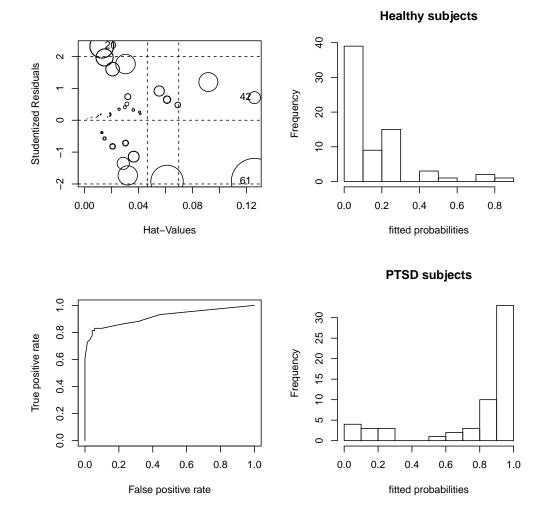
i	Т	Ρ	Е	\mathbf{S}	\mathbf{Sex}	PTSD	st_i	h_i	C_i	MODEL
20	0	0	0	0	0	1	2.65	0.01	0.08	T+E+S
43	7	2	0	1	0	1	0.99	0.18	0.04	T+E+S
61	2	1	0	3	0	0	-1.83	0.14	0.15	T+E+S
20	0	0	0	0	0	1	2.32	0.01	0.06	E+S
42	3	0	0	3	0	1	0.71	0.13	0.01	E+S
61	2	1	0	3	0	0	-1.92	0.13	0.21	E+S
52	0	1	4	3	0	1	0.81	0.15	0.02	T+S
61	2	1	0	3	0	0	-2.35	0.06	0.22	T+S
72	7	0	0	1	0	0	-2.56	0.03	0.20	T+S

Results

Since no model was rejected by the goodness-of-fit statistics, the model selection was based on descriminative power and parameter significance. All parameters in the three models T+E+S, E+S and T+S were significant on the 0.05 level. The T+E+S had an AUC on 0.92, E+S and T+S had both an AUC on 0.89. These models showed that ETI-SR has a strong discriminative capacity between healthy subjects and subjects diagnosed with PTSD.

Discussion

The main goal of the analysis was to investigate the validity of ETI-SR and propose strategies for future research. Validity was investigated by examining whether or not ETI-SR scores could discriminate between healthy subjects and subjects diagnosed with PTSD. Strategies for future research were extracted through observed patterns of association between variables and examining Figure 1: Visual Diagnostics of model E+S: Influence plot (top left), ROC-curve (bottom left) and histograms over predicted probabilities given outcome.



influential observations.

Validity

ETI-SR scores had a strong capacity to discriminate between healthy subjects and subjects diagnosed with PTSD. This fact clearly strengthens the validity of ETI-SR scores as a measure of trauma. The sexual trauma score (S) had the largest impact on the probability of being classified as a PTSD subject. This further increased the validity since sexual trauma is an explicit part of the diagnostic criteria of PTSD.

Associations

The first association observed was the negative parameter estimation of P (odds ratio below one) in the T+P+E+S+P*E model. The negative effect of P upon PTSD in the presence of T,E and S can be explained as follows: P has the lowest capacity to predict PTSD subjects of all predictors. When all predictors are present in the model, P becomes a variable that is used to predict healthy subjects instead of PTSD subjects. This is possible since there are correlations between the predictors. The fact that physical trauma had a poor discriminative capacity could also be investigated in future studies. The second and third pattern of assocations were the correlations between T and E as well as between P and E. The fact that emotional trauma seems to be present when both general and physical trauma are observed could also be investigated in future studies. From a theoretical point of view, it could be argued that emotional trauma is often present whenever another trauma category is observed.

Influential observations

The influential observations gave insight into where the model did not account for observed data. Observations 20,61 and 72 could all be explained as proper outliers. Observations 42 and 52 could not be considered influential given their Cook's distance. Finally, observation 43 should be considered a high leverage point given its high score on the trauma variable.

Sample

The studied sample consisted of only healthy subjects or subjects diagnosed with PTSD and can, for that reason, only be considered to represent a purely hypothetical target population. In future studies, a sample representing a real target population of scientific interest should be of primary focus.

Future studies

Future studies should primarly focus on defining a target population. When a target population is defined and a proper sample collected, a modern analytical approach would employ structural equation modeling (SEM). SEM can partition model variance into a measurement and a regression part, making it easier to detect significant parameters. SEM also offers an in-depth study of correlational patterns such as mediating, moderating as well as causal analysis. The relationship between emotional trauma and the remaining trauma categories could merit for such an in-depth correlational analysis.

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