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A quantile regression analysis of the impact of climate change on the seasonal pollen release in Sweden

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June 2023


#### Abstract

It is well established that climate change has led to significant changes in the timing and intensity of phenological events around the world. One such event is the release of pollen by plants, which is a major cause of allergies in humans. This study aims to investigate the impact of climate change on the timing and intensity of pollen release in Sweden. We also look at whether or not the latitude of plants has a significant influence on seasonal pollen release. Using data provided by the Swedish Museum of Natural History, we compare nonparametric quantile regression and linear regression on empirical quantiles to model the changes in pollen season. We find that empirical quantiles produce better results. Our findings show that the start and peak dates of various species of pollen advance at a similar rate over time in a warming climate while the ending dates move at a slower rate. The amount of days a year the pollen season is deemed to be active thus increases over time. Furthermore, the effects mentioned appear to be intensified at higher latitudes, apart from the extension of seasonal length for grass pollen, which was found to be reduced at more northerly latitudes. To conclude, our findings suggest that climate change is having a significant impact on the timing andintensity of pollen release in Sweden, which is of concern for reasons of public health and potentially for wildlife and agriculture, thus it is important to monitor these changes in pollen release patterns and develop strategies to mitigate their impact.


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Using data provided by the Swedish Museum of Natural History, we compare nonparametric quantile regression and linear regression on empirical quantiles to model the changes in pollen season. We find that empirical quantiles produce better results. Our findings show that the start and peak dates of various species of pollen advance at a similar rate over time in a warming climate while the ending dates move at a slower rate. The amount of days a year the pollen season is deemed to be active thus increases over time. Furthermore, the effects mentioned appear to be intensified at higher latitudes, apart from the extension of seasonal length for grass pollen, which was found to be reduced at more northerly latitudes.

To conclude, our findings suggest that climate change is having a significant impact on the timing and intensity of pollen release in Sweden, which is of concern for reasons of public health and potentially for wildlife and agriculture, thus it is important to monitor these changes in pollen release patterns and develop strategies to mitigate their impact.


Keywords: Pollen, phenology, climate change, quantile regression, linear regression, empirical quantiles.

## PREFACE

I have conducted this research paper as my bachelor's thesis in mathematical statistics at Stockholm University. I want to thank my advisor Martin Sköld who has been of great value throughout this semester by providing me the guidance necessary to complete this thesis. I must acknowledge that I have used
artificial intelligence tools. More specifically OpenAI's ChatGPT has been a valuable tool for gaining a deeper understanding of certain concepts. However I have not used any AI tools to produce any of the contents in my texts, nor has it been used as a source for any in-text information.

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## 1. INTRODUCTION

Over the past couple of decades it has been made clear that our climate is changing rapidly in various ways. Most notably the global average temperature has risen by ca $0.2^{\circ} \mathrm{C}$ per decade since the mid seventies, this constitutes an almost $1^{\circ} \mathrm{C}$ increase over the past half century (Hansen et al., 2006). Limiting our view to only Sweden, we also see significant shifts in this regions' otherwise stable climate. Several studies in recent decades draw the conclusion that plant phenology is impacted by this increase in temperature. Van Vliet et al. (2002) discusses in a study of the seasonal pollen shift in the Netherlands that an advance in the start of the pollen season by 3-22 days took place in the latter third of the 20th century. Likewise this paper strives to understand what seasonal changes have occurred to the pollen season in Sweden. Results
may differ for various species of pollen. Lind et al. (2016) found a stark difference in duration among arboreal plant species compared to herbaceous ones, with the former trending towards an earlier end date, while the latter was pushed to a further date and thus have a longer seasonal duration. Grass pollen, being herbaceous, is the leading cause of pollen allergy in many developed countries, meaning a lot of people suffer from these seasonal changes for an extended time (García-Mozo 2017). In Sweden and other parts of northern Europe however, due to differences in temperature and overall climate, the arboreal types like birch (betula) are the most common cause of pollinosis (D'Amato et al., 2017). Continous monitoring of
pollen conducted by the Swedish Museum of Natural History (NRM) began in 1973 at the Palynological laboratory in Stockholm. Since then multiple other stations have been included in the scope of NRM's continous pollen monitoring program. As of 2022 there are 20 active stations involved (of Natural History, 2023), monitoring the release of several unique species of pollen. In this paper we consider 7 of the most allergenic species, these are the arboreal pollen of alder (alnus), birch (betula), hazel (corylus), oak (quercus), willow (salix), elm (ulmus) and the herbaceous species of grass pollen (poaceae). In this paper
we will attempt to determine the shift in dates of the start and end of the pollen season in Sweden as an effect of global warming of Earth's climate. We will consider global warming as a linear trend over the researched time period as to simplify the process of analyzing pollen patterns. We can do this as research has shown acceptable fitting of linear models over anthropogenic climate change, in regard to temperature (Hansen et al. 2006). An analysis will be conducted based on two separate frequentist quantile regression models, namely linear regression on empirical quantiles (EQ) and nonparametric quantile regression (QR). In a study of seasonal shifts of migratory birds, Karlsson and Hössjer (2022) performs both these methods. This paper covers the majority of the theory in regards to the construction of the statistical models. We also make good use of Takeuchi et al. (2006), a paper on nonparametric quantile estimations, for a more in depth description of the QR method. By its conclusion this research paper aims to have
built a statistical model that can explain the historic shift in pollen seasons for each of the 7 species and also possesses the ability to predict expected further changes in the Swedish pollen season. We begin
our research with the section 2. Litterature review, where we present the findings of what we consider to be the most relevant studies of similar character to what we envisage our own research to look like. The following section is 3. Data, in which all aspects of the data used in the research is presented. We
cover these aspects in four smaller sections. Firstly 3.1 Data collection where a light description of both the physical and digital methods used to collect the data. An extensive description of how we use the data is later presented in the section 3.2 Understanding the data. Here the reader is granted a look at the structure of the data in the form of easily-to-read tables and which variables are to be considered in the research. In 3.3 Data transformation we follow with a description of how the data has been restructured. 3.4 Data selection is where we conduct an explanation of how we deal with missing values and how the data sets are reduced. We conclude the Data section with 3.5 Annual distribution of pollen in which we use density ridge plots to glance at what sort of results to expect. 4. Methods encompasses all the
theoretical definitions of the methods included in this paper. In order to strengthen the significance of our results, we have used two independent methods. Linear regression on empirical quantiles and Nonparametric quantile regression. Linear regression on empirical quantiles, which we refer to as

EQ, is a simple method commonly used to perform statistical analyses on different quantile levels of data, while nonparametric quantile regression $(\mathrm{QR})$ is a more complex method which substitutes the conditional mean of the loss function for the conditional median and requires numerical approaches to the resulting optimization problem. The sections (4.1 Nonparametric quantile regression) and (4.2 Linear regression on empirical quantiles) explain the theoretical aspects of respective method, while simultaneously providing descriptions of how we have applied them in our models. The most
important findings of our models are presented in 5. Results. This part of the paper is divided in to 3 sections for increased readability. The first, 5.1 Statistical model performance across quantiles contains an analysis of how well each of our methods perform across quantiles of the data. it is in this section we determine which method to primarily base our conclusions from in the following sections. There are a lot of interesting results to share from this statistical analysis, the most important of which are presented as a table in 5.2 Estimations and predictions per species. Here we analyze which species of pollen see the most significant seasonal shift and make estimations of which dates to expect the pollen season to be active at certain years. One of our assumptions prior to performing our analysis is that high latitudes correspond to larger shifts in the climate and would thus affect the pollen season to a greater extent than at lower latitudes. In the section 5.3 Geographic influence we attempt to find out whether this assumption is correct or not in regards to predicting seasonal trends in pollen release in Sweden. Results that are not of central importance to our discussion and conclusions are nonetheless accessible in Appendix C. Section 7. Discussion, is divided in to (7.1 Comparison with prior studies)
and (7.2 Further improvements). The former section covers a comparative analysis of our results with other, similar studies. For each of our questions at issue, we may be able obtain a definitive conclusion, in which case we present this in (6. Conclusions). If we are not able to obtain a definitive conclusion, a description of what we is missing for us to draw a conclusion follows. Since there are limits to what we can expect to answer in this thesis, we have set aside a section (7.2 Further improvements) on which parts of our work is inconclusive and how they can be improved upon. This paper concludes with the an

Appendix section. Additional or complementary information which the reader may find interesting, but not vital for the results of which we form our conclusions. Each appendix is refered to by, and refers back to a result or explanation.

## 2. LITERATURE REVIEW

In a paper from the journal 'Proceedings of the National Academy of Sciences" (Hansen et al., 2006), James Hansen and his team were able to deduce that global surface temperature has increased $\sim 0.2^{\circ} \mathrm{C}$ per decade between the mid 1970's and mid 2000's. Assuming this trend also holds for the past 2 decades, we estimate the global average temperature to be almost $1^{\circ} \mathrm{C}$ warmer than half a century ago. Using these trends in global average temperature over time, we are able to form conclusions about how plant phenology, and more specifically pollen release, is affected by a warming climate. A Dutch study
analyzing a 31 year long collection of pollen in the Netherlands saw an advance in the pollen season by 3-22 days, depending on species. With species such as elm, oak and alder advancing between 15 and 18 days, while they recorded lesser advances in willow, birch and grass pollen at 12,10 and 6 days respectively (Van Vliet et al., 2002). In a more local study conducted by the Palynological laboratory
in Stockholm, there was found to have been an advance in the seasonal starting dates for many arboreal species of pollen such as birch, oak, pine etc. Peak dates of pollen release were found to have largely followed the same trajectories as the starting dates, while the end dates for herbaceous pollen species like grass, were seen to have in fact moved to later dates. The length of pollen seasons had increased for some species, grass, mugwort and birch being among them, while most arboreal pollen types saw a decrease in overall seasonal length (Lind et al., 2016). A study of plant phenology in high latitude environments found that their advancement in annual phenology tends to be greater than equivalent plants at lower latitudes (Alecrim et al., 2023). This is consistent with findings that show a greater increase in average yearly temperature near the poles than closer to the equator (Hisano et al., 2021). Research has found that
the ongoing increase in temperature extremes may be contributing to an extended seasonal duration of airborne allergenic pollen across the northern hemisphere (Ziska et al., 2019). For those whom suffer from
pollen allergies, the length and intensity of pollen release would preferably be as low as possible. Not all types of pollen are as allergenic as each other however. People may react differently to various species of pollen. García-Mozo (2017) concludes that grass pollen are the most allergenic worldwide, whereas in an Italian study, it was revealed that birch pollen was the leading cause of allergic pollen reactions in northern europe (D'Amato et al. 2017), due to the fact that this type of pollen is more prevalent in this region. What we can conclude from these findings is that seasonal changes in grass and birch pollen affect the population to a greater extent than other types and thus may be of greater interest to follow. Our
primary source of material covering the theoretical background of our methods can be found in a study of migratory birds (Karlsson and Hössjer, 2022). They came to the conclusion that quantile regression (QR) is more effective than linear regression on quantiles (EQ) at predicting seasonal shifts in migratory birds as a response to climate change. We compare our results in the Discussion section. Another piece of work we have used to build our models is Takeuchi et al. (2006) in which we find more detailed definitions and properties of quantile regression. Many of the most central aspects of quantile regression, like the pinball loss function among other things, are thoroughly explained in Koenker (2005).

## 3. DATA

### 3.1 Data collection

As mentioned earlier in this paper, the monitoring of pollen in the Stockholm region is conducted by the Palynological laboratory at NRM. The laboratory in question uses a Burkard Seven Day Volumetric Spore Trap (HIRST, 1952) to capture pollen and spores from the air through a small entrance meant to resemble the human airways. Thus approximately 10 liters of air passes through the machine each minute, which is what humans tend to consume. In order to capture the pollen particles carried by the passing air, a sticky tape is mounted to a drum rotating at 2 mm per hour. As only a small portion of the tape is exposed to the air at each point in time, this method grants us a good indication of the volume of pollen in the passing air at any given moment. Each captured pollen is individually counted with regular intervals using microscopes. It must be noted that not all stations possess the same equipment. In particular, differing microscope sizes are used across the country. Consequently, the measured values of the pollen counts are biased towards the larger microscopes, thus showing a somewhat inaccurate representation of the true pollen counts (of Natural History, 2022) (of Natural History, 2017). However considering the structure of the data set and the consequent data analysis being relativistic, for which a descriptive presentation follows in the Data section, this phenomenon has been ignored.

### 3.2 Understanding the data

The data set that we have at our disposal contains 5 unique variables: date, station, name, count and factor. Of which all but the factor variable are used in this research paper. We have also added a latitude variable since it is known that higher latitudes contribute to more extreme climate changes (Alecrim et al. 2023). This variable is however entirely dependent on station. A short description of the meaning of each variable is shown in table 1. Equipment for data collection is active only during the period of the predicted pollen season, estimated through using predictive models for pollen activity based on historic results.

| Variable | Type | Description |
| :--- | :--- | :--- |
| Station | Categorical | Geographic location of the pollen monitoring station. |
| Species | Categorical | Genus of the recorded pollen counts. |
| Date | Continuous | Gregorian calendar date on which the airborne pollen were registered. |
| Count | Continuous | Number of individual pollen counted. |
| Factor | Continuous | Reference variable for the size of the microscope used. |
| Latitude | Continuous | Northern latitudinal coordinates of said station. |

Table 1. Description of the variables present in the data sets used in this research paper.

Viewing the years of availability in table 2 we conclude that not all data points are present in the data set. The stations had differing opening dates and not all species tend to be available to begin with. If no consideration for the location of said data points are made, we may observed skewed results, as the geographic distribution of monitored pollen changes over time due to availability. Thus analyzing the data in geographic categories of where they were collected is a necessary consideration. For the English and Swedish names of each pollen genus, see Appendix A.

| Station | Latitude | Pollen genus (since the year) |
| :--- | :---: | :--- |
| Umeå | 62.83 | Alnus, Betula, Poaceae, Ulmus (1979), Salix (1981), Corylus (1987), Quercus (1995) |
| Eskilstuna | 59.37 | Alnus, Betula, Corylus, Poaceae, Quercus, Salix, Ulmus (1976) |
| Stockholm | 59.33 | Alnus, Betula, Corylus, Poaceae, Quercus, Ulmus (1973), Salix (1977) |
| Norrköping | 58.59 | Alnus, Betula, Corylus, Poaceae, Quercus, Salix, Ulmus (1987) |
| Jönköping | 57.78 | Alnus, Betula, Poaceae, Quercus, Salix, Ulmus (1988), Corylus (1989) |
| Västervik | 57.76 | Alnus, Betula, Corylus, Poaceae, Quercus, Salix, Ulmus (1987) |
| Göteborg | 57.71 | Alnus, Betula, Corylus, Poaceae, Quercus, Salix, Ulmus (1979) |
| Malmö | 55.60 | Alnus, Betula, Corylus, Poaceae, Quercus, Salix, Ulmus (1979) |

Table 2. Years for which data for different pollen species are available at the pollen monitoring stations used in our research, as well as their latidudal coordinates.

### 3.3 Data transformation

In order to be able to perform methods like quantile regression and linear regression on quantiles, a transformation of the given data structure is necessary. To be more precise, we must turn each individual pollen, quantified by the 'count' variable, in to its own data point. Using the function uncount () in $R$ we are able to perform the described transformation of data. The daily counts of certain pollen types at any given station can be in the thousands. Each pollen season usually lasting around a month, and the data covering multiple species of pollen at a multitude of geographic locations over a number of years, means in return, that the size of the data set increases dramatically and as a consequence slows down computation time.

### 3.4 Data selection

After altering the structure of the data in to an individualistic format, we first remove any observations with missing values in any of the columns date, station or species, since these are fundamental parameters to perform the following data analysis. For the purposes of EQ regression, the time it takes to compute the models on the entire data set in R is negligible. For nonparametric quantile regression however, the computational time appears to grow very quickly with the amount of observations. This has led us to the decision to reduce the amount of content in each data set (one data set containing all observations of any combination of station and species) to below a fixed limit. Without reducing
any data, the largest data sets contain $\sim 500000$ observations, while some combinations of data contain considerably fewer. By testing and optimizing the models for various sizes of the data sets we decided to set the limit to 5000 observations per combination of data. This reduces the running time considerably, making the process of analyzing our results much easier. In order to minimize the effect this has on our results the rows must be removed uniformly over the time of year. If the number of observations in a data set is smaller than $n \cdot 5000$, we want to select every $n$th element and discard the rest. We can do this in $R$ by using the slice () and bind_rows () functions from the dplyr package. By performing these modifications to our data, we are sure to keep the shape of the distributions mostly the same and thus the results of the quantile regression. Information about by how much each data set was reduced by can be found in Appendix B.

### 3.5 Annual distribution of pollen

Before constructing the models, we can take a glance at what sort of results to expect from the subsequent regression analysis. In figure 1 we present a ridge plot containing information about the distribution of observed pollen by date of year. The geom_density_ridges() function from the R library
ggridges uses kernel density estimation (KDE) to fit a continuous density function to what is, in essence, a histogram of the frequency of pollen on each date. KDE is a non-parametric method used to estimate the probability density function of a random variable. This is achieved by placing a kernel at each data point. In our case the kernels are Gaussian distributed. We then summarise each of the kernels' distributions in order to obtain a smooth continuous approximation of the density. The width of the kernel, known as the bandwidth, determines by how much each data point contributes to the density estimate. Larger bandwidths result in smoother but less detailed density estimates, while smaller bandwidths result in more detailed but noisier density estimates (Duong, 2001). A common way to find the optimal bandwidth is to use the Asymptotic Mean Integrated Squared Error (AMISE). The function geom_density_ridges () uses an alternative method based on the [@10.2307/2345597] plug-in bandwidth selection method. By looking at the average distribution from the first and last 5 years in the
data set respectively, we construct a somewhat consistent average in terms of the state of the pollen season at a given location, reducing the risk of falsely identifying outlier data as patterned occurrences. We can conclude that, in the Stockholm area, the pollen season seemed to begin earlier for all the arboreal pollen species while grass pollen (Poaceae) did not appear to show those tendencies. In general the shape of the pollen distribution was not seen to have been altered over time. A visual analysis of the distribution of annual pollen release may suggest that the average pollen season now begins earlier by up to a month compared to 45 years ago.


Figure 1. Ridge-line figures approximating the density functions of the distribution of annual pollen. Comparing the observed dates of pollen capture for the average of the years 1973-1978 (green) with the average for 2018-2022 (red).)

## 4. METHODS

In this section we explain the underlying theory behind the statistical approaches used for our data analysis. We will compare "linear regression on empirical quantiles" (EQ), as used in Karlsson and Hössjer (2022) with the supposedly more powerful method "nonparametric quantile regression" which we refer to as QR. In the QR approach, we estimate the response variable as the conditional median of the predictor
variables, of which the median can be substituted to any other quantile of data. In the EQ method however, we use the method of ordinary least squares (OLS) to estimate the conditional mean of a subset of the predictor variables corresponding to the desired quantile level. Ordinary linear regression is the preferred method for many research purposes due to its inherent simplicity. In this research paper however, we are more interested in patterns for certain quantiles of data than the mean. To do this effectively we have conducted our research using quantile regression methods instead. An aspect in which quantile regression
performs better than linear regression is when data is not homoscedastic or normally distributed. Linear regression models perform poorly for data that is heteroscadastic and/or non-normally distributed. By estimating the conditional median however, as opposed to the mean, we get a model that yields better predictions for data with these properties. Another advantage of using quantile based methods is that they are less sensitive to outliers, since nonlinear tendencies may lead to abnormal behaviors for more extreme observations, which can be more accurately accounted for when looking at quantiles rather than the mean. All models in this paper will use 'date' as the response variable, or a yearly quantile of 'date'.

The release date of pollen grain $i \in\{1, \ldots, n\}$, where $n$ is the total amount of pollen grains released, is predicted by the models as the response $y_{i}$. We form a covariate vector $x_{i}=\left(1, t_{i}\right)$ which includes an intercept and the predictor variable $t_{i} \in \mathscr{T}=\{1, \ldots, T\}$ which refers to the gregorian year of observation with $T$ being the amount of years monitored. In order to make an educated analysis of the behaviour of
pollen release we need a strict definition for what constitutes a pollen season. There have been many attempts by various authors to find an optimal definition for the dates of which each pollen season covers. In this paper we classify the dates within which the pollen season is deemed active by referring to the EAN definition (Bastl et al. 2018). The EAN database contains a lot of data on pollen release in which they define the pollen season as the date at which at least $1 \%$ of the annual pollen have been counted. By the same definition, the pollen season is said to end when $95 \%$ of all pollen has been released. A reason as to why the starting quantile level and the one at the end of the season are not symmetric (i.e. why $Q_{\text {start }} \neq 1-Q_{\text {end }}$ ) may be similar to how infection transmission models tend to behave, namely that the beginning of the season tends to be very intense while the distribution decreases more slowly towards the end of the season.

When performing calculations of the arithmetic mean of a vector, we use the following formula:

$$
\begin{equation*}
\bar{x}=\underset{\mu \in \mathbb{R}}{\arg \min } \sum_{i=1}^{n}\left(y_{i}-\mu\right)^{2}, \tag{1}
\end{equation*}
$$

where $y$ corresponds to the vector of observed values and $\mu$ is a scalar value that minimizes the sum of squares.

If we instead want to calculate the arithmetic median, we use the absolute values between the vector of observations and the average instead of the square of these values. The median is thus provided by

$$
\begin{equation*}
\hat{x}=\underset{\mu \in \mathbb{R}}{\arg \min } \sum_{i=1}^{n}\left|y_{i}-\hat{x}\right| . \tag{2}
\end{equation*}
$$

Furthermore, if one wants to compute any given quantile $\tau$ given a vector of observations, one may use the minimization algorithm

$$
\begin{equation*}
x_{\tau}=\underset{\mu_{\tau} \in \mathbb{R}}{\arg \min } \sum_{i=1}^{n} \mathscr{L}_{\tau}\left(y_{i}-\mu_{\tau}\right) \tag{3}
\end{equation*}
$$

where $\mathscr{L}_{\tau}(\xi)$ represents the pinball loss function (Koenker, 2005). This is a convex, piecewise linear function which computes the deviation between the predicted quantile and the actual value of the target variable. It is defined by

$$
\mathscr{L}_{\tau}(\xi)= \begin{cases}\xi \cdot \tau & \text { if }(\xi) \geq 0  \tag{4}\\ \xi \cdot(\tau-1) & \text { if }(\xi)<0\end{cases}
$$

The name of the pinball loss function derives from the fact that it's shape somewhat resembles that of a pinball game as shown in figure 2. Rather trivially, the loss is minimized at $\xi=0$, since that means the predicted quantile exactly attains the value of the observed value of the target variable. Overprediction is more heavily penalized than underprediciton for quantiles $\tau<0$, while the opposite is true when $\tau>0$. This is because the loss function places more weight on the residuals above the predicted value for overprediction and below the predicted value for underprediction (Takeuchi et al., 2006).


Figure 2. Illustration of the pinball loss function. Here $\xi$ represents the offset of predicted values from the true values. As the quantile level $\tau$ increases, the slope for underestimation errors becomes steeper (more negative), while the slope for overestimation errors becomes shallower (less negative)

### 4.1 Nonparametric quantile regression

By nonparametric regression models we refer to models that do not make any parametric assumptions about the form of the conditional distribution function, the relationship between the response variable and predictor variable(s) are linear however. By applying the theory of nonparametric models to a linear predictor of the conditional median, we arrive at the basis of a nonparametric quantile regression model.

For a nonparametric QR model, consider the observed response date $y_{i}$ of grain $i$ and the predictor variable year $t_{i}$ with an intercept in the $(n \times 2)$-matrix $X$.

$$
X=\left(\begin{array}{cccc}
1 & 1 & \ldots & 1 \\
t_{1} & t_{2} & \ldots & t_{n}
\end{array}\right)
$$

Let $Y_{i}$ be a stochastic variable corresponding to the observed value of $y_{i}$ with the conditional distribution function

$$
\begin{equation*}
F_{Y_{i} \mid x_{i}}(y)=\mathbb{P}\left(Y_{i} \leq y \mid x_{i}\right), \quad-\infty<y<\infty \tag{5}
\end{equation*}
$$

In order to find the $\tau$-quantile of the conditional distribution of $Y_{i} \mid x_{i}$ we need to find the smallest value of $y$ such that the conditional cumulative distribution function is greater or equal to $\tau$.

So, for each quantile $0<\tau<1$ the inverse

$$
\begin{equation*}
Q\left(\tau \mid x_{i}\right)=\inf \left\{y ; F_{Y_{i} \mid x_{i}}(y) \geq \tau\right\} \tag{6}
\end{equation*}
$$

of the conditional distribution function in (5) represents the conditional quantile function.
Moving forward, our model takes the form

$$
\begin{equation*}
Q(\tau \mid X)=X \beta(\tau)+\varepsilon(\tau) \tag{7}
\end{equation*}
$$

whereby the column vector $\beta(\tau)=\left(\beta_{0}(\tau), \beta_{t}(\tau)\right)^{\top}$ contains the intercept $\beta_{0}(\tau)$ and year as the slope $\beta_{t}(\tau)$ of quantile $\tau$. We define $\varepsilon(\tau)$ as a non-parameterised vector of the error terms Karlsson and Hössjer, 2022).

An optimization problem is then constructed as to minimize the objective function in (5). In ordinary linear regression one typically uses the method of least squared errors (LSE) to find the model that leads to the lowest amount of loss of information. In our case however, we want to learn the parameters of a quantile regression model that can accurately predict the conditional quantiles of the target variable. To do this, we minimize the pinball loss function $\mathscr{L}_{\tau}(\xi)$, which we previously introduced in 4. Methods, over a set of data $X, y$ (Koenker, 2005, Takeuchi et al., 2006).

The resulting regression parameter estimates of the $\tau$-quantile is thus given by the solution to the minimization problem

$$
\begin{equation*}
\beta(\tau)=\underset{b \in \mathbb{R}^{\neq}}{\arg \min } \sum_{i=1}^{n} \mathscr{L}_{\tau}\left(y_{i}-x_{i} b\right) \tag{8}
\end{equation*}
$$

Comparing equation 8 with equation 3 , note that we replace the scalar $\mu_{\tau}$ with $x_{i} b$ in the loss function and minimize over $b$ leading to an estimation of the $\beta$-coefficient which coupled with a vector of data $x$ grants a predictive linear model Karlsson and Hössjer 2022). Since the function (8) is non-differentiable
at 0 , no direct solution exists. Rather we use numerical estimation methods provided in the R library quantreg, such as the Frisch-Newton interior point method to find the optimal point along the $\xi$-axis (Koenker, 2023).

### 4.2 Linear regression on emprical quantiles

Another approach we used is to perform linear regression on empirical quantiles, a statistical method that combines linear regression with year-wise empirical quantiles to model the relationship between a response variable and one or more predictor variables. The basic gist of EQ regression is to fit a linear regression model to the empirical quantiles of the response variable, rather than to the mean. Most source material and notations in this section are collected from (Karlsson and Hössjer 2022). This method is called "empirical" quantile regression because it estimates quantiles of the response variable based on the empirical distribution of the data, rather than assuming a specific distribution for the response variable.

Rather than using the raw gregorian date $y_{i}$ as response variable, we predict an empirical quantile of the dates of a subset of observations. For each year $t \in \mathscr{T}$ we extract a set of observations $\mathscr{Y}_{t}$. Let $\tau \in(0,1)$ be a quantile. For each set $\mathscr{Y} \sqcup$ we let $\hat{F}_{(t)}$ be the empirical distribution function formed by the elements of its set. The corresponding empirical quantile is defined as

$$
\begin{equation*}
\hat{Q}_{(t)}(\tau)=\inf \left\{y \in \mathscr{Y}_{t}: \hat{F}_{t}(y) \geq \tau \mid \mathscr{Y}_{t} \neq \emptyset\right\} \tag{9}
\end{equation*}
$$

As in the previous method we construct the $(n \times 2)$ matrix $X$ by stacking the intercept and vector of years $(1, t)$ on top of each other. For all $t \in \mathscr{T}$, we stack the quantiles into the vector of observations $Y(\tau)$. Next we formulate the linear model

$$
\begin{equation*}
Y(\tau)=X \beta(\tau)+\varepsilon(\tau) \tag{10}
\end{equation*}
$$

As before $\beta=\left(\beta_{0}, \beta_{t}\right)$ defines the regression parameters for the intercept $\beta_{0}$ and year $\beta_{t}$. Recall that in the nonparametric method, no assumptions were made about the error terms. In this approach however,
we assume a normal distribution of the error terms $\varepsilon(\tau) \sim N\left(0, \sigma^{2}(\tau) I_{T}\right)$, where $I_{T}$ is the identity matrix of rank $T$.

The log-likelihood of the model is given by

$$
\begin{equation*}
l\left(\beta(\tau), \sigma^{2}(\tau) \mid Y(\tau), X\right)=\sum_{t \in \mathscr{T}} \log f\left(\hat{Q}_{t}(\tau) \mid(X \beta(\tau))_{t}, \sigma^{2}(\tau)\right) \tag{11}
\end{equation*}
$$

So far in this approach the model grants each year $t$ the same weight, not each individual pollen. Since the amount of pollen observed each year does not remain constant. A reweighting of the log-likelihood may be conducted as to grant each pollen the same weight. To get even weighting of each pollen, we add a weight factor $w_{t}=\left|\mathscr{Y}_{t}\right|$ to each empirical quantile $\hat{Q}_{t}(\tau)$. The reweighed log-likelihood, takes the form

$$
\begin{equation*}
l_{w}\left(\beta(\tau), \sigma^{2}(\tau) \mid Y(\tau), X\right)=\sum_{t \in \mathscr{T}} w_{t} \log f\left(\hat{Q}_{t}(\tau) \mid(X \beta(\tau))_{t}, \sigma^{2}(\tau)\right) \tag{12}
\end{equation*}
$$

This can easily be implemented with the weights argument of the lm function where we set the weight to be $\frac{1}{N_{t}}$ where $N_{t}$ represents the number of observations at year $t$.

To proceed fitting the model, we attempt to find the maximum likelihood estimate (MLE) of $\beta(\tau)$ and $\sigma^{2}(\tau)$ by optimizing

$$
\begin{equation*}
\operatorname{MLE}\left(\beta(\tau), \sigma^{2}(\tau)\right)=\underset{\beta, \sigma^{2} \in \mathbb{R}^{\neq}}{\arg \min }\left\{l_{w}\left(\beta(\tau), \sigma^{2}(\tau) \mid Y(\tau), X\right)\right\} \tag{13}
\end{equation*}
$$

for a given quantile $\tau$. Since the function is continuous and twice differentiable, the solution can easily be found using conventional methods (Karlsson and Hössjer, 2022).

## 5. RESULTS

### 5.1 Statistical model performance across quantiles

Putting both these methods to the test, we take a look at how each method behaves for any quantile of data. In figure 3 we limit the scope of our analysis to birch pollen monitored in Stockholm, similar patterns are however observable for data of various species found at other stations. We observe that the intercept begins around 25 days earlier for the QR model but increases more rapidly over the low quantiles than the EQ model, eventually converging as the quantile level increases, leading to a significantly earlier estimation of when the pollen season begins (low quantiles) using the QR approach. Another phenomenon visible in figure 3 is that the direction of the slopes are different in parts of the second graph. What we observe is that the QR estimates of the Year variable move from positive, to negative values across the quantiles. This would imply that we should expect a later start of the pollen season each year, but an advance in dates across most if the season. Note that the ridge plots in figure 1 show an advance in dates for birch (betula) pollen in Stockholm. The EQ method appears to do a better job at modelling this seasonal shift that QR .

An aspect in which QR appears to be a stronger alternative is the fact that the $95 \%$-confidence intervals are consistently lower using this method. Based on this and prior mentioned information, one may ask oneself which of these approaches we deem to be the most fit to represent the behaviour of the pollen season. The differing sizes of the confidence intervals make tempting to chose QR as the most viable method, however the differing direction of slopes in the QR model is concerning as our previous glances at the data tell us otherwise, this pattern is neither intuitive nor is it consistent among stations and species, meaning we find this pattern in some combinations of data, but not others. Based on these finding, we choose to continue with what we consider to be the safer option which is linear regression on empirical quantiles (sacrificing precision for accuracy). Admittedly, the kernel density ridges in figure 1 may not be able to highlight opposing directions of seasonal shift particularly well, as a smoothed density is fitted onto the distribution, leading to a loss of information and therefore accuracy.

In order to understand the stark differences in coefficients for low quantiles between the two methods, let us take a look at how the models look overlayed on the set of observations.


Figure 3. Line graph over the coefficient and intecept estimates for each quantile of betula pollen in Stockholm. The thicker, opaque bounaries define the $95 \%$-confidence intervals for each method. The red line shows the linear regression coefficient without quantile parameterization. As we conduct a comparison between the methods $E Q$ and $Q R$, we find that the $E Q$ method is more in line with the red ordinary linear regression line than QR in both figures, implying there is a more uniform shift in dates across all quantiles. QR produces lower confidence intervals. Note that the year parameter using QR estimates positive coefficients (pollen season moving to later dates) for low quantiles.

Figure 4 tells us the difference in fitting of both our models. The quantile regression method fits a linear model to encompass a given quantile, in this case the first $1 \%$ of annual pollen release, below the regression line. We can observe that year 17 appears to be a heavy outlier in which the entire pollen season came significantly earlier than other years. Since the QR method tries to fit a linear trend with a specific amount of observations below it, and a large amount of these first $1 \%$ of observations are from the 17th year, the model is heavily skewed by the observations from this year and thus does not generate a good fit. EQ does not succumb to this issue since we in this approach aggregate the observations from each year to a specific date value (green points) and then perform ordinary linear regression on these aggregated observations. In this case the one outlier year corresponds to only $2 \%$ of the total observations while for QR this effect appears to be well over $50 \%$. These results explain why the slopes for low quantiles of QR are so different to the equivalent measure of EQ . Thus we conclude that linear regression on empirical quantiles is a more fitting approach to use in the case of estimating the annual seasonal shift of pollen.


Figure 4. Jitter plot over the annual distribution of betula pollen in Stockholm comparing QR and EQ at the $1 \%$ quantile level. Blue line is the fitted nonparametric quantile regression $(\mathrm{QR})$ line which keeps $1 \%$ of observations (black dots) below it self, while the green line represents the linear regression (EQ) slope fitted on to the averaged points highlighted in green.

### 5.2 Estimations and predictions per species

| Species | Avg slope (1\%) | Avg slope (50\%) | Avg slope (95\%) | Slope (season length) |
| :--- | :---: | :---: | :---: | :---: |
| Corylus | -0.755 | -0.539 | -0.408 | 0.42 |
| Alnus | -0.490 | -0.418 | -0.384 | 0.30 |
| Salix | -0.432 | -0.396 | -0.178 | 0.30 |
| Ulmus | -0.336 | -0.184 | -0.303 | 0.16 |
| Quercus | -0.326 | -0.217 | -0.163 | 0.18 |
| Betula | -0.188 | -0.135 | -0.144 | 0.00 |
| Poaceae | -0.092 | -0.201 | -0.132 | 0.28 |

Table 3. Modelled pollen season shift by species averaged over all monitored stations.

| Species | Estimated <br> start 1973 | Predicted start <br> 2023 |  | Predicted start <br> 2050 | Duration <br> $(1973)$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Corylus | April 7 | February 23 | January 31 | Duration <br> $(2023)$ | Duration <br> $(2050)$ |  |
| Alnus | April 4 | March 1 | February 11 | 21 days | 40 days | 52 days |
| Salix | April 20 | April 1 | March 22 | 35 days | 50 days | 44 days |
| Ulmus | April 25 | April 1 | March 19 | 17 days | 25 days | 30 days |
| Quercus | May 29 | May 7 | April 25 | 11 days | 20 days | 26 days |
| Betula | April 30 | April 20 | April 14 | 25 days | 25 days | 26 days |
| Poaceae | June 1 | May 28 | May 26 | 58 days | 72 days | 80 days |

Table 4. Estimated starting dates and seasonal duration averaged over all monitored stations.

| Species | Avg slope (1\%) | Avg slope (50\%) | Avg slope (95\%) | Slope (season length) |
| :--- | :---: | :---: | :---: | :---: |
| Alnus | -0.777 | -0.501 | -0.351 | 0.70 |
| Corylus | -0.648 | -0.347 | -0.332 | 0.36 |
| Salix | -0.473 | -0.432 | -0.231 | 0.24 |
| Ulmus | -0.420 | -0.373 | -0.335 | 0.18 |
| Betula | -0.349 | -0.298 | -0.204 | 0.14 |
| Quercus | -0.322 | -0.266 | -0.257 | 0.12 |
| Poaceae | -0.042 | -0.128 | 0.205 | 0.30 |

Table 5. Modelled pollen season shift by species averaged in the Stockholm region.

| Species | Estimated <br> start 1973 | Predicted start <br> 2023 |  | Predicted start <br> 2050 | Duration <br> $(1973)$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Alnus | March 31 | February 26 | February 8 | Duration <br> $(2023)$ | Duration <br> $(2050)$ |  |
| Corylus | March 27 | February 17 | January 28 | 25 days | 51 days | 70 days |
| Salix | April 24 | April 1 | March 19 | 36 days | 48 days | 53 days |
| Ulmus | April 23 | April 2 | March 22 | 14 days | 23 days | 28 days |
| Betula | May 3 | April 16 | April 7 | 23 days | 30 days | 33 days |
| Quercus | May 26 | May 10 | May 2 | 11 days | 17 days | 20 days |
| Poaceae | June 2 | May 27 | May 27 | 66 days | 81 days | 88 days |

Table 6. Estimated starting dates and seasonal duration averaged in the Stockholm region.

Table 3 grants us information about how our EQ models predict the pollen seasons, for all monitored species, to respond to increasing temperatures in the atmosphere. The entries in the columns named Slope are the average of the $\beta$-coefficient values generated at different combinations of species and station, for each species of pollen. Although some pollen monitoring stations generated results with positive coefficients for certain species, such a phenomenon appears not to be present when we take the mean of our regression coefficients for each species from all stations. The exception to this however would be the $95 \%$-quantile estimate for grass pollen (poaceae), meaning the grass pollen season is moving to later dates over time, while its start and peak shifts to earlier dates. This brings us to the conclusion that the pollen season unanimously is headed towards an earlier start, as well as peak, as the climate warms. The date at which $95 \%$ of all annual pollen gets monitored also seems to arrive earlier each year. The exception to this pattern is, as we point out, grass pollen. The slope estimates for the end of season arrival dates are however lower than the starting dates across all species. By using the linear regression coefficients shown in the table, we can predict the dates at which quantiles of the annual pollen release are expected to appear at. Using the $1 \%$ quantile estimate, we observe in table 4 that the predicted dates for
hazel and alder pollen moved from early April back in 1973 to late February as of 2023 and are expected to reach late January and early February respectively by 2050. The willow and elm seasons appear to shift at a slightly slower pace moving from late to early April over the same 50-year time span. Meanwhile birch pollen only accumulated a movement of 10 days over the past half century. Oak pollen now starts to appear in early May rather than towards the end of the month, which appears to have been the case in the 1970's. Just as we saw in the ridge plots in section 3.5 Annual distribution of pollen, the starting dates of the grass pollen season are not seen to be as greatly affected by climate change as for arboreal pollen, although we find the peak to advance by 0.20 days annually, which is faster than at least one arboreal plant (birch). Something we can observe across the board in regards to the monitored pollen species however, is
that the season lengths (difference in dates between the $95 \%$-quantile and $1 \%$-quantile) become longer over time. This phenomenon is present in all species we have covered in this paper. The largest increase in pollen season length among the arboreal species is seen in hazel pollen ( 21 days in 50 years and 33 days in 77 years) while grass pollen, which saw negligible changes to its starting dates, still saw a significant increase in season length ( 14 days in 50 years and 22 days in 77 years), largely owing to the fact that the seasonal end moves to later dates by approximately 0.13 days a year. Birch pollen saw the smallest increase as the season length is extends by barely a day after 77 years of increasing temperatures. Looking
at the equivalent results from the Stockholm region in tables 5 and 6, we see similar slopes and predicted starting dates, although season lengths appear somewhat longer than average. In this region we find alder to be the species most heavily affected by climate change and a smaller advance in the start and peak of annual grass pollen distributions. While the slopes tell us a great deal about alterations in the dates of pollen release, it is important to consider that the average P -values for models of some species are above $5 \%$, meaning they are not statistically significant. Looking at the average values for the adjusted $R^{2}$ of these models, we see that none are negative, meaning our models are a better fit that a horizontal line. There could be many explanations for why the annual distribution of pollen behaves the way we have
observed. In this paper we are not aiming to try and answer this question. Some quick thoughts however are that with an ever warmer climate, the temperatures at which plants are able to pollinate are present more days of the year, meaning the window of opportunity for trees and grass to release their pollen gets longer over time. The early pollen is what, for most species, gets shifted most heavily to earlier dates. An idea to why this may be the case is that since the spring arrives earlier over time, plants are able to begin the process of pollination earlier and consequently release most of their pollen (which always occurs in the spring months for arboreal species) at earlier dates, leaving the later parts of the year with less relative amounts of pollen. Results for each individual station can be accessed in Appendix C.

### 5.3 Geographic influence



Figure 5. Visualization of how higher latitudes impact the regression coefficient estimations per species (season start, $1 \%$ quantile). For reference Sweden spans approximately the latitudes $55^{\circ} \mathrm{N}$ to $69^{\circ} \mathrm{N}$.


Figure 6. Visualization of how higher latitudes impact the regression coefficient estimations per species (season peak, $50 \%$ quantile). For reference Sweden spans approximately the latitudes $55^{\circ} \mathrm{N}$ to $69^{\circ} \mathrm{N}$.


Figure 7. Visualization of how higher latitudes impact the regression coefficient estimations per species (season end, $95 \%$ quantile). For reference Sweden spans approximately the latitudes $55^{\circ} \mathrm{N}$ to $69^{\circ} \mathrm{N}$.

In figures 5 through 7 we can view the regression coefficients for each species and how they differ based on the latitude of the station the pollen were collected at. Thus we can determine whether pollen in colder climates are affected by climate change by a greater extent than in warmer areas, or vice versa. Figure 5 tells us how the beginning of the pollen season shifts at the observed locations throughout Sweden. Alder, hazel and oak all appear to have more dramatic seasonal shifts at northerly latitudes, while grass, birch, willow and elm show the opposite effect. The median quantile, which we also refer to as the peak of the pollen season was not observed to have had a significant impact of the latitude of where it took place, as seen in figure 6 . The steep regression slope of the peak of the elm pollen seasonal shift comes down to the fact that the coefficient at the highest latitude is vastly different to the others. By that metric, a conclusion can not be made about whether this is proof of climate change impacting elm pollen at higher latitudes simply an outlier. It is worth noting that the p-value of the aforementioned elm coefficient in Umeå is 0.981 . By all accounts, this points to the data point being an outlier. By figure 7 we see how the coefficients for the $95 \%$-quantiles are estimated. We now observe, in contrast to the $1 \%$-quantiles, that grass pollen attributes to a quicker movement toward earlier dates, the further north one goes. To summarise, the length of the grass pollen season appears to be reduced at higher latitudes. The remaining pollen species' seasonal ends do not appear to be strongly affected by latitude. What we can conclude from this is that the seasonal window for grass pollen does not expand as much at high latitudes than what can be observed at lower latitudes. Alder pollen appears to, somewhat consistently, attain more strongly negative coefficients, for any quantile, at the higher latitudes. Other species were not observed to be affected by the latitude of their release by any greater extent.

## 6. Conclusions

The results in this study point towards EQ being a more fitting model than QR when predicting changes in seasonal pollen distributions. Largely, this has to do with the fact that the spread, or variance of dates is fairly low since the linear regression lines are based on yearly averages. Nonparametric quantile regression models struggle to predict extreme quantiles due to clustering of extreme observations in the case of abnormally hot or cold years, as discussed in 5.1 Statistical model performance across quantiles. Based on the slopes of our predictive linear models, we conclude that an advance in the seasonal start
of pollen release among arboreal species occurs by between 0.19 and 0.76 calendar days per year. The seasonal peak in arboreal pollen appears to also be heavily affected by climate change, as an advance of 0.14 to 0.54 days per year is observed. The 95 th percentile of annual observed pollen was found to not advance as dramatically, resulting in longer periods of seasonal activity. Since 1973, season lengths of arboreal pollen have increased by 8-21 days on average with the exception of birch, for which no
noticeable change in season length was observed. Grass pollen saw an increase of 14 days over the same 50 -year time frame. Grass pollen is the only species to attain a positive slope (pollination occurring later over time) as we see ending dates regress by 0.13 days annually. A comparison of seasonal pollen
distributions between various locations, as seen in 5.3 Geographic influence reveals that the slopes can differ quite dramatically between latitudes. In particular we see that alder, hazel and oak pollen see more dramatic advances at higher latitudes, while herbaceous species appear less affected by climate change further north.

## 7. DISCUSSION

### 7.1 Comparison with prior studies

The conclusion made in (Karlsson and Hössjer 2022) was that nonparametric quantile regression (QR) is the most well suited for single as well as multiple species analyses of migratory birds. What we can conclude from this study however is that linear regression on empirical quantiles (EQ) appears to be the preferred method between the two when it comes to modelling the advance in seasonal distribution of pollen. We observed how outlier years influence the slope of QR models more than EQ models. Perhaps this is where we can find an explanation as to why our findings differ in our respective studies. One reason could be that the variance in observed dates among pollen is lower than for migratory birds, leading to a more heavily skewed slope if a cluster of observations falls a lot earlier or later than normal, which tends to be the case for years with abnormal average temperatures. As we recall the results observed in the

Estimations and predictions per species section, our findings reveal that hazel pollen is subject to the largest advance in starting dates as well as overall dates of seasonal pollinosis. Grass pollen, which is not arboreal, we find to not advance its end of season date, rather the opposite effect is observed as there is a push back of 0.13 days per year on average. We find birch to be the species with the smallest change in season length and the shortest advance in starting dates among the arboreal pollen. As mentioned in
2. Literature review, a similar Stockholm based study on seasonal pollen shift in response to climate change (Lind et al., 2016) found that end-of-season dates for herbaceous species of pollen, like grass, had moved to later parts of the year but showed no significant change in starting or peak dates. This is consistent with our findings as we see a push back of 0.21 days per year on average in Stockholm. Just like in (Lind et al. 2016) we see a significant advance in starting, as well as peak dates for all arboreal species. Our averaged results agree on the fact that hazel was the species with the greatest advance in dates whereas if we look exclusively at the phenology in Stockholm, alder appears to have advanced more rapidly. We found oak to be the species with the smallest change in season length and the shortest advance in starting dates among the arboreal pollen in Stockholm, whereas the model provided in (Lind et al., 2016) found lower estimates for both elm and willow trees.

### 7.2 Further improvements

The QR approach struggles with clusters of outliers such as a year with a significantly earlier pollen season. One way to look at it is that there exists a random effect of year that we do not consider in this approach, thus deeming the assumption of independent observations incorrect in the case of our QR model. In order to improve upon these models, one may consider adding a random variable to deal with this yearly effect, resulting in a linear quantile mixed model, which may grant more significant slope estimations than these QR and EQ models. This study only tells us about the distribution of pollen, we do not make
any considerations about the volume of pollen, which may be a telling factor in determining why and how plants' pollination cycle reacts to a warming climate. Thus, further studies of this character may consider integrating pollen volume in their models. A substantial amount of our models are not significant at $95 \%$
confidence level. The majority of these are for combinations of species and station that do not have extensive sets of data, leading to less accurate models. In particular, grass pollen and observations from the Umeå laboratory are overrepresented in poorly fitted models. To increase the accuracy of such models, larger data sets are necessary.

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## APPENDIX

## Appendix A: Translation of the latin names of pollen species

| Latin name | English name | Swedish name |
| :---: | :---: | :---: |
| Alnus | Alder | Al |
| Betula | Birch | Björk |
| Corylus | Hazel | Hassel |
| Poaceae | Grass | Gräs |
| Quercus | Oak | Ek |
| Salix | Willow | Viden |
| Ulmus | Elm | Alm |

Table 7. Pollen species translation in to English and Swedish.

## Appendix B: Information about reduced data sets

In order to make the process of analyzing results a lot quicker, we have reduced the size of each data set to less than 5000 observations. Each data set is made up of observations of individual pollen for all combinations of species and monitoring station. Some data sets are already below this size limit, like all the hazel (corylus) data sets and a a few more at the Umeå station, while others need massive reductions in size. Table 8 denotes sizes of the data sets for combinations of species and station. The denominators of which we reduce the data sets with are represented in brackets. As mentioned in the section Data selection, a reduction in the size of each data set does not affect the shape of their distribution since we use the slice () function to remove observations uniformly across all dates.

| Stations | Alnus | Betula | Corylus | Poaceae | Quercus | Salix | Ulmus |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Eskilstuna | $45362(10)$ | $494464(99)$ | $3594(1)$ | $50537(18)$ | $26899(6)$ | $15398(4)$ | $167(1)$ |
| Göteborg | $31722(7)$ | $451397(91)$ | $3374(1)$ | $31676(7)$ | $42482(9)$ | $14715(3)$ | $33284(7)$ |
| Jönköping | $16454(4)$ | $231147(47)$ | $3812(1)$ | $74372(15)$ | $26482(6)$ | $17940(4)$ | $6509(2)$ |
| Malmö | $55346(12)$ | $212232(43)$ | $4254(1)$ | $48265(10)$ | $54957(11)$ | $13929(3)$ | $35560(8)$ |
| Norrköping | $24089(5)$ | $372668(75)$ | $113(1)$ | $37613(8)$ | $57914(12)$ | $19285(4)$ | $4884(1)$ |
| Stockholm | $31596(7)$ | $286630(58)$ | $5848(1)$ | $22639(5)$ | $40942(2)$ | $31227(7)$ | $11645(3)$ |
| Umeå | $42217(9)$ | $203475(41)$ | $3812(1)$ | $52569(11)$ | $182(1)$ | $8127(2)$ | $9779(2)$ |
| Västervik | $33653(7)$ | $239006(48)$ | $4484(1)$ | $48265(11)$ | $81161(17)$ | $8875(2)$ | $9821(2)$ |

Table 8. Data set sizes for combinations of species and location. Brackets denote the factor of which each data set is reduced by.

## Appendix C: Results per species and location

| Species | Location | Slope | P -value | Adj. R^2 | Estimated start 1973 | Predicted start 2023 | Predcited start 2050 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Corylus | Umeå | -1.08 | 0.00 | 0.35 | May 22 | March 15 | February 6 |
| Corylus | Eskilstuna | -0.87 | 0.00 | 0.42 | April 12 | February 21 | January 25 |
| Corylus | Västervik | -0.85 | 0.00 | 0.24 | April 7 | February 18 | January 23 |
| Corylus | Norrköping | -0.83 | 0.00 | 0.24 | April 18 | February 18 | January 17 |
| Corylus | Malmö | -0.82 | 0.00 | 0.28 | March 27 | February 14 | January 24 |
| Alnus | Stockholm | -0.78 | 0.00 | 0.38 | March 31 | February 26 | February 9 |
| Quercus | Umeå | -0.76 | 0.02 | 0.41 | July 13 | May 12 | April 8 |
| Alnus | Eskilstuna | -0.75 | 0.00 | 0.51 | April 12 | March 2 | February 7 |
| Salix | Eskilstuna | -0.75 | 0.00 | 0.38 | April 24 | April 2 | March 21 |
| Corylus | Stockholm | -0.65 | 0.00 | 0.18 | March 27 | February 18 | January 29 |
| Ulmus | Malmö | -0.64 | 0.05 | 0.08 | April 12 | March 19 | March 6 |
| Corylus | Jönköping | -0.58 | 0.03 | 0.11 | March 18 | February 21 | February 8 |
| Alnus | Malmö | -0.56 | 0.00 | 0.17 | March 26 | February 18 | January 29 |
| Alnus | Umeå | -0.54 | 0.01 | 0.13 | May 2 | March 13 | February 14 |
| Salix | Göteborg | -0.51 | 0.00 | 0.30 | April 19 | March 25 | March 12 |
| Salix | Norrköping | -0.51 | 0.00 | 0.31 | April 19 | March 30 | March 19 |
| Ulmus | Norrköping | -0.48 | 0.00 | 0.19 | April 24 | March 27 | March 12 |
| Salix | Stockholm | -0.47 | 0.00 | 0.24 | April 24 | April 1 | March 20 |
| Ulmus | Eskilstuna | -0.47 | 0.00 | 0.17 | April 25 | April 1 | March 19 |
| Alnus | Norrköping | -0.43 | 0.04 | 0.09 | April 5 | March 1 | February 10 |
| Salix | Malmö | -0.42 | 0.01 | 0.13 | April 8 | March 23 | March 14 |
| Ulmus | Stockholm | -0.42 | 0.02 | 0.09 | April 23 | April 3 | March 23 |
| Betula | Malmö | -0.39 | 0.00 | 0.30 | April 28 | April 10 | March 31 |
| Corylus | Göteborg | -0.36 | 0.22 | 0.01 | March 20 | March 6 | February 26 |
| Quercus | Norrköping | -0.35 | 0.01 | 0.17 | May 26 | May 6 | April 25 |
| Betula | Stockholm | -0.35 | 0.00 | 0.21 | May 3 | April 16 | April 7 |
| Salix | Västervik | -0.35 | 0.04 | 0.09 | April 9 | March 31 | March 25 |
| Alnus | Göteborg | -0.34 | 0.10 | 0.04 | March 27 | March 12 | March 4 |
| Salix | Jönköping | -0.32 | 0.09 | 0.06 | April 15 | March 31 | March 23 |
| Quercus | Stockholm | -0.32 | 0.00 | 0.35 | May 26 | May 11 | May 2 |
| Betula | Eskilstuna | -0.32 | 0.00 | 0.25 | May 5 | April 19 | April 10 |
| Quercus | Malmö | -0.32 | 0.00 | 0.21 | May 18 | April 28 | April 17 |
| Betula | Göteborg | -0.32 | 0.00 | 0.26 | April 30 | April 17 | April 10 |
| Alnus | Jönköping | -0.31 | 0.17 | 0.03 | March 27 | February 21 | February 2 |
| Ulmus | Göteborg | -0.31 | 0.10 | 0.04 | April 18 | March 25 | March 12 |
| Poaceae | Jönköping | -0.29 | 0.02 | 0.12 | June 7 | May 22 | May 14 |
| Quercus | Eskilstuna | -0.29 | 0.00 | 0.17 | May 25 | May 8 | April 29 |
| Quercus | Göteborg | -0.28 | 0.01 | 0.13 | May 26 | May 5 | April 25 |
| Alnus | Västervik | -0.22 | 0.38 | -0.01 | March 28 | March 1 | February 14 |
| Ulmus | Västervik | -0.22 | 0.29 | 0.00 | April 9 | March 31 | March 26 |
| Poaceae | Malmö | -0.21 | 0.06 | 0.07 | May 29 | May 16 | May 10 |
| Quercus | Jönköping | -0.19 | 0.15 | 0.04 | May 20 | May 9 | May 3 |
| Poaceae | Norrköping | -0.19 | 0.12 | 0.04 | June 2 | May 29 | May 26 |
| Betula | Norrköping | -0.18 | 0.10 | 0.05 | April 29 | April 18 | April 13 |
| Ulmus | Jönköping | -0.16 | 0.56 | -0.02 | April 15 | March 29 | March 20 |
| Salix | Umeå | -0.12 | 0.44 | -0.01 | May 7 | April 21 | April 13 |
| Poaceae | Göteborg | -0.12 | 0.20 | 0.02 | June 1 | May 23 | May 18 |
| Quercus | Västervik | -0.09 | 0.50 | -0.02 | May 17 | May 9 | May 5 |
| Poaceae | Stockholm | -0.04 | 0.50 | -0.01 | June 2 | May 29 | May 27 |
| Betula | Västervik | -0.01 | 0.94 | -0.03 | April 22 | April 20 | April 19 |
| Poaceae | Umeå | -0.01 | 0.95 | -0.02 | June 15 | June 14 | June 13 |
| Betula | Umeå | -0.00 | 0.99 | -0.02 | May 12 | May 3 | April 28 |
| Poaceae | Eskilstuna | -0.00 | 0.99 | -0.02 | May 29 | May 29 | May 29 |
| Ulmus | Umeå | 0.01 | 0.98 | -0.05 | June 8 | April 26 | April 3 |
| Betula | Jönköping | 0.07 | 0.58 | -0.02 | April 21 | April 24 | April 25 |
| Poaceae | Västervik | 0.13 | 0.32 | 0.00 | May 21 | May 30 | June 4 |

Table 9. Start of pollen season for different species and locations

| Species | Location | Slope | P -value | Adj. R^2 | Estimated peak 1973 | Predcited peak 2023 | Predicted end 2050 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Corylus | Umeå | -1.13 | 0.00 | 0.41 | May 27 | March 19 | February 9 |
| Alnus | Umeå | -0.77 | 0.00 | 0.28 | May 10 | March 23 | February 25 |
| Salix | Eskilstuna | -0.75 | 0.00 | 0.47 | May 19 | April 20 | April 5 |
| Corylus | Västervik | -0.69 | 0.01 | 0.15 | April 9 | March 11 | February 23 |
| Quercus | Umeå | -0.61 | 0.03 | 0.38 | July 11 | May 20 | April 22 |
| Corylus | Norrköping | -0.53 | 0.05 | 0.08 | April 23 | March 11 | February 16 |
| Alnus | Eskilstuna | -0.52 | 0.00 | 0.43 | April 18 | March 19 | March 2 |
| Corylus | Jönköping | -0.50 | 0.12 | 0.05 | March 26 | March 15 | March 10 |
| Alnus | Stockholm | -0.50 | 0.00 | 0.32 | April 9 | March 21 | March 10 |
| Corylus | Eskilstuna | -0.47 | 0.00 | 0.19 | April 14 | March 15 | February 28 |
| Corylus | Malmö | -0.44 | 0.06 | 0.07 | April 2 | March 9 | February 25 |
| Salix | Stockholm | -0.43 | 0.00 | 0.27 | May 10 | April 16 | April 4 |
| Salix | Malmö | -0.43 | 0.00 | 0.26 | May 14 | April 26 | April 16 |
| Salix | Norrköping | -0.42 | 0.00 | 0.31 | May 3 | April 15 | April 5 |
| Ulmus | Västervik | -0.42 | 0.03 | 0.10 | April 24 | April 5 | March 26 |
| Ulmus | Eskilstuna | -0.41 | 0.00 | 0.26 | May 3 | April 13 | April 2 |
| Alnus | Malmö | -0.39 | 0.05 | 0.08 | April 1 | March 12 | March 2 |
| Alnus | Göteborg | -0.38 | 0.02 | 0.11 | April 9 | March 29 | March 23 |
| Salix | Göteborg | -0.37 | 0.00 | 0.25 | May 3 | April 15 | April 6 |
| Ulmus | Stockholm | -0.37 | 0.00 | 0.15 | April 30 | April 13 | April 4 |
| Corylus | Stockholm | -0.35 | 0.05 | 0.06 | April 7 | March 15 | March 3 |
| Alnus | Norrköping | -0.34 | 0.04 | 0.10 | April 11 | March 21 | March 10 |
| Ulmus | Göteborg | -0.34 | 0.02 | 0.11 | April 28 | April 6 | March 25 |
| Ulmus | Malmö | -0.34 | 0.23 | 0.01 | April 20 | April 8 | April 2 |
| Salix | Umeå | -0.32 | 0.10 | 0.04 | May 27 | May 21 | May 18 |
| Poaceae | Malmö | -0.30 | 0.00 | 0.22 | July 2 | June 17 | June 9 |
| Betula | Stockholm | -0.30 | 0.00 | 0.28 | May 15 | April 30 | April 22 |
| Poaceae | Jönköping | -0.28 | 0.01 | 0.17 | July 10 | June 25 | June 17 |
| Salix | Jönköping | -0.28 | 0.07 | 0.07 | April 30 | April 19 | April 14 |
| Betula | Eskilstuna | -0.27 | 0.00 | 0.19 | May 13 | April 29 | April 21 |
| Poaceae | Göteborg | -0.27 | 0.01 | 0.13 | July 7 | June 23 | June 15 |
| Quercus | Stockholm | -0.27 | 0.00 | 0.27 | May 31 | May 19 | May 12 |
| Poaceae | Eskilstuna | -0.26 | 0.00 | 0.23 | July 8 | June 25 | June 18 |
| Betula | Malmö | -0.25 | 0.00 | 0.18 | May 9 | April 23 | April 15 |
| Betula | Göteborg | -0.24 | 0.00 | 0.17 | May 9 | April 28 | April 22 |
| Alnus | Jönköping | -0.23 | 0.29 | 0.00 | April 4 | March 15 | March 5 |
| Ulmus | Jönköping | -0.23 | 0.27 | 0.01 | April 29 | April 11 | April 2 |
| Poaceae | Norrköping | -0.21 | 0.06 | 0.07 | June 30 | June 24 | June 21 |
| Alnus | Västervik | -0.20 | 0.32 | 0.00 | April 2 | March 20 | March 13 |
| Corylus | Göteborg | -0.20 | 0.41 | -0.01 | April 4 | March 29 | March 27 |
| Quercus | Malmö | -0.19 | 0.08 | 0.06 | May 26 | May 14 | May 7 |
| Quercus | Göteborg | -0.18 | 0.07 | 0.05 | May 29 | May 18 | May 12 |
| Quercus | Norrköping | -0.18 | 0.11 | 0.04 | May 27 | May 19 | May 15 |
| Quercus | Västervik | -0.18 | 0.12 | 0.04 | May 31 | May 20 | May 14 |
| Salix | Västervik | -0.16 | 0.39 | -0.01 | April 25 | April 22 | April 21 |
| Ulmus | Norrköping | -0.15 | 0.57 | -0.02 | April 21 | April 25 | April 27 |
| Poaceae | Stockholm | -0.13 | 0.06 | 0.05 | July 5 | June 28 | June 24 |
| Quercus | Eskilstuna | -0.13 | 0.14 | 0.03 | May 28 | May 19 | May 14 |
| Poaceae | Umeå | -0.11 | 0.15 | 0.03 | July 14 | July 10 | July 8 |
| Betula | Umeå | -0.05 | 0.66 | -0.02 | May 27 | May 18 | May 12 |
| Poaceae | Västervik | -0.04 | 0.74 | -0.03 | June 23 | June 30 | July 4 |
| Betula | Västervik | -0.02 | 0.85 | -0.03 | May 8 | May 1 | April 27 |
| Quercus | Jönköping | -0.01 | 0.95 | -0.03 | May 27 | May 23 | May 20 |
| Betula | Norrköping | -0.00 | 0.97 | -0.03 | May 7 | April 30 | April 26 |
| Betula | Jönköping | 0.05 | 0.66 | -0.02 | May 8 | May 2 | April 29 |
| Ulmus | Umeå | 0.80 | 0.11 | 0.08 | May 25 | May 17 | May 13 |

Table 10. Peak of pollen season for different species and locations

| Species | Location | Slope | P -value | Adj. R^2 | Estimated end 1973 | $\begin{gathered} \text { Predicted end } \\ 2023 \end{gathered}$ | Predicted end 2050 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Corylus | Norrköping | -0.75 | 0.00 | 0.35 | May 6 | March 27 | March 5 |
| Alnus | Norrköping | -0.58 | 0.00 | 0.29 | April 28 | April 5 | March 23 |
| Alnus | Umeå | -0.57 | 0.00 | 0.30 | May 21 | April 8 | March 16 |
| Corylus | Umeå | -0.56 | 0.28 | 0.01 | June 7 | March 29 | February 19 |
| Alnus | Eskilstuna | -0.52 | 0.00 | 0.55 | May 6 | March 30 | March 11 |
| Corylus | Jönköping | -0.48 | 0.10 | 0.05 | April 9 | April 8 | April 8 |
| Ulmus | Västervik | -0.45 | 0.01 | 0.17 | May 9 | April 18 | April 7 |
| Corylus | Eskilstuna | -0.44 | 0.00 | 0.22 | April 26 | March 31 | March 16 |
| Salix | Malmö | -0.44 | 0.00 | 0.26 | June 7 | May 12 | April 28 |
| Corylus | Västervik | -0.44 | 0.03 | 0.11 | April 23 | March 30 | March 17 |
| Ulmus | Eskilstuna | -0.43 | 0.00 | 0.33 | May 13 | April 21 | April 9 |
| Ulmus | Umeå | -0.38 | 0.48 | -0.02 | June 10 | May 19 | May 7 |
| Alnus | Stockholm | -0.35 | 0.00 | 0.16 | April 16 | April 18 | April 20 |
| Alnus | Malmö | -0.34 | 0.03 | 0.09 | April 16 | April 1 | March 23 |
| Alnus | Göteborg | -0.34 | 0.02 | 0.11 | April 22 | April 10 | April 4 |
| Ulmus | Jönköping | -0.34 | 0.28 | 0.00 | May 28 | April 24 | April 5 |
| Ulmus | Stockholm | -0.34 | 0.01 | 0.11 | May 7 | April 26 | April 20 |
| Corylus | Stockholm | -0.33 | 0.01 | 0.11 | April 21 | April 2 | March 23 |
| Poaceae | Eskilstuna | -0.33 | 0.00 | 0.16 | August 17 | July 31 | July 21 |
| Corylus | Malmö | -0.32 | 0.10 | 0.05 | April 14 | April 4 | March 29 |
| Betula | Malmö | -0.29 | 0.01 | 0.16 | May 21 | May 9 | May 2 |
| Ulmus | Malmö | -0.29 | 0.19 | 0.02 | April 28 | April 21 | April 17 |
| Ulmus | Göteborg | -0.29 | 0.01 | 0.14 | May 3 | April 17 | April 8 |
| Salix | Jönköping | -0.28 | 0.10 | 0.05 | May 26 | May 18 | May 15 |
| Alnus | Jönköping | -0.26 | 0.09 | 0.06 | April 17 | April 3 | March 27 |
| Quercus | Stockholm | -0.26 | 0.00 | 0.34 | June 7 | May 27 | May 22 |
| Betula | Göteborg | -0.25 | 0.01 | 0.14 | May 20 | May 11 | May 6 |
| Betula | Eskilstuna | -0.24 | 0.00 | 0.17 | May 26 | May 14 | May 7 |
| Salix | Stockholm | -0.23 | 0.00 | 0.17 | May 30 | May 19 | May 13 |
| Salix | Umeå | -0.22 | 0.04 | 0.08 | June 9 | June 10 | June 11 |
| Quercus | Umeå | -0.21 | 0.44 | -0.04 | July 1 | May 31 | May 15 |
| Quercus | Malmö | -0.20 | 0.10 | 0.05 | June 7 | May 22 | May 14 |
| Betula | Stockholm | -0.20 | 0.00 | 0.17 | May 27 | May 16 | May 10 |
| Salix | Eskilstuna | -0.20 | 0.00 | 0.17 | May 27 | May 20 | May 16 |
| Poaceae | Umeå | -0.20 | 0.02 | 0.11 | August 3 | July 27 | July 22 |
| Quercus | Göteborg | -0.19 | 0.05 | 0.07 | June 6 | May 25 | May 19 |
| Quercus | Västervik | -0.18 | 0.10 | 0.05 | June 8 | May 27 | May 21 |
| Quercus | Eskilstuna | -0.18 | 0.07 | 0.05 | June 7 | May 27 | May 20 |
| Salix | Norrköping | -0.17 | 0.28 | 0.01 | May 13 | May 15 | May 17 |
| Salix | Göteborg | -0.15 | 0.15 | 0.03 | May 20 | May 15 | May 11 |
| Alnus | Västervik | -0.11 | 0.56 | -0.02 | April 14 | April 6 | April 2 |
| Betula | Umeå | -0.09 | 0.41 | -0.01 | June 9 | June 1 | May 28 |
| Quercus | Norrköping | -0.08 | 0.45 | -0.01 | May 31 | May 29 | May 28 |
| Betula | Norrköping | -0.08 | 0.45 | -0.01 | May 21 | May 13 | May 9 |
| Poaceae | Jönköping | -0.03 | 0.84 | -0.03 | August 8 | August 1 | July 29 |
| Betula | Västervik | -0.01 | 0.97 | -0.03 | May 22 | May 16 | May 13 |
| Betula | Jönköping | 0.00 | 0.98 | -0.03 | May 22 | May 13 | May 9 |
| Quercus | Jönköping | 0.01 | 0.92 | -0.03 | June 2 | May 31 | May 30 |
| Corylus | Göteborg | 0.07 | 0.77 | -0.02 | April 13 | April 29 | May 7 |
| Ulmus | Norrköping | 0.09 | 0.76 | -0.03 | April 29 | May 6 | May 10 |
| Poaceae | Stockholm | 0.20 | 0.00 | 0.14 | August 8 | August 18 | August 24 |
| Poaceae | Malmö | 0.22 | 0.16 | 0.03 | July 27 | August 9 | August 17 |
| Poaceae | Norrköping | 0.26 | 0.15 | 0.03 | July 17 | August 9 | August 22 |
| Salix | Västervik | 0.27 | 0.12 | 0.04 | May 8 | May 26 | June 5 |
| Poaceae | Göteborg | 0.29 | 0.06 | 0.06 | July 29 | August 14 | August 23 |
| Poaceae | Västervik | 0.64 | 0.00 | 0.18 | June 29 | August 19 | September 16 |

Table 11. End of pollen season for different species and locations


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