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Abstract

In this work we have used data provided by The Swedish Museum of Natural History from necropsies of over 800 harbour porpoises (Phocoena phocoena) found dead or that have been caught as by-catch in fishing gear between 1975 and 2022. The data have been collected as part of the project "Health and Disease Monitoring of Marine Mammals" by The Swedish Museum of Natural History in collaboration with the (Swedish) National Veterinary Institute. We have used this provided data for three different purposes: 1) To get an insight in morphological differences between populations, the weight-length relationship for the Swedish population have been calculated and compared to the weight-length relationships calculated for other populations of porpoises. 2) As an aid in determine the health status of dead individuals residual body condition indices have been constructed using the animals weight, length and the day of the year the animal had been found. 3) By the means of log linear regression and partial correlation, we have used measurements for circumference and blubber thickness to investigate how these relate to the animals weight and nutritional status. Results and conclusions: 1) Although no hypothesis testing was made to statistically confirm differences between populations, the weight-length relationships for other populations seemed to follow a steeper curve than those calculated for the Swedish populations. These differences were believed to mainly relate to the difference in growth between juveniles and adults and, to the fact that weight-length relationships were calculated for different classes of animals. 2) In testing, the residual body condition indices were shown to be useful in estimating an individuals nutritional status. However, there were indications that the residuals overall were to low and further testing may be necessary. 3) In the log linear regression the circumferences measured anterior of anus were found to correlate strongly with the animals weight. Blubber thickness were found to correlated poorly with weight in the log linear regression. The partial correlation was found to not be a very useful method to explain the animals weight with measurements of circumference and blubber thickness.

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Contents

1 Introduction

The harbour porpoise (Phocoena phocoena) is one of the smallest odontocetes, with adults reaching a length of between 150 and 190 cm and weighing between 50 and 70 kg. In Swedish waters, three separate populations exist with limited genetic exchange: the population of the Baltic Sea, one in the Danish straits, and one in the North Sea (artdatabanken: Phocoena Phocoena). While the Atlantic populations were listed as 'Least Concern' by IUCN in 2020, the population in the Baltic Sea was listed as 'Critically Endangered' in 2008 (IUCN 2020; IUCN 2008).

The health status of the harbour porpoise population in Swedish waters is being monitored by the Swedish Museum of Natural History (SMNH) and the (Swedish) National Veterinary Institute (NVI) as part of the project "Health and Disease Monitoring of Marine Mammals," funded by the Swedish Agency for Marine and Water Management. To survey the health status, porpoises that are found deceased or those accidentally caught in fishing gear are sent to NVI for further examination. A necropsy is performed in collaboration with SMNH personnel, during which various measurements are taken, including age, weight, length, girth, and thickness of blubber at different sites on the animal.

1.1 How to determine the health status of an individual

Blubber thickness is commonly used in marine mammals to assess nutritional status (Kauhala et al., 2019; Marón et al., 2021). However, blubber thickness alone is not sufficient to determine nutritional status as starvation involves the loss of both fat and muscle mass (IJsseldijk et al., 2019). Therefore, it should be complemented with other species-specific parameters of health status (Siebert et al., 2022). Another measurement, girth, is useful in assessing nutritional status, and in toothed whales, starvation is often first observed as depressions in areas lateral to the dorsal fin (Kastelein & Van Battum, 1990).

Blubber thickness and girth are examples of measurements that can be used, to varying degrees, to directly infer nutritional status. However, variables such as age, length, day of the year, and gender, known as *confounders*, co-vary with weight but do not provide direct information about nutritional status. Nevertheless, these confounders, including variables like girth that have a causal relationship with nutritional status, can be used to create a *body condition index*. By comparing this index to an individual's weight, an estimation of its nutritional status can be obtained.

Indices for body condition based on mass and morphometric variables generally fall into two categories: Ratio indices and residual indices (Labocha, Schutz & Hayes, 2014). Of the ratio indices, body mass index (BMI) (for humans) is probably the one best known to the public. It is calculated by dividing mass (in kg) by the square of length (in meters). For humans, a BMI under 18.5 kg/m^2 indicates underweight, 18.5-24.9 kg/m^2 is considered normal weight, 25-29.9 kg/m^2 is overweight, and a BMI over 30.0 kg/m^2 is classified as obesity (National Heart, Lung and Blood Institute).

A residual body condition index, on the other hand, involves a linear regression model with weight as the response variable and length, girth, or other covariates as explanatory variables;

$$\hat{Y} = f(\mathbf{X}) = \alpha + \beta_1 f_1(X_1) + \beta_2 f_2(X_2) \; + \; ... \; + \; \beta_n f_n(X_n), \tag{1}$$

where $\hat{Y}$ denotes the animals weight or a transformation thereof, X_1, X_2, ..., X_n are the explanatory variables such as length or girth and, $f_i()$ is a known transformation of covariate i. The coefficients α, β_1, β_2, ... β_n can be estimated using statistical methods (e.g. maximum likelihood or least square approximation).

When a new animal is found we can calculate the residual for that individual as

$$e = y - \hat{y}, \tag{2}$$

where y is the animals weight or a transformation thereof and $\hat{y} = \alpha + \beta_1 f_1(x_1) + \beta_2 f_2(x_2) \; + \; ... \; + \; \beta_n f_n(x_n)$ is the animals predicted weight (or transformation thereof) using measurements x_1, x_2, ..., x_n corresponding to this individual. A positive residual would indicate that the individual is well nourished whereas a negative one would indicate that it is malnourished or even emaciated.

1.2 Weight - length relationship for harbour porpoises

In line with residual indices, the weight-length relationship for harbour porpoises have been studied in several articles including Bryden (1986), Van Utrecht (1978), Kastelein and van Battum (1990) and Bilgin, Kose and Yesilcicek (2019) (Bilgin, Kose & Yesilcicek, 2019). These weight - length relationships are important tools for marine biologists and fisheries managers and have many applications in stock assessments and ecological studies (Bilgin, Kose & Yesilcicek, 2019). The standard formula when studying weight - length relationships in harbour porpoises is

$$W = \alpha \cdot L^{\beta}, \tag{3}$$

where W is the animals weight and L its length. Taking the logarithm of equation (3) we obtain

$$\log(W) = \log(\alpha) + \beta \cdot \log(L) = \alpha_{log} + \beta \cdot \log(L), \tag{4}$$

and with linear regression can the coefficients α_{log} and β be estimated. In at least two of the aforementioned articles (Kastelein & van Battum 1990; Bilgin, Kose & Yesilcicek, 2019) the weight-length relationships have been used (among other uses) as a way of predicting an animals weight in situations where it's unfeasible to measure the animals weight, these models, however, could also be used to create a residual body condition index.

1.3 Purpose of this project

This work serves two main purposes. Firstly, we aim to develop and test a residual body condition index using regression models with weight as the response variable and the animal's length and/or time of the year as explanatory variables. Secondly, we intend to investigate the relationship between the animal's girth and weight, as well as the relationship between blubber thickness and weight. As confounders such as length, season, age class, and gender may contribute to a significant portion of the correlation in these relationships, partial correlation will be utilized to account for their influence.

In addition to the aforementioned purposes, the weight - length relationships for the Swedish populations will also be investigated.

2 Method and Data

2.1 Data

Variable	Type	Description
Accession, id	discrete	A unique identification number given to each porpoise
Date	discrete	The date the porpoise was found
Source	categorical	How the porpoise was encountered, e.g. 'found dead' or 'fishing'
Nourishment	categorical	For example: 'Well nourished' or 'Less than well nourished'
Decay	categorical	Indication in which state of decay the porpoise was found
Gender	categorical	The animals gender
Age	discrete	The estimated age of the animal
Totallength	continuous	The animals overall length in centimeters
Weight	continuous	The animals weight in kg
Fat, neck ventral	continuous	Blubber thickness at underside of the neck (cm)
Fat, breast ventral	continuous	Blubber thickness at underside of the breast (cm)
Fat, abdomen ventral	continuous	Blubber thickness at underside of the abdomen (cm)
Fat, hip ventral	continuous	Blubber thickness at underside of the hip (cm)
Fat, neck right	continuous	Blubber thickness on the right side of the neck (cm)
Fat, breast right	continuous	Blubber thickness on the right side of the breast (cm)
Fat, abdomen right	continuous	Blubber thickness on the right side of the abdomen (cm)
Fat, hip right	continuous	Blubber thickness on the right side of the hip (cm)
Fat, neck back	continuous	Blubber thickness on the dorsal side of the neck (cm)
Fat, breast back	continuous	Blubber thickness on the dorsal side of the breast (cm)
Fat, abdomen back	continuous	Blubber thickness on the dorsal side of the abdomen (cm)
Fat, hip back	continuous	Blubber thickness on the dorsal side of the hip (cm)
Circumference, neck	continuous	The girth in cm measured around the neck
Circumference, breast	continuous	The girth in cm measured around the breast
Circumference, abdomen	continuous	The girth in cm measured around the abdomen
Circumference, hip	continuous	The girth in cm measured around the hip

Table 1: Names of the different variables found in the data set and a short explication of each one.

The data consisted of measurements from 857 porpoises collected from 1975 to 2022 of these, 40 observations were deemed unreliable due to written comments in the original set and excluded from testing. The data set consisted of 25 variables; in figure 1 we see that we have 3 discrete variables; *accession id*, *date* and *age*, 4 categorical variables; *source*, *nourishment*, *decay*, *gender* and, 18 continuous ones; *total length*, *weight*, the blubber thickness measured at 12 different places and the circumference measured at 4 sites on the animal. Table 1 will give you a short description of each variable.

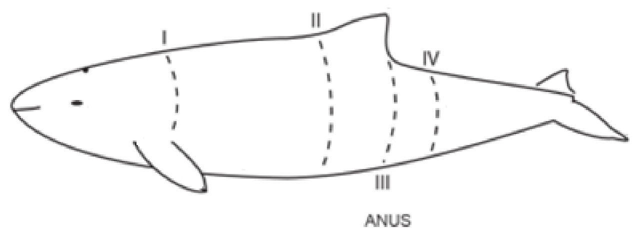


Figure 1: The figure show the sites where circumference and blubber thickness have been measured. The dashed lines denoted by the roman numerals I-IV corresponds to the positions *neck*, *breast*, *abdomen* and *hip*. *Ventral* refers to the chest or bottom part of the animal, *back* to the upper part and *right* to the (righthand) side.

The variable *weight* was used as response variable in our regression models and *total length*, the animals circumference and blubber thickness as well as *date* (by converting it to day of the year) were used as explanatory variables. The variables *gender*, *age* and *total length* were used to divide the data set in age classes, *nourishment* was used to test the residual body condition index and, lastly, *accession id* was useful to match the residuals in the partial correlation. The variables *source* and *decay* were not used in this work.

The data set had a lot of missing values - out of 857 rows only 16 were complete. The main reason for this was that measurements for blubber thickness and circumference were first collected from 2006 (although some measurements date back to 1996). This presents a challenge since regression models with different data set are hard to compare, to counter this we have not chosen to use imputation methods, but instead taking care to use the same subsets of the data when comparing different methods.

In figure 2 below is shown how the missing values are distributed in the data set. Almost 90% of all values in *fat, hip ventral* and nearly 80% of all values in *nourishment* are missing. Furthermore, between 75% and 90% of all values for measurements for girth and blubber thickness are absent.

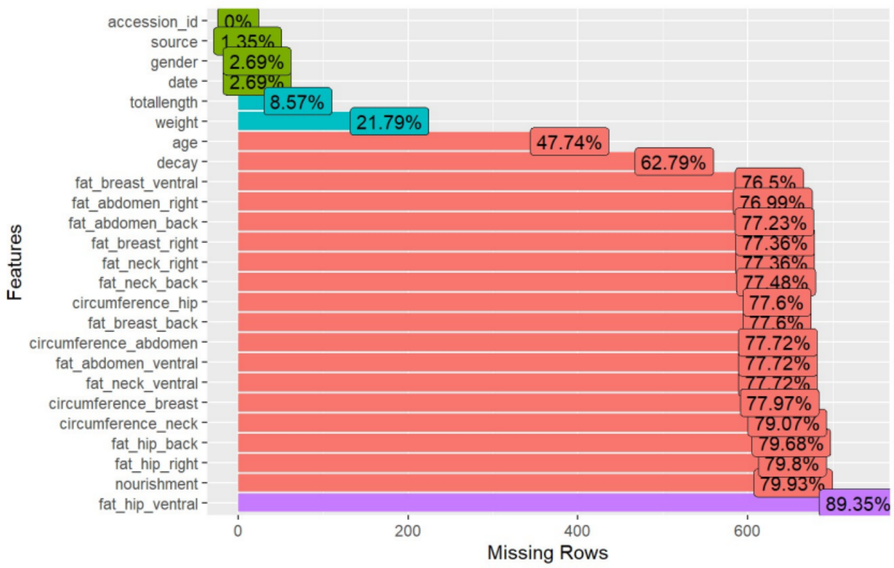


Figure 2: Percentage of missing values by variable

2.1.1 Partitioning of the data

It was assumed that porpoises in different stages of maturity would have different intercepts and slopes in our regression models due to biological factors (e.g. pregnancy and breastfeeding for adult females and, growth in juveniles). Ideally we would have liked to classify the porpoises into groups depending on sexual maturity (calf, inmature and mature) and gender, based on anatomical or biological observations (e.g. Koopman (Koopman, 1998) classified porpoises as a 'calf' if it had unerupted teeth or presence of milk in the stomach). Also dividing the mature females into pregnant, lactating and non-pregnant less lactating would have been desirable since pregnancy and lactation are two factors that affect energy consumption and weight.

Unfortunately there were no information in the data about sexual maturity or if a female were pregnant or lactating. The variables that we had at hand to classify porpoises as mature or inmature were primarily *age* and the animals size (i.e. *total length*). Lockyer (Lockyer, 1995) had in her article partitioned the observations into seven groups based on gender and length (Neonates: < 91cm, Inmature less neonates: $91 - 130$cm, Matures less lactating (Male + Female): > 130cm, Females > 140cm less lactating) and, she further divided females in pregnant, lactating and, pregnant and lactating. In a similar way we decided to divide the data set in "juveniles", "adult females" and "adult males" based on length (juveniles: 100-130 cm, adults > 130cm) or on age (juveniles 1-5 years old, adults > 5 years of age). Factor variables named *length_sex_class* and *age_sex_class*, respectively, were constructed for this purpose. Animals shorter than 100cm or younger than 1 year were classed as "neonatals" (or "yearlings") and excluded from testing. In the rest of the the work we will use these classifications in our testing.

2.2 Theory

2.2.1 Generalized Additive models (GAM)

Examples and theory in this section is derived from a webinar by Gavin Simpson (https://www.youtube.com/watch?v=sgw4cu8hrZM/) and from chapter 3.1, 3.2 and 4.1 of the book *Generalized Additive Models - An Introduction with R* (2006) by Simon N. Wood.

Overview

In linear models we try to model the response variable y_i as a weighted sum of some covariates $x_i, \dots x_n$:

$$y_i = \beta_0 + \sum_{j=1}^{n} \beta_j x_{j,i} + \varepsilon_i, \tag{5}$$

where $\beta_0, \dots, \beta_n$ are coefficients and $\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$ are error terms. Instead in GAM we assume that $Y \sim$ some exponential family distribution and, the linear predictor is, instead of a sum of covariates, made up of a sum of smoothing functions ("smooths", often used when smoothing y using splines)

$$y_i = f(x_i) + \varepsilon_i = \beta_0 + \sum_{j=1}^{n} f_j(x_{j,i}) + \varepsilon_i. \tag{6}$$

Basis functions

Splines and polynomials are examples of functions made up of simpler *basis functions*, $b_{j,k}$, and a set of basis functions are called a *basis.* When using splines in GAM each basis function has a coefficient $\beta_{j,k}$ and, the spline is calculated as the weighted sum of these basis functions

$$f_j(x_j) = \sum_{k=1}^{K} \beta_{j,k} b_{j,k}(x_j), \tag{7}$$

where x_j is the j:th covariate of the model, $j = 1, \dots, n$ and $k = 1, \dots, K$, where n denotes the number of covariates and K the number of basis functions for fj. If f_j was to be, for example, a second degree polynomial then a basis for f_j could be $\{x^0 = 1, x, x^2\}$.

Penalized log-likelihood and penalized least square

In using GAM we want to avoid over fitting and Runges phenomenon in our fitted line. One way to avoid oscillations is to introduce a penalty to our estimated function $f(x)$ when fitting our model.

We measure the smoothing function f tendency to oscillate (it's "wigglines") as:

$$\mathrm{W} = \int_{\mathbb{R}} (f''(x))^2 dx = \boldsymbol{\beta}^T \mathbf{S} \boldsymbol{\beta}, \tag{8}$$

where $f(x)$ is the estimation of y (i.e. the sum of the basis functions with estimated coefficients), $\boldsymbol{\beta} = (\beta_{1,1}, \ldots, \beta_{n,k})^T$ is the parameter vector and $\mathbf{S}$ is the penalty matrix.

In linear models when using a least square approach we minimize $||\mathbf{y} - \mathbf{X}\boldsymbol{\beta}||^2$ to estimate the coefficients β_j, in GAM when estimating the coefficients $\beta_{j,k}$ we want to penalize $f(x)$ for wiggliness. One way to do that is to instead of minimising

$$||\mathbf{y} - \mathbf{B}\boldsymbol{\beta}||^2,$$

where $\mathbf{B}$ are a matrix with the basis functions $b_{j,k}$ evaluated at $x_{j,i}$ and $\boldsymbol{\beta} = (\beta_{1,1}, \ldots, \beta_{j,k})^T$, we minimize

$$||\mathbf{y} - \mathbf{B}\boldsymbol{\beta}||^2 + \lambda \mathrm{W},$$

where $\lambda \in \mathbb{R}$ is a smoothing parameter we can adjust to control the smoothness of our smoothing function $f(x)$. A higher λ will produce a $f(x)$ with less tendency to oscillate. With a too high λ we run the risk of underfitting data and very high λ will result in $f(x)$ being a straight line. A low λ, on the other hand, will give $f(x)$ more room to oscillate with the risk of overfitting data.

In a similar way, when estimating $\boldsymbol{\beta}$ using a likelihood approach, it's possible to define the penalized log-likelihood by penalize the log-likelihood with the term $\lambda \mathrm{W}$.

Cubic splines

There are several types of smoothing functions that can be used in GAM e.g. thin plates or cubic splines. In this work we will be using cubic cyclic splines and, because of this, we will here make a short presentation of cubic splines.

Natural cubic splines

Let $\{(x_i,\, y_i)\}, \quad i = 1,\, \ldots,\, n$, be a set of points where $x_i < x_{i+1}$. The natural cubic spline, $g(x)$, interpolating these points is a function made up of piecewise cubic polynomials, $p_i(x) = \alpha_i + \beta_{i,1}x + \beta_{i,2}x^2 + \beta_{i,3}x^3$, one for each interval $[x_i,\, x_{i+1}], \quad i = 1,\, \ldots,\, n-1$ and, $p_i(x) = 0$ outside of $[x_i,\, x_{i+1}]$. Furthermore, $g(x) = \sum_{i=1}^{n} p_i(x)$ are constructed so that the first and second derivative of $g(x)$ are continuous on $[x_1,\, x_n]$ and, so that at the end points we have $g''(x_1) = g''(x_n) = 0$.

As mentioned earlier, wiggliness of a function $f(x)$ on an interval $[x_1,\, x_n]$ can be measured as

$$\mathrm{W}(f) = \int_{x_1}^{x_n} (f''(x))^2 dx. \tag{9}$$

Now suppose that $f(x)$ is continuous on $[x_1,\, x_n]$, have an absolute continuous first derivative and, interpolate the points $(x_i, y_i), \quad i = 1, \ldots, n$. Of all possible functions $f(x)$ it can be shown that $g(x)$ is, in fact, the function that minimize equation (9), meaning that of all possible functions $f(x)$, the natural cubic spline $g(x)$ is the smoothest.

Cubic regression splines
Usually y_i is measured with noise and it's usually better to smooth the points $(x_i,\ y_i)$ than to interpolate them. There are several equivalent ways to define a basis for cubic regression splines, we will here show one example of how to define one such basis as well as the resulting penalized equations that can be solved with least square estimations.

As mentioned before, cubic splines are made up of piecewise cubic polynomials, the points where these polynomials start and end are called *knots*. In natural cubic splines these knots were the interpolation points $(x_i,\ y_i)$, in cubic regression splines, however, we don't seek to interpolate the data points and, instead we have to choose other knots on the interval $[x_1,\ x_n]$. Two common ways to choose these knots is to either to to place them at an even distance on the x-axis (within $[x_1,\ x_n]$), not necessarily where there is a datum or, place them at quantiles of the distribution of unique x values.

A simple example of a cubic spline basis
As mentioned earlier, there are several equivalent ways of defining the basis functions for the same basis. Here is an example of how to define a cubic spline basis on the interval $x = [0, 1]$. Consider points (x_i, y_i) on the interval $x = [0, 1]$ that we want to smooth using GAM. We denote the knots as $x_i^*, i = 1, \dots, q-2$, and we define the basis functions as

$$\begin{cases} b_1(x) = 1 \\ b_2(x) = x \\ b_{i+2}(x) = R_i(x,\ x_i^*), \qquad i = 1,\ \dots, q-2, \end{cases} \tag{10}$$

where

$$R_i(x,\ x_i^*) = \frac{\left(\left(x_i^* - \frac{1}{2}\right)^2 - \frac{1}{12}\right)\left(\left(x - \frac{1}{2}\right)^2 - \frac{1}{12}\right)}{4} - \frac{\left(|x - x_i^*| - \frac{1}{2}\right)^4 - \frac{1}{2}\left(|x - x_i^*| - \frac{1}{2}\right)^2 + \frac{7}{240}}{24}.$$

Observe that for $q-2$ knots we will have q basis functions. With this model we model y_j as

$$y_j = \sum_{i=1}^{q} \beta_i b_i(x_j) + \varepsilon_j. \tag{11}$$

Which results in the equation system

$$\begin{cases} \hat{y}_1 = \beta_1 + \beta_2 x_i + \cdots + \beta_q R_{q-2}(x_n,\, x^*_{q-2}) \\ \vdots \\ \hat{y}_n = \beta_1 + \beta_2 x_i + \cdots + \beta_q R_{q-2}(x_n,\, x^*_{q-2}) \end{cases} \Leftrightarrow \mathbf{y} = \mathbf{X}\boldsymbol{\beta}. \tag{12}$$

Since (11) is linear we can use least squares to estimate the coefficients β_i, however, we also want to introduce a term to penalize wiggliness, so instead of minimizing

$$||\mathbf{y} - \mathbf{X}\boldsymbol{\beta}||^2,$$

we could minimize

$$||\mathbf{y} - \mathbf{X}\boldsymbol{\beta}||^2 - \lambda \int_{x_1}^{x_n} |f''(x)|^2 dx = ||\mathbf{y} - \mathbf{X}\boldsymbol{\beta}||^2 - \boldsymbol{\beta}^T \mathbf{S} \boldsymbol{\beta}.$$

With this way of writing down the basis the elements in the penalty matrix $\mathbf{S}$ can be written as $\mathbf{S} = R(x^*_i,\, x^*_j), \quad$ for $i,\, j = 1,\, \ldots, q-2$ and with only zeros in the first two rows and columns.

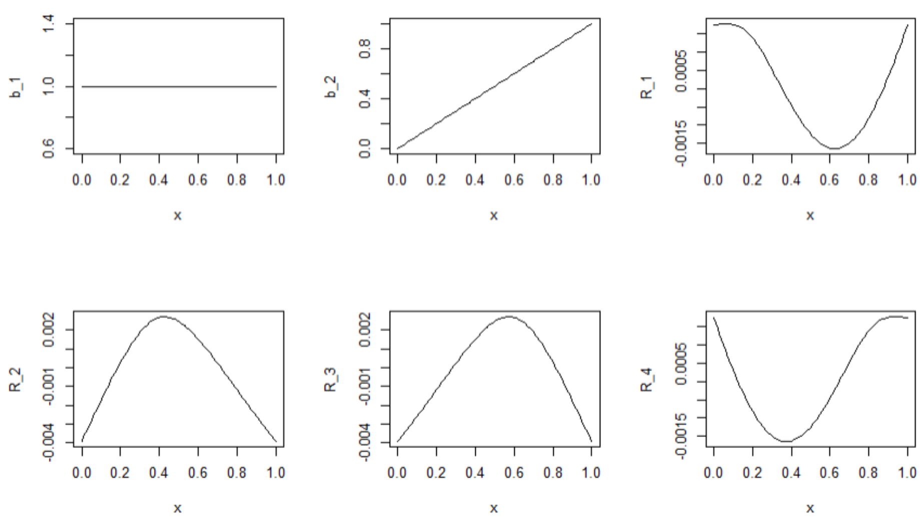


Figure 3: The basis functions described in example above using four knots (own work).

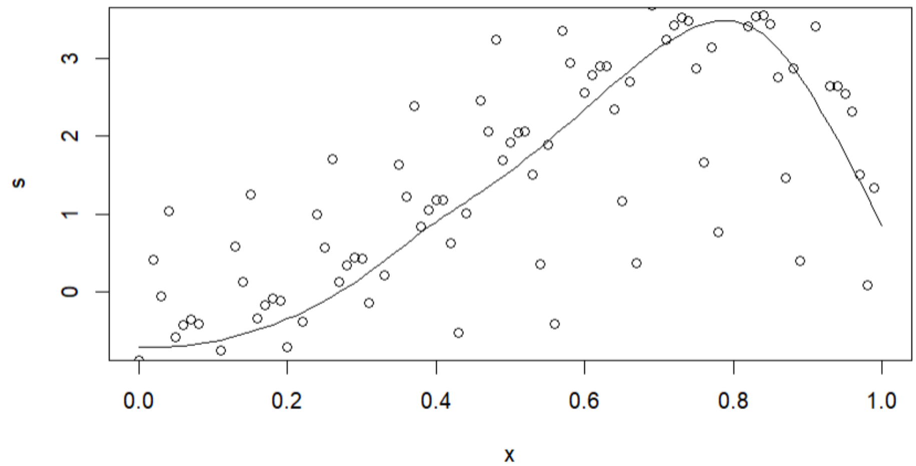


Figure 4: The result when using the basis functions shown above and linear regression to smooth the points shown in the figure (own work).

Figure 3 show basis functions as in the example when using four knots at $x_1^* = 1/6$, $x_2^* = 7/18$, $x_3^* = 11/18$, and $x_4^* = 5/6$. In figure 4 I have used these basis functions to fit (without using the penalty term λW) cubic splines to a set of points.

The example shown above is not a very practical one and is only useful on the interval [0, 1], it can however, give the reader an idea on how it's possible to use linear regression and sums of basis function to fit models in GAM.

Cubic cyclic splines
In this work we have used the day of the year as a covariate in our GAM:s and it would desirable to not have a discontinuity at the end of the year. Cubic cyclic splines are, just as cubic splines, made up of piecewise third degree polynomials and have continuous first and second derivatives on $[x_1, x_n]$. Unlike cubic splines, cyclic cubic splines also have the condition that $g(x_1) = g(x_n)$, $g'(x_1) = g'(x_n)$ and, $g''(x_1) = g''(x_n)$.

2.2.2 Partial Correlation

The theory in this section is derived from the chapter 4.3 of the book *Multivariate statistics - High-Dimensional and Large-Sample Approximations* (2010) by Fujikoshi, Ulynaov & Shimizu.

Let X, Y be two random variables of interest and let $Z_1, \ldots, Z_n$ be confounding random variables that covariate with X and Y. The correlation between X and Y, ρ_{XY}, can be viewed as in part due to the direct correlation between X and Y and, in part due to the indirect correlation with other confounding variables. The partial correlation between X and Y is the direct correlation between these two variables after removing the effects of the variables $Z_1, \ldots, Z_n$.

Let $\mathbf{X} = (X,\, Y,\, Z_1,\, \ldots, Z_n)^T = (X,\, ,Y,\, \mathbf{Z}^T)^T$ be a random vector and let

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_X \\ \mu_Y \\ \boldsymbol{\mu}_Z \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} \sigma_{XX} & \sigma_{XY} & \boldsymbol{\sigma}_{ZX}^T \\ \sigma_{YX} & \sigma_{YY} & \boldsymbol{\sigma}_{ZY}^T \\ \boldsymbol{\sigma}_{ZX} & \boldsymbol{\sigma}_{ZY} & \Sigma_{ZZ} \end{pmatrix}$$

be the mean vector and covariance matrix of $\mathbf{X}$, respectively.

The best linear predictor of X and Y by linear function of $\mathbf{Z}$ is given by

$$\begin{cases} l_X(\mathbf{Z}) & = \mu_X + \boldsymbol{\sigma}_{XZ}^T \Sigma_{ZZ}^{-1}(\mathbf{Z} - \boldsymbol{\mu}_Z) \\ l_Y(\mathbf{Z}) & = \mu_Y + \boldsymbol{\sigma}_{YZ}^T \Sigma_{ZZ}^{-1}(\mathbf{Z} - \boldsymbol{\mu}_Z). \end{cases}$$

The residuals $\mathbf{e} = (\mathbf{e}_X, \mathbf{e}_Y)$ can be regarded as the portion of X and Y left after removing the effects of $\mathbf{Z}$.

$$\begin{aligned} \mathbf{e} = \begin{pmatrix} \mathbf{e}_X \\ \mathbf{e}_Y \end{pmatrix} = \begin{pmatrix} X - l_X(\mathbf{Z}) \\ Y - l_Y(\mathbf{Z}) \end{pmatrix} = \\ \begin{pmatrix} X - \mu_X \\ Y - \mu_Y \end{pmatrix} - \begin{pmatrix} \boldsymbol{\sigma}_{ZX}^T \\ \boldsymbol{\sigma}_{ZY}^T \end{pmatrix} \Sigma_{ZZ}^{-1}(\mathbf{Z} - \boldsymbol{\mu}_Z) = \\ \begin{pmatrix} 1 & 0 & \boldsymbol{\sigma}_{ZX}^T \Sigma_{ZZ}^{-1} \\ 0 & 1 & \boldsymbol{\sigma}_{ZY}^T \Sigma_{ZZ}^{-1} \end{pmatrix} \begin{pmatrix} X - \mu_X \\ Y - \mu_Y \\ \mathbf{Z} - \boldsymbol{\mu}_Z \end{pmatrix}. \end{aligned}$$

Then the covariance matrix of $\mathbf{e}$ is given by

$$\begin{aligned} \mathrm{Var}(\mathbf{e}) &= \begin{pmatrix} \sigma_{XX} & \sigma_{XY} \\ \sigma_{YX} & \sigma_{YY} \end{pmatrix} - \begin{pmatrix} \boldsymbol{\sigma}_{ZX} \\ \boldsymbol{\sigma}_{ZY} \end{pmatrix} \Sigma_{ZZ}^{-1} (\boldsymbol{\sigma}_{ZX} \quad \boldsymbol{\sigma}_{ZX}) \\ &\equiv \Sigma_{(XY)(XY)\cdot\mathbf{Z}} = \begin{pmatrix} \sigma_{XX\cdot\mathbf{Z}} & \sigma_{XY\cdot\mathbf{Z}} \\ \sigma_{YX\cdot\mathbf{Z}} & \sigma_{YY\cdot\mathbf{Z}} \end{pmatrix} \end{aligned}$$

Definition

With the notation used above, the partial correlation (coefficient) between X and Y given $\mathbf{Z}$ is

$$\rho_{XY\cdot Z} = \frac{\sigma_{XY\cdot\mathbf{Z}}}{\sqrt{\sigma_{XX\cdot\mathbf{Z}} \cdot \sigma_{YY\cdot\mathbf{Z}}}}$$

□

Calculating the partial correlation of a sample

Calculating the partial correlation of a random sample can be made in two steps.

Let $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_n)$ be two random samples from two random variables X and Y of interest and, let

$$\begin{cases} \mathbf{z}_1 = (z_{1,1},\ \ldots, z_{n,1})^T \\ \vdots \\ \mathbf{z}_p = (z_{1,p},\ \ldots, z_{n,p})^T \end{cases}$$

be random samples of confounding random variables $Z_1, \ldots, Z_p$.

Step 1

Use linear regression to fit the two models

$$\hat{X} = \alpha_X + \beta_{X,1}Z_1 + \beta_{X,2}Z_2 + \cdots + \beta_{X,p}Z_p \tag{13}$$

$$\hat{Y} = \alpha_Y + \beta_{Y,1}Z_1 + \beta_{Y,2}Z_2 + \cdots + \beta_{Y,p}Z_p \tag{14}$$

Step 2

Calculate the sample partial correlation between X and Y given $\mathbf{Z}$ by calculating the correlation between the residuals from model 13 and 14.

2.3 Method

2.3.1 Weight - length relationships

To be able to compare the weight - length relationships for the Swedish populations of harbor porpoises to those of Bryden (1986), Van Utrecht (1978), Kastelein and van Battum (1990) and Bilgin, Kose and Yesilcicek (2019) (Bilgin, Kose & Yesilcicek 2019), linear regressions of weight against total length with logarithmized values were made for the different age classes described in section 2.1.1.

2.3.2 A residual measure of body condition

To construct a residual body condition index using *total length* as explanatory variable and *weight* as response, linear regression with logarithmized variables (using the 10 logarithm) were used. To be able to compare the results with the models including seasonal variation three models with and without a factor variable for age class for three subsets of the data were made; one corresponding to all complete observations (rows in the data set) available for the variables *weight*, *total length* and *day*, one with all complete observations available for *weight*, *total length*, *day* and *gender* and, one with all complete observations available for *weight*, *total length*, *day*, *gender* and *age*.

To test the influence of seasonal variation in weight GAM with cubic cyclic splines were used. Computing of the GAM models were made with and without the animals length as a linear element, in addition, the models were computed with or without a factor variable for age class dividing the data set in juveniles, adult males and adult females as described in section 2.1.1. The difference between each models adjusted R^2 and the adjusted R^2 for the log linear regression with *total length* were calculated.

For cross validation, three new GAM models with *total length* included as a linear element, with and without the factor variables for age classes were made, but this time only using observations for which *nourishment* had missing values (remember that 79.93% of it's values were missing). When the models had been fitted, the predicted weight, $\hat{y}$, were calculated for each observation were *nourishment* weren't missing. The residuals, $e = y - \hat{y}$, were calculated by taking the difference between the animals actual weight, y, and the predicted weight, $\hat{y}$. The distribution of the residuals for each level of the variable *nourishment* (the four levels being: "Emaciated", "Less than average nourishment", "Average nourished" and, "Well nourished") were then shown in box plots.

2.3.3 Circumference and blubber thickness

Due to many missing values of the variable *age* only the age class variable based on length and gender (*length_sex_class*) was used in this section. Here the relationship between weight and blubber thickness and circumference was tested on four different groups of animals; "juveniles and adults" (> 100cm), "juveniles" (100-130cm), "adult males" (>130 cm) and, "adult females" (> 130 cm).

It was perceived during the initial analysis that the thickness of blubber may show a greater correlation with weight within different groups of animals based on gender, age, length, weight or season. Other authors had also shown that blubber thickness varied between different groups of animals, for example, Koopman (Koopman, 1998) notes that in the thoracic-abdominal region, calves had the thickest blubber, lactating females had the thinnest blubber and porpoises in other groups had thicknesses of blubber intermediate between those of calves and lactating females. Lockyer (Lockyer, 1995) also showed that the blubber thickness differed between different sexes and ages, whereas Siebert et al. (Siebert et al., 2022) showed that the thickness of blubber varied depending on the season. To account for the influence of these confounding variables partial correlation was used, with the logarithmized values of *total length* as a linear element, the class variable *length_sex_class* as a factor variable and *day of the year* as smoothing element.

First a model was fitted where the animals weight was explained by its length, the class variable and day of the year:

Model 1:

$$\begin{aligned} \log_{10}(\text{weight}) \; = \; & \log_{10}(\text{totallength}) \; + \; \text{length\_sex\_class} \; + \\ & + \; s(\text{day of the year, by} = \text{length\_sex\_class}), \end{aligned}$$

where $s(\text{day of the year, by} = \text{length\_sex\_class})$ is the cyclic cubic spline element of the GAM model that is allowed to vary with length_sex_class. In this model all available observations were used (N = 482), the residuals from this model were added as a new column in the tibble used to fit the model.

As a second step, models were fitted for each of the circumference- or fat variables against the same explanatory variables:

Model 2:

$$\log_{10}(\text{circumference or fat variable}) = \log_{10}(\text{totallength}) + \text{length\_sex\_class} + \\ + s(\text{day of the year, by} = \text{length\_sex\_class}).$$

Since the variables for circumference and blubber thickness had a lot of missing data (between 76.5% and 89.35%, see figure 2) these models had a lot fewer observations at hand (N between 70 and 145) than for model 1.

Residuals from these models were matched to the corresponding residuals in model 1 via an inner join by the variable *accession id*. Residuals from model 1 and 2 were partitioned in juveniles, adult males and adult females and, fitted against each other to calculate the partial correlation between weight and the circumference and fat variables.

To have a basis of comparison, log linear regression models with weight against each one of the variables for circumference and fat were also made.

3 Results

3.1 Weight - length relationship

Class	R^2	Pr(>F)	No of observations
Juveniles and adults (> 100cm and older than 0 years)	0.706	0.00000	493
Juvenile (1-5 years of age)	0.673	0.00000	232
Juvenile (100-130cm)	0.549	0.00000	262
Adult Female (> 130cm)	0.424	0.00000	129
Adult Male (> 130cm)	0.145	0.00008	102
Adult Male (older than 5 years)	0.088	0.08301	35
Adult Female (older than 5 years)	0.062	0.12764	39

Note:

R^2 rounded to 3 decimal places and p values rounded to 5

Table 2: Results for linear regression for the model $\log(\text{weight}) = \alpha + \beta \log(\text{total length})$ for different groups of animals. Ordered with descending R^2.

Table 2 is a table over the results of the weight - length relationships for seven different groups of animals. Of the seven models five were significant, only the two models with less than 100 observations were not. The model with the most observations, "Juveniles and adults", yielded the highest R^2 of 0.706, the two groups with juvenile harbor porpoises with more than 200 observations gave an R^2 between 0.549 and 0.673 and, two groups with between 102 and 129 observations ("Adult Female (> 130 cm)" and "Adult Male (> 130 cm)") yielded R^2 of 0.424 and 0.145, respectively.

3.2 A residual measure of body condition

No	Model	R^2	Adj. R^2	Pr(>F)	Observations
1	$\log_{10}$(weight) = $\log_{10}$(totallength)	0.701	0.700	0	482
2	$\log_{10}$(weight) = $\log_{10}$(totallength) + length_sex_class	0.701	0.699	0	482
3	$\log_{10}$(weight) = $\log_{10}$(totallength) + age_sex_class	0.671	0.667	0	304

Note:
R^2 rounded to 3 decimal places and p values rounded to 5

Table 3: The result of the regression model $\log_{10}(\text{weight}) = \alpha + \beta \cdot \log_{10}(\text{total length})$ for subsets of the data corresponding to the different GAM models below

Formula	R^2	Adj. R^2	P-value for the smooth element	Pr(> $\lvert t \rvert$) for totallength	N	ΔR^{2*}
log10(weight) = log10(totallength) + s(day)	0.737	0.734	0.00000	0	482	0.033
log10(weight) = s(day)	0.089	0.001	0.30934	NA	482	-0.700

Note:
R^2 rounded to 3 decimal places and p values rounded to 5. N denotes no of observations.
* Adj. R^2 compared to the R^2 of model 1 (log(weight)=log(total length)), i.e. Adj. R^2 - R^2

Table 4: All porpoises >100 cm and older than 0 years without further division in age classes

Formula	R^2	Adj. R^2	P-values for the smooth elements: Adult female	Adult male	Juvenile	$\Pr(>|t|)$ for totallength	N	ΔR^{2*}
$\log_{10}$(weight) = $\log_{10}$(totallength) + length_sex_class + s(day, by = length_sex_class)	0.740	0.733	0.00000	0.01550	0.00003	0	482	0.034
$\log_{10}$(weight) = length_sex_class + s(day, by = length_sex_class)	0.502	0.495	0.08855	0.14406	0.00046	NA	482	-0.204

Note:
R^2 rounded to 3 decimal places and p values rounded to 5. N denotes no of observations.
* Adj. R^2 compared to the R^2 of model 2 ($\log_{10}$(weight)=$\log_{10}$(total length) + length_sex_class), i.e. Adj. R^2 - adj. R^2

Table 5: Porpoises divided in groups based on length and gender (juveniles: 100-130cm, adults: >130 cm)

Formula	R^2	Adj. R^2	P-values for the smooth elements: Adult female	Adult male	Juvenile	$\Pr(>|t|)$ for totallength	N	ΔR^{2*}
$\log_{10}$(weight) = $\log_{10}$(totallength) + age_sex_class + s(day, by = age_sex_class)	0.740	0.729	0.00003	0.00001	0.00007	0	304	0.062
$\log_{10}$(weight) = age_sex_class + s(day, by = age_sex_class)	0.339	0.264	0.21584	0.01748	0.64074	NA	304	-0.403

Note:
R^2 rounded to 3 deimal places, P values rounded to 5 decimal places. N = no of observations.
* Adj. R^2 compared to the adj. R^2 of model 3 (log(weight)=log(total length) + age_sex_class), i.e. adj. R^2 - adj. R^2

Table 6: Porpoises divided in groups based on age and gender (juveniles: 1-5 years old, adults: older than 5 years)

In table 3 we see the result of three regression models used as a base line for comparison with the GAM models in tables 4 through 6. The three models yielded a (multiple) R^2 between 0.671 and 0.701 and, an adjusted R^2 between 0.667 and 0.700. Furthermore, the p-value for the F-statistica showed that all models were highly significant, although the p-values for the t-statistica in model 2 and 3 were not significant for "Adult male" and "Juvenile" (not shown in the table).

Tables 4 through 6 shows the results of the GAM models used to construct our residual body condition indices. Using only year day and, in appropriate models, a factor variable for age class, we got an adjusted R^2 between 0.001 and 0.495 and, compared to the models in table 3, we got a Δ(adjusted) R^2

between -0.700 and -0.203. Looking at the approximate significance of each smooth element ("P value for the smooth element" in the table), we see that only two out of seven smooth elements were significant at the 2.5% level.

In the GAM models where we also used *total length* as a linear element, we got an adjusted R^2 between 0.729 and 0.734 and, we also see an increase in (adjusted) R^2 of between 0.034 and 0.062 compared to the models in table 3. In these GAM models the smooth elements were all approximate significant at the 2.5% level and, all but one, "Adult male" in table 5, were significant at the 1% level.

3.2.1 Testing the residual index

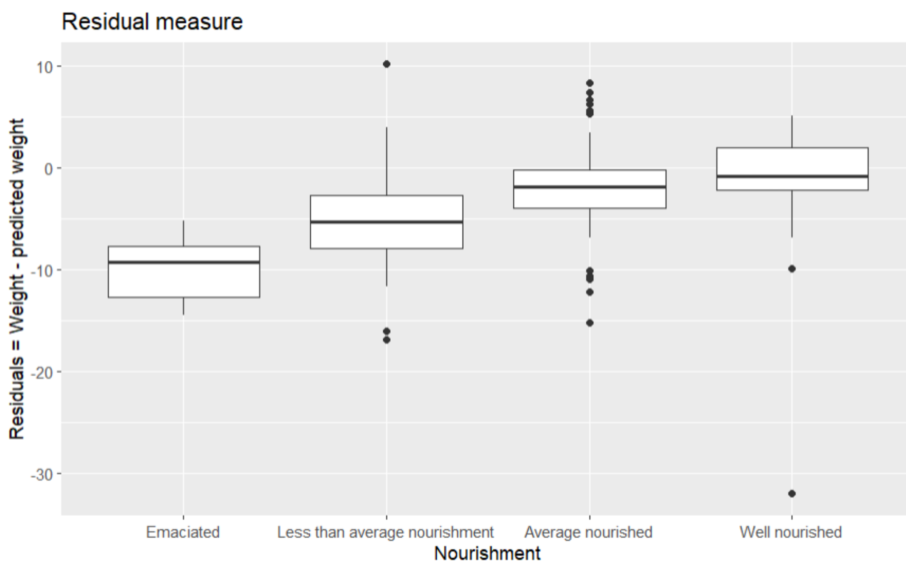


Figure 5: The distributions of residuals when testing the residual index based on the model $\log_{10}(\text{weight}) = \log_{10}(\text{total length}) + s(\text{day})$

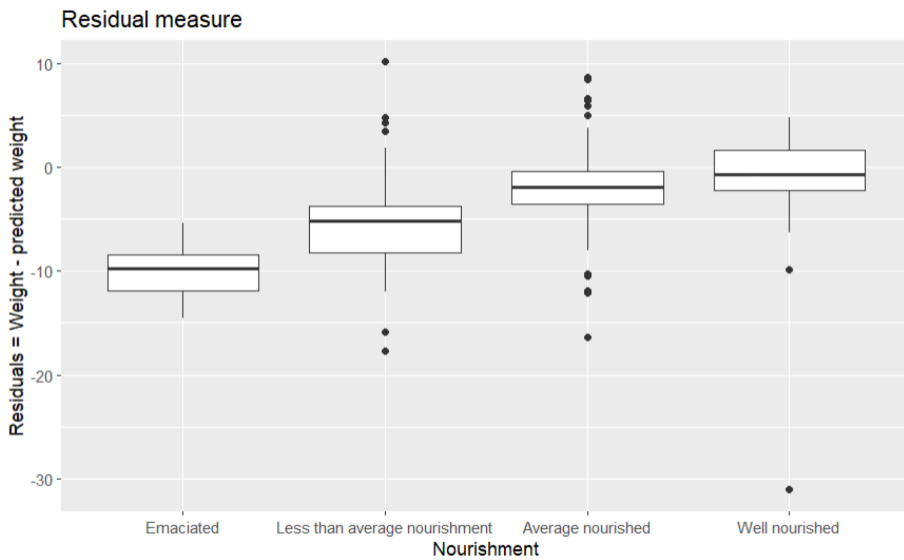


Figure 6: The distributions of residuals when testing the residual index based on the model $\log_{10}(\text{weight}) = \log_{10}(\text{total length}) + \text{length\_sex\_class} + s(\text{day}, \text{by} = \text{length\_sex\_class})$

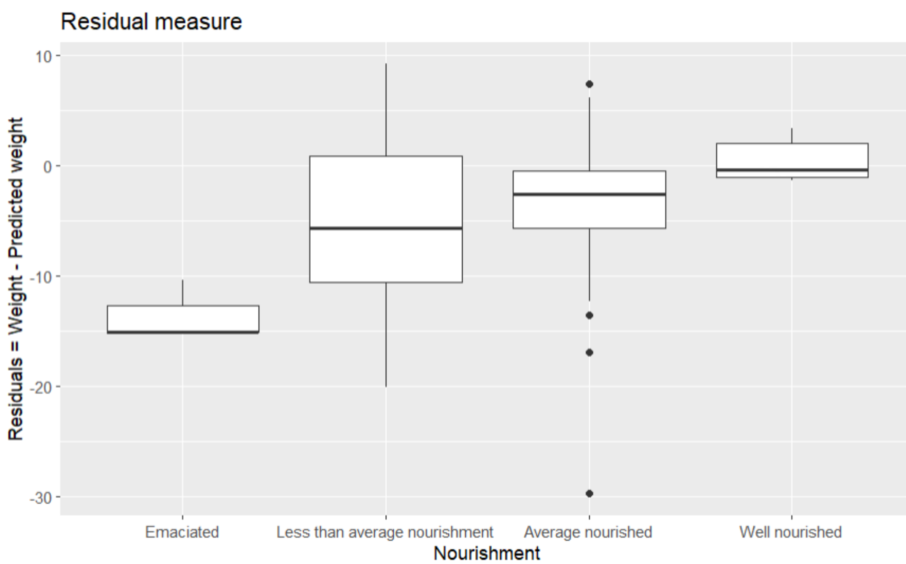


Figure 7: The distributions of residuals when testing the residual index based on the model $\log_{10}(\text{weight}) = \log_{10}(\text{total length}) + \text{age\_sex\_class} + s(\text{day}, \text{by} = \text{age\_sex\_class})$

In figure 5 through 7 we see box plots of the of the distribution of residuals after testing three different residual body condition indices. For all indices we see that the median, the 25% and the 75% quantile increase as the level of nutritional status increases, the only exception being the group "Less than average nourishment" in figure 7, where the 75% quantile was higher than that for the group "Average nourished".

	Quantile for residual = 0 (in %)			
Nourisment level	All porpoises > 100cm	With age class variable length_sex_class	With age class variable age_sex_class	N
Well nourished	53.6	57.1	60.0	5
Average nourished	75.8	77.4	81.5	27
Less than average nourishment	80.0	80.0	75.0	12
Emaciated	100.0	100.0	100.0	3

Table 7: The table list the percentage of negative residuals for each of the residual indices tested.

Table 7 above shows the percentage of negative residuals for our tested indices. For all nourishment levels and for all indices the majority of the residuals was negative, for the levels "Emaciated" and "Less than average nourished" it was between 75% and 100% that were negative and, for the levels "Average nourished" and "Well nourished" between 53.6% and 81.5% of the residuals were negative.

3.2.2 Using the residual indices

In figure 8 through 14 below we see curves of the predicted weight using the three different models for our residual indices; $\log_{10}$(weight) = $\log_{10}$(total length) + s(day), $\log_{10}$(weight)= $\log_{10}$(total length) + age_sex_class + s(day, by = age_sex_class) and, $\log_{10}$(weight)= $\log_{10}$(total length)+ length_sex_class + s(day, by = length_sex_class). In the figures the predicted weight are shown on the y-axis and on the x-axis is the day of the year, the predicted weights are calculated for different animals lengths in 10 cm increments.

These graphs can be useful in determine an animals nutritional status. By knowing the date and the animal's weight and length and, for models with an age class variable the animals gender (for the model with the factor variable *age_sex_class* it is also necessary to determine the animal's age) it's easy to visually compare the animal's weight to the adequate line in the graphs; an actual weight over the predicted curve indicates that it is a healthy animal and, in contrast, an actual weight below the predicted curve indicates that the animal is malnourished.

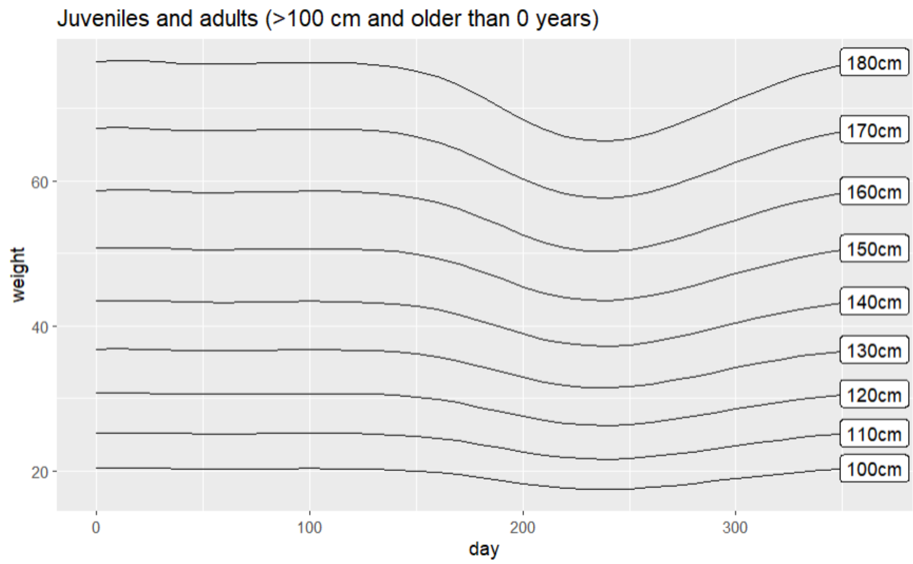


Figure 8: The figure shows the predicted weight for porpoises of different lengths using the model $\log_{10}(\text{weight}) = \log_{10}(\text{total length}) + \text{s(day)}$

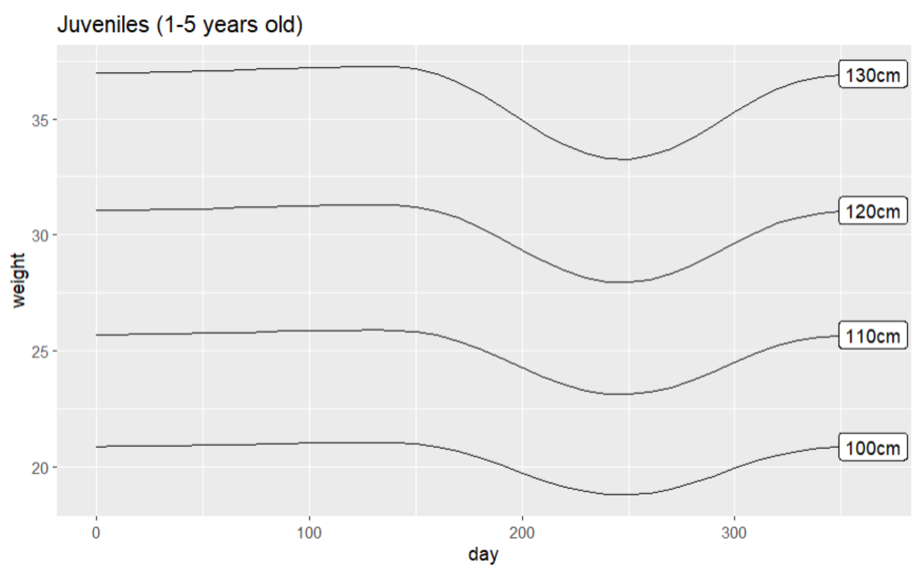


Figure 9: The figure shows the predicted weight for juvenile porpoises of different lengths using the model $\log_{10}(\text{weight}) = \log_{10}(\text{total length}) + \text{age\_sex\_class} + \text{s(day, by = age\_sex\_class)}$

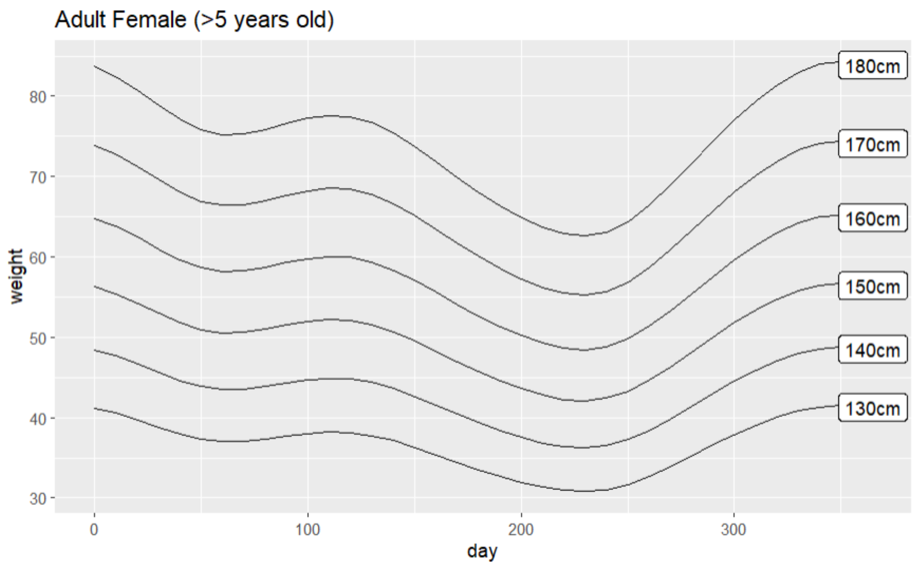


Figure 10: The figure shows the predicted weight for adult female porpoises of different lengths using the model $\log_{10}(\text{weight}) = \log_{10}(\text{total length}) + \text{age\_sex\_class} + \text{s(day, by} = \text{age\_sex\_class)}$

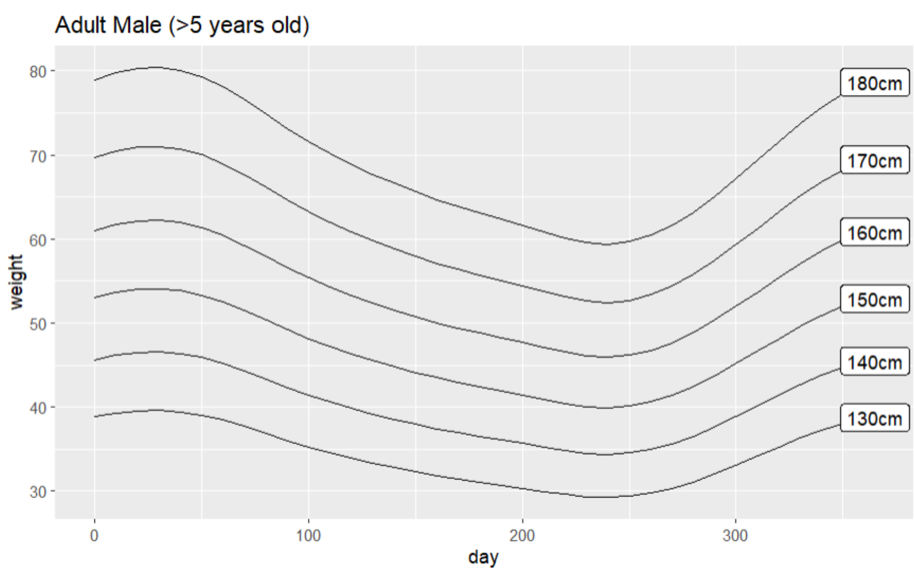


Figure 11: The figure shows the predicted weight for adult male porpoises of different lengths using the model $\log_{10}(\text{weight}) = \log_{10}(\text{total length}) + \text{age\_sex\_class} + \text{s(day, by} = \text{age\_sex\_class)}$

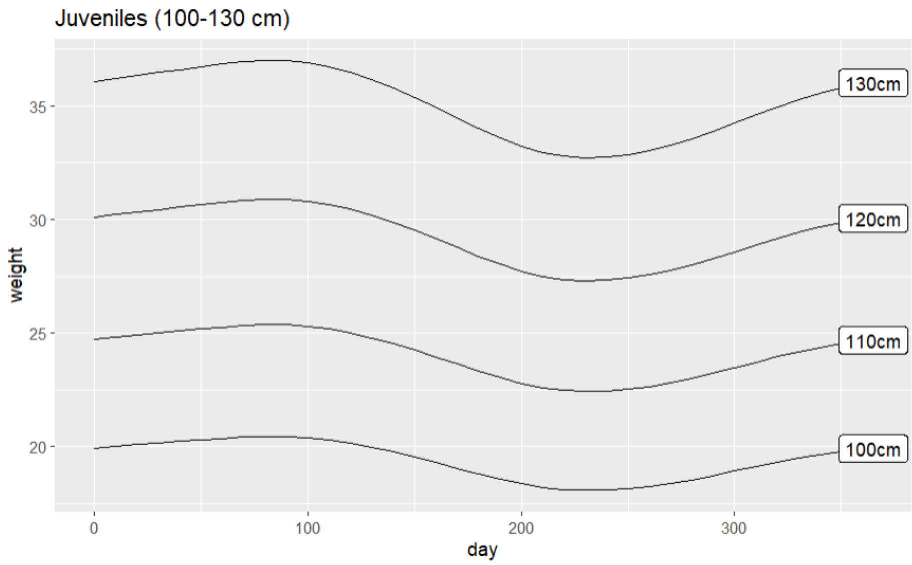


Figure 12: The figure shows the predicted weight for juvenile porpoises of different lengths using the model $\log_{10}(\text{weight}) = \log_{10}(\text{total length}) + \text{length\_sex\_class} + \text{s(day, by = length\_sex\_class)}$

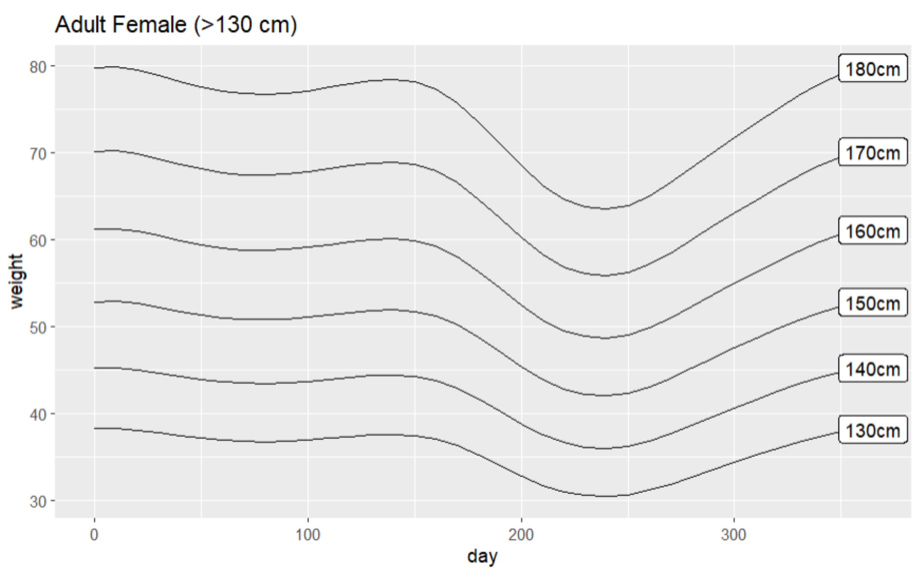


Figure 13: The figure shows the predicted weight for adult female porpoises of different lengths using the model $\log_{10}(\text{weight}) = \log_{10}(\text{total length}) + \text{length\_sex\_class} + \text{s(day, by = length\_sex\_class)}$

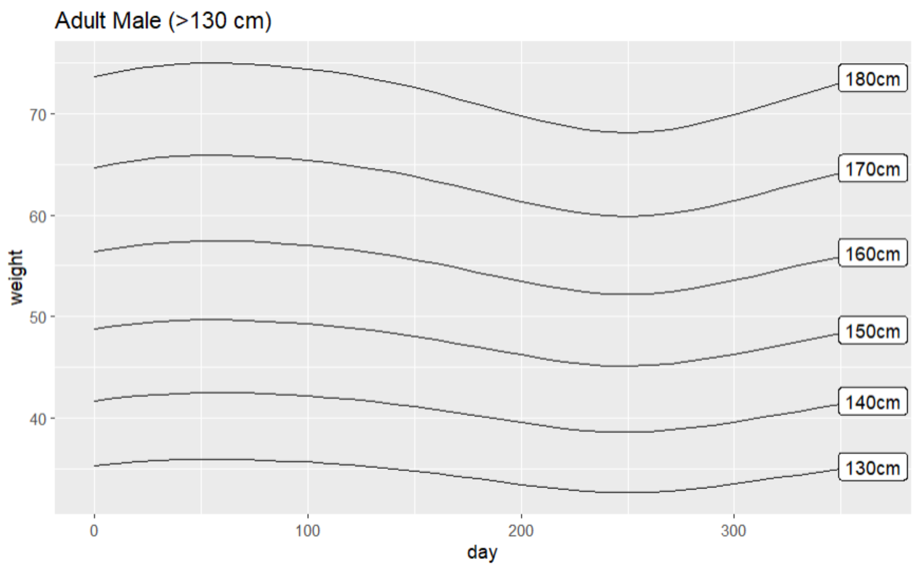


Figure 14: The figure shows the predicted weight for adult male porpoises of different lengths using the model $\log_{10}(\text{weight}) = \log_{10}(\text{total length}) + \text{length\_sex\_class} + \text{s(day, by} = \text{length\_sex\_class)}$

3.3 Circumference and blubber thickness

3.3.1 Juveniles and adults (>100 cm and older than 0 years)

Explanatory variable	R^2	Pr(>F)	N
circumference_breast	0.777	0.00000	134
circumference_neck	0.751	0.00000	128
circumference_abdomen	0.591	0.00000	138
circumference_hip	0.337	0.00000	139
fat_abdomen_right	0.015	0.14632	143
fat_hip_right	0.011	0.24252	127
fat_breast_right	0.009	0.28318	136
fat_abdomen_ventral	0.005	0.40005	142
fat_hip_ventral	0.004	0.59462	70
fat_neck_right	0.003	0.55158	139
fat_abdomen_back	0.003	0.48867	141
fat_breast_back	0.001	0.73832	139
fat_hip_back	0.001	0.73075	128
fat_neck_ventral	0.000	0.87927	138
fat_breast_ventral	0.000	0.79167	145
fat_neck_back	0.000	0.83587	138

(a) Log linear regression, ordered by R^2

Explanatory variable	R^2	Pr(>F)	N	ΔR^2
fat, hip right	0.278	0.00000	127	0.267
fat, abdomen right	0.266	0.00000	143	0.251
fat, breast back	0.240	0.00000	139	0.239
fat, neck ventral	0.222	0.00000	138	0.222
fat, abdomen ventral	0.203	0.00000	142	0.198
fat, hip back	0.192	0.00000	128	0.191
fat, neck right	0.186	0.00000	139	0.183
fat, breast ventral	0.154	0.00000	145	0.154
fat, breast right	0.162	0.00000	136	0.153
fat, neck back	0.143	0.00001	138	0.143
fat, abdomen back	0.134	0.00001	141	0.131
fat, hip ventral	0.124	0.00276	70	0.120
circumference, breast	0.649	0.00000	134	-0.128
circumference, neck	0.622	0.00000	128	-0.129
circumference, hip	0.154	0.00000	139	-0.183
circumference, abdomen	0.327	0.00000	138	-0.264

(b) Partial correlation, ordered by ΔR^2

Table 8: The results of the log linear regression for all porpoises >100cm. The difference in R^2 is measured as $\Delta R^2 = R^2_{\text{log lin regression}} - R^2_{\text{partial corr.}}$

3.3.2 Juveniles (100-130cm)

Explanatory variable	R^2	Pr(>F)	N
circumference, breast	0.777	0.00000	85
circumference, neck	0.774	0.00000	78
circumference, abdomen	0.362	0.00000	86
fat, neck ventral	0.305	0.00000	89
fat, hip right	0.260	0.00000	79
fat, neck right	0.251	0.00000	88
fat, breast back	0.242	0.00000	91
fat, breast right	0.227	0.00000	89
fat, hip back	0.214	0.00001	81
circumference, hip	0.197	0.00002	86
fat, abdomen right	0.194	0.00001	93
fat, abdomen back	0.193	0.00001	90
fat, neck back	0.183	0.00003	89
fat, breast ventral	0.182	0.00002	95
fat, hip ventral	0.153	0.00499	50
fat, abdomen ventral	0.136	0.00028	93

(a) Log linear regression, ordered by R^2

Explanatory variable	R^2	Pr(>F)	N	ΔR^2
fat, abdomen back	0.397	0.00000	90	0.204
fat, hip right	0.429	0.00000	79	0.169
fat, breast ventral	0.349	0.00000	95	0.167
fat, abdomen ventral	0.299	0.00000	93	0.163
fat, abdomen right	0.354	0.00000	93	0.160
fat, breast back	0.387	0.00000	91	0.145
fat, hip ventral	0.295	0.00005	50	0.142
fat, neck right	0.385	0.00000	88	0.134
fat, neck ventral	0.425	0.00000	89	0.120
fat, hip back	0.328	0.00000	81	0.114
fat, breast right	0.332	0.00000	89	0.105
fat, neck back	0.285	0.00000	89	0.102
circumference, abdomen	0.304	0.00000	86	-0.058
circumference, neck	0.694	0.00000	78	-0.080
circumference, hip	0.116	0.00136	86	-0.081
circumference, breast	0.676	0.00000	85	-0.101

(b) Partial correlation, ordered by ΔR^2

Table 9: The results of the log linear regression and partial correlation for juveniles (100 - 130cm). The difference in R^2 is measured as $\Delta R^2 = R^2_{\text{log lin regression}} - R^2_{\text{partial corr.}}$

3.3.3 Adult males (>130cm)

Explanatory variable	R^2	Pr(>F)	N
circumference, abdomen	0.442	0.00021	26
circumference, breast	0.407	0.00060	25
circumference, neck	0.392	0.00062	26
circumference, hip	0.258	0.00685	27
fat, breast ventral	0.225	0.01435	26
fat, breast right	0.069	0.20398	25
fat, neck ventral	0.059	0.24255	25
fat, breast back	0.054	0.26143	25
fat, abdomen back	0.049	0.27705	26
fat, hip right	0.041	0.34305	24
fat, neck right	0.035	0.36146	26
fat, abdomen right	0.028	0.42136	25
fat, abdomen ventral	0.008	0.66529	25
fat, hip ventral	0.008	0.79117	11
fat, neck back	0.001	0.85880	24
fat, hip back	0.000	0.97034	24

(a) Log liner regression, ordered by R^2

Explanatory variable	R^2	Pr(>F)	N	ΔR^2
circumference, neck	0.557	0.00001	26	0.165
circumference, breast	0.531	0.00004	25	0.124
fat, hip ventral	0.080	0.39831	11	0.072
circumference, abdomen	0.490	0.00007	26	0.048
fat, neck back	0.004	0.75617	24	0.003
fat, hip back	0.003	0.80278	24	0.003
fat, abdomen right	0.027	0.43647	25	-0.001
fat, abdomen ventral	0.006	0.71951	25	-0.002
fat, hip right	0.038	0.36378	24	-0.003
circumference, hip	0.233	0.01075	27	-0.025
fat, neck right	0.002	0.83816	26	-0.033
fat, abdomen back	0.004	0.76245	26	-0.045
fat, breast back	0.003	0.80514	25	-0.051
fat, neck ventral	0.000	0.96767	25	-0.059
fat, breast right	0.004	0.76063	25	-0.065
fat, breast ventral	0.068	0.19696	26	-0.157

(b) Partial correlation, , ordered by ΔR^2

Table 10: The results of the log linear regression and the partial correlation for males longer than 130cm. The difference in R^2 is measured as $\Delta R^2 = R^2_{\text{log lin regression}} - R^2_{\text{partial corr.}}$

3.3.4 Adult females (>130cm)

Explanatory variable	R^2	Pr(>F)	N
circumference, neck	0.818	0.00000	24
circumference, breast	0.797	0.00000	24
circumference, abdomen	0.588	0.00000	26
circumference, hip	0.368	0.00101	26
fat, breast back	0.208	0.02876	23
fat, breast ventral	0.207	0.02534	24
fat, hip ventral	0.206	0.21924	9
fat, neck ventral	0.181	0.03816	24
fat, neck back	0.158	0.04949	25
fat, abdomen ventral	0.147	0.06456	24
fat, neck right	0.117	0.09444	25
fat, breast right	0.097	0.15914	22
fat, abdomen right	0.084	0.16045	25
fat, hip back	0.003	0.78916	23
fat, hip right	0.000	0.97277	24
fat, abdomen back	0.000	0.92869	25

(a) Log linear regression, , ordered by R^2

Explanatory variable	R^2	Pr(>F)	N	ΔR^2
fat, hip right	0.244	0.01424	24	0.244
fat, abdomen right	0.287	0.00578	25	0.203
fat, neck back	0.352	0.00176	25	0.194
fat, breast back	0.399	0.00123	23	0.191
fat, breast ventral	0.397	0.00096	24	0.190
fat, neck ventral	0.370	0.00162	24	0.189
fat, neck right	0.273	0.00735	25	0.156
fat, abdomen ventral	0.280	0.00782	24	0.133
fat, breast right	0.219	0.02825	22	0.122
fat, hip back	0.114	0.11456	23	0.111
fat, abdomen back	0.025	0.45023	25	0.025
fat, hip ventral	0.147	0.30882	9	-0.059
circumference, breast	0.710	0.00000	24	-0.087
circumference, hip	0.193	0.02486	26	-0.175
circumference, neck	0.640	0.00000	24	-0.178
circumference, abdomen	0.250	0.00926	26	-0.338

(b) Partial correlation, , ordered by ΔR^2

Table 11: The results of the log linear regression and the partial correlation for adult females (>100cm). The difference in R^2 is measured as $\Delta R^2 = R^2_{\text{log lin regression}} - R^2_{\text{partial corr.}}$

In table 8 through 11 we see the results of the log linear regression and partial correlation with circumference and blubber thickness as explanatory variables.

In the log linear regression, models with the circumference variables yielded an R^2 between 0.197 and 0.818 and were all highly significant. Models with the four circumference variables gave the highest R^2, except in the group “juveniles”, where the model with *circumference, hip* not was among the four highest ranking models. The models based on blubber thickness yielded an R^2 between 0.000 (i.e. < 0.0005) and 0.305, the significance of these models varied greatly between the different groups; in the group “juveniles” were all twelve models highly significant, whereas in the other three groups only five out of 36 models were significant at the 5% level.

Looking at the partial correlation we see that models based on variables for circumference yielded an ΔR^2 between -0.338 and 0.165 and, that for these models $\Delta R^2 < 0$ for all groups, except for the group “Adult Males” where three of the models based on circumference variables had an $\Delta R^2 > 0$. All of the models with variables measuring girth were highly significant. Models with blubber thickness as explanatory variable yielded a ΔR^2 between -0.157 and 0.267, 10 of these models had a negative ΔR^2 and 38 had a positive one. The significance for the models based on variables for blubber thickness varied greatly between the different groups; none of the models where significant in the group “Adult males”, whereas in the other three groups 33 out of 36 models were significant at the 5% level.

4 Discussion

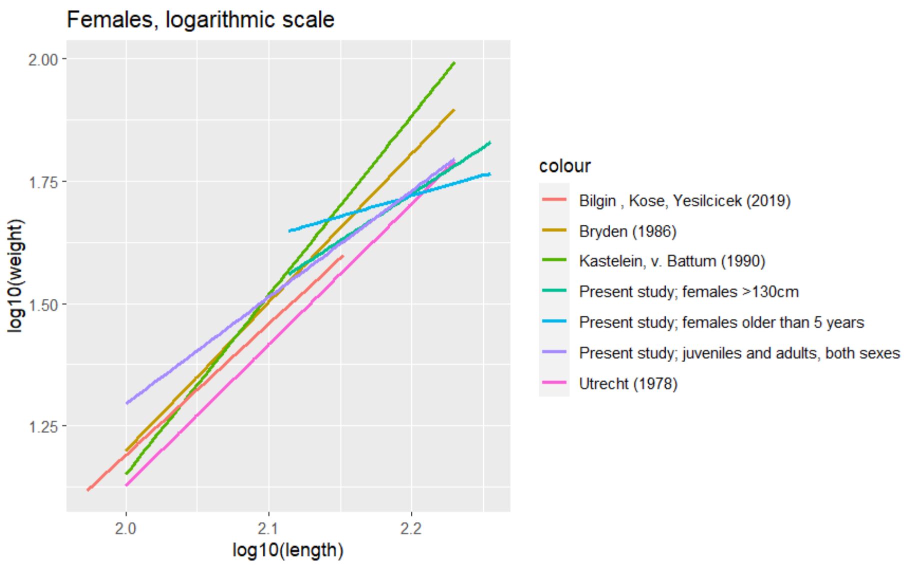
4.1 Weight - length relationship

As of 2019 three subspecies of harbour porpoises are being recognized by Society for Marine Mammalogy’s Committee on Taxonomy; the Pacific Harbor Porpoise (Phocoena phocoena vomerina), the Atlantic Harbor Porpoise (P. p. phocoena) and the Black Sea Harbor Porpoise (P. p. relicta) (IUCN, 2020). In several articles it have also been reported a difference in morphology between porpoise populations of different geographical locations. Kastelein and Van Battum (Kastelein & Van Battum, 1990) reports that: “Animals of East Canadian population are larger than animals of the same age from the North Sea population (Gaskin et al., 1984), while animals of the Baltic population are more slender, but have more body fat than those from the North Sea (van Utrecht, 1960)”. In the inner Danish and adjacent Swedish and German waters it have been reported that females grow to lengths of 161 cm and males to lengths of 148 cm (Galatius, 2005), whereas in the Black Sea males attain a maximum lengths of 120 cm and females a maximum length of 130 cm (Tonay, Dede & Öztürk, 2017). Furthermore, animals of the Sea of Azov are reported to be longer than animals of the same age from the Black Sea (Gol’din, 2004).

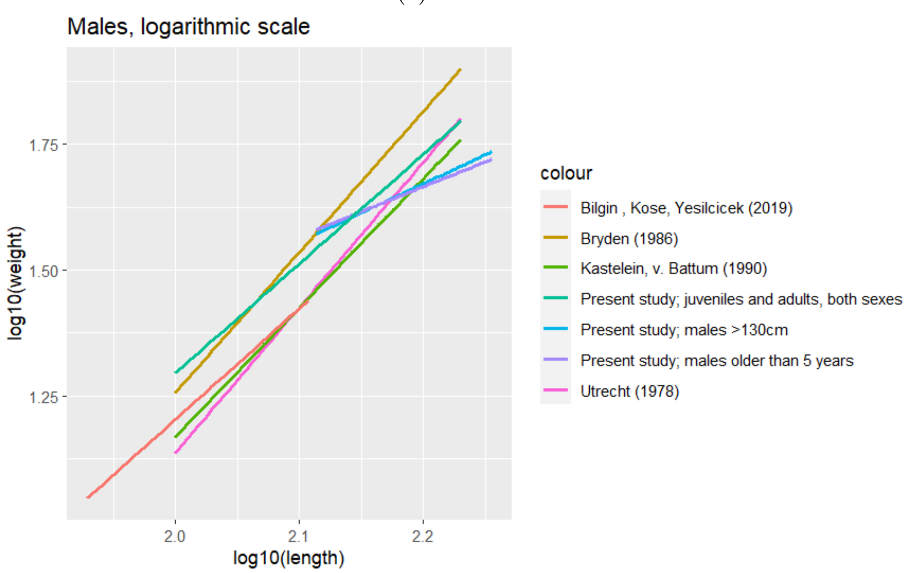
As mentioned in the introduction, weight - length relationships are important tools for marine biologist and fisheries managers and, due to differences in morphology between populations, it is important to have weight - length relations at hand for the population of interest. Furthermore, comparing weight - length relationships may be useful to determine morphological differences between populations.

Coefficients	Sex/ Age class	Region	R^2	Author	Year	N
$\alpha = -4.3473$ $\beta = 2.8011$	Males	Baltic Sea	NA	**Data:** Møhl-Hansen (1954). **Formula:** Bryden (1986)	1954	208
$\alpha = -4.8814$ $\beta = 3.0395$	Females	Baltic Sea	NA	**Data:** Møhl-Hansen (1954). **Formula:** Bryden (1986)	1954	164
$\alpha = -4.6445$ $\beta = 2.8902$	Males	North Sea	NA	Van Utrecht	1978	41
$\alpha = -4.6369$ $\beta = 2.8818$	Females	North Sea	NA	Van Utrecht	1978	58
$\alpha = -3.9743$ $\beta = 2.5707$	Males	North Sea	0.97	**Data:** Kastelein & Van Battum (1990). **Formula:** Bilgin et al. (2019)	1990	7
$\alpha = -6.1758$ $\beta = 3.6630$	Females	North Sea	0.88	**Data:** Kastelein & Van Battum (1990). **Formula:** Bilgin et al. (2019)	1990	18
$\alpha = -3.2198$ $\beta = 2.2109$	Males	Black Sea	0.83	Bilgin et al.	2019	31
$\alpha = -4.1719$ $\beta = 2.6807$	Females (Pregnant + non-pregnant)	Black Sea	0.89	Bilgin et al.	2019	37
$\alpha = -0.8666$ $\beta = 1.1537$	Males (> 130cm)	Swedish Waters	0.15	Present study	2023	102
$\alpha = -0.5088$ $\beta = 0.9884$	Males (> 5 years old)	Swedish Waters	0.09	Present study	2023	35
$\alpha = -2.4781$ $\beta = 1.9100$	Females (> 130cm)	Swedish Waters	0.42	Present study	2023	129
$\alpha = -0.1227$ $\beta = 0.8375$	Females (> 5 years old)	Swedish Waters	0.06	Present study	2023	39
$\alpha = -3.0605$ $\beta = 2.1775$	Both sexes, juveniles and adults	Swedish Waters	0.71	Present study	2023	493
$\alpha = -4.3315$ $\beta = 2.7919$	Juveniles (100 − 130cm), both sexes	Swedish Waters	0.55	Present study	2023	262
$\alpha = -2.2296$ $\beta = 3.1551$	Juveniles (1 − 5 years old), both sexes	Swedish Waters	0.67	Present study	2023	232

Table 12: Weight - length relationship from various studies showing the coefficients for the formula $\log_{10}(\text{Weight}) = \alpha + \beta \cdot \log_{10}(\text{Length})$ (Bilgin, Kose & Yesilcicek 2019).



(a) Females



(b) Males

Figure 15: Comparing the length-weight relationships of the present study to earlier studies

Table 12 show the coefficients for the weight - length relationship calculated with linear regression from data gathered from five different years and from different geographical locations (these weight - length relationships are retrieved from

(Bilgin, Kose & Yesilcicek, 2019)). Figure 15 show lines of the different weight - length relationships from table 12 (juveniles are not included). Comparing the lines it seems that the slopes for males and females of the this study are lower than those of earlier studies (I have, however, not tested hypothesis of β to show differences). I, however, don't only attribute these differences to differences between populations. Rather, I think the main reason for this is because we have calculated weight-length relationships for adult animals, whereas the other studies have used juveniles and adults together and, due to growth, it is likely that juveniles have steeper weight-length curves. Looking at the curve for juveniles and adults together from this study we see a higher slope, reinforcing my belief that the main difference is due to difference in growth between adults and juveniles. However, the slope for the group juveniles and adults in this study is lower than the slope of all earlier studies, indicating that the weight-length relationship curve for populations in Swedish waters may be flatter than for other populations.

4.2 A residual measure of body condition

To construct residual body condition indices we tried using *total length* as a linear regressor and GAM models to model the seasonal changes in weight and, in addition, to account for differences in weight between animals of different gender and in different stages of maturity, we used factor variables based on length and gender or, on age and gender.

Of these variables it was evident that the variable *total length* contributed to the biggest part of the coefficient of determination in our residual indices, our baseline models (table 3) yielded an R^2 between 0.671 and 0.701. Also, looking at the models for our residual indices (tables 4 through 6), the models including *total length* were more significant and had an increase in R^2 of between 0.238 and 0.734 compared to models which only were based on seasonal variation and an age class variable. Likewise, Lockyer (Lockyer, 1995) showed in her article that the animals weight related strongly to the animals length, both for male and female porpoises, whereas Spotte (Spotte, 1978) came to the conclusion that length alone wasn't enough to determine an animal's weight and, Møhl-Hansen (Møhl-Hansen, 1954) showed that the weight of adult animals of the same length could differ as much as 25kg. Having that in mind a residual body condition index used to determine if an individual is well nourished or not should not solely rely on length as explanatory variable, nonetheless, my conclusion is that such an index should benefit from including it.

As discussed above, *total length* is by it self not sufficient to explain weight and should be complemented with other variables to construct a residual index. In addition to the animals length, we have in this work also used the day of the year to construct our residual indices. Looking at the ΔR^2 in the tables 4, 5 and, 6, we see an increase in the adjusted R^2 of between 0.033 and 0.062 indicating that the seasonal variation had a small influence on the coefficient of determination. However, all smooth elements in the models including both *total length* and day of the year showed a significant influence, leading me to believe that the seasonal variation do is useful in our residual indices.

In the interest of not over complicate the residual indices, we should ask ourselves if there is any variable we can exclude from the models. My conclusion is that for these residual indices to be useful we need *total length* and *day* (or, perhaps, an another variable instead of *day* but that lies outside of the scope of this work), which only leaves us with the factor variables for age classes to exclude. Compared to the model $\log_{10}(\text{weight}) = \log_{10}(\text{total length}) + \text{s(day)}$ the variables for the age classes doesn't contribute to a higher adjusted R^2 and only an increase of in R^2 of 0.03, indicating that is not necessary to include them in the model.

4.2.1 Testing the residual index

The testing of the models for the residual indices shows that the median for the residuals increases as the level of nutritional status increases, which is what we would expect of well constructed indices. However, since we have used zero as our baseline ideally we would like to see positive residuals for the levels "Well nourished" and "Averege nourished" and, negative ones for "Less than average nourished" and "Emaciated", but table 7 shows that all medians are negative suggesting that the residuals are overall to low. One way to correct for this could be to use a negative constant as a new baseline to compare the residuals to or, equivalently, adjust the residual index with a constant $c > 0$ so that the residuals would be calculated as $e = y - \hat{y} + c$. Judging by the box plots (figure 5 through 6) a new baseline around the median for residuals for the group "Average nourished", i.e. around -2.5 or, equivalently $c = 2.5$, might give a better indication of nutritional status. Adjusting the residual indices with a constant, $c \approx 2.5$, would mean, at least for the observations tested, that residuals for the group "Average nourished" would be centered around zero, the group "Well nourished" would have mostly positive residuals and, that the groups "Less than average nourishment" and "Emaciated" would have negative medians.

Comparing the models tested we see that the model without a variable for age class ("All porpoises > 100cm", table 7) had the best performance of the three, i.e. most positive residuals for "Well nourished", a median that was closest to zero for "Average nourished" and, equally or more negative residuals for the groups "Less than average nourishment" and "Emaciated". This, once again, indicates that it's not necessary or even preferable to include a factor variable for age class in the model.

It is, however, important to note that there are some limitations to how accurate the testing of the models might be. Firstly, we had a small sample size when we tested our models, only 47 of the observations in data had an observation for the variable *Nourishment* present and, secondly, we had no information of how the the level of nourishment was estimated. It is not inconceivable that a larger sample size or different methods to measure nourishment level would generate different distributions of residuals. Therefor further testing of the residual indices could be of interest.

4.3 Circumference and blubber thickness

In testing how well the variables for blubber thickness and circumference could explain the animals weight we used two methods; log linear regression and partial correlation.

4.3.1 Log linear regression

Circumference variables

As discussed in the introduction, girth measurements are important in assessing the nutritional status of an individual (Kastelein & Van Battum, 1990). Our findings in the log linear regression seems to support this statement and, in the results we found that three of the four variables for girth, namely *circumference, breast*, *circumference, neck* and *circumference, abdomen* were all very useful to explain the variation i weight. They all produced models which all were highly significant and with the highest R^2 (between 0.362 and 0.818) of all models in all groups. Further, of these three variables, *circumference, neck* and *circumference, breast*, yielded the highest R^2 in three of the groups of between 0.751 and 0.818.

Lockyer (Lockyer, 1995) found that weight and the girth around the center of the animals related strongly to each other and, that weight and mid-girth were more strongly related than that for length and weight. Likewise, comparing our results with the table with weight - length relationships (figure 2, section 3.1) we see for that all comparable groups that, although we had more available observations for *total length*, variables for girth can explain weight better than *total length*. She (Lockyer, 1995), also made multiple regression models with length and mid-girth against weight with logarithmaized variables for three different groups of animals that all yielded an R^2 over 0.97, she further notes that there is a strong correlation of body weight with both length and mid-girth indicating that body weight can be predicted by log-linear regression using weight on both length and mid-girth. Mid-girth alone may also be useful to to predict body weight in damaged carcasses where flukes or head are missing (Lockyer, 1995).

Koopman (Koopman, 1998) hypothesized that blubber in the thoracic-abdominal region acts primarily as an energy reserve and isolation, whereas posterior blubber acts to maintain hydrodynamic shape om the peduncle, also an examination of nine emaciated porpoises showed that they had used 20-40% of their blubber layers in the thoracic-abdominal area but had not used any of the blubber posterior to the anus. Since the posterior blubber layer not is affected by famine *circumference, hip* may not be a good indicator whether an animal is well nourished or not and, we also saw in that this variable yielded a lower R^2 than the other models with circumference variables.

Variables for blubber thickness

As discussed in the introduction, blubber thickness is commonly used in marine mammals to asses nutritional status (Kauhala et al., 2019; Marón et al., 2021) although it shouldn't be used as standalone metric (IJsseldijk et al., 2019) and, Siebert et al. (Siebert et al., 2022) also note that while blubber thickness is an important screening tool one cannot rely on these measurements to asses the overall health in marine mammals. Our results showed that using blubber thickness as a regressor yielded, with one exception, lower R^2 and, except for the group "juveniles", less significant models than those that relied on variables for circumference. All this coupled with what was discussed in the paragraph above suggest that girth measurements anterior of the anus might be a better tool to infer nutritional status.

There are findings from former studies suggesting that blubber thickness has more to do with stage of maturity than nutritional status. Lockyer (Lockyer, 1995) found that blubber thickness varied greatly with development stage and body size in harbour porpoises in British waters. As an animal grow into an adult muscle becomes a more dominant tissue and, at the same time, blubber appears to be of less importance (Lockyer, 1995). Small and juvenile animals are both relatively and actually fatter than adults which may be explained by the greater surface/volume of young and their need for insulation and thermoregulation (Lockyer, 1995). Although Lockyer (Lockyer, 1995) had limited data, her findings among females indicated that pregnant females were heaviest and fattest, lactating females had the thinnest blubber and, that anoestrous females had blubber thickness in between the two other groups.

Heather N. Koopman (Koopman, 1998) who used linear regression on body length against blubber thickness measured at various sites on the animals from the Bay of Fundy, Canada, and Gulf of Maine and mid-Atlantic coast of the United States, also found that blubber thickness varied considerably between reproductive classes but that there was no apparent relation between blubber thickness and length within the groups. She found that blubber thickness correlated negatively with body length at sites measured anterior of the anus and draw the conclusion that harbour porpoises are the only marine mammal in which the blubber thickness is enantiometric; meaning that in absolute terms blubber thickness decrease with increasing length. This negative relationship in harbour porpoises was first observed by Møhl-Hansen (Møhl-Hansen, 1954) in harbour porpoises in Danish waters who found that blubber in calves was thicker than that in pregnant and simultaneously lactating females. Similarly to Lockyer, Koopman (Koopman, 1998) observed that calves had the thickest blubber, lactating females the thinnest and, for both sexes, that juveniles tended to have thicker blubber than older animals.

Other findings do suggest that nutritional status do affect blubber thickness. Siebert et al. (Siebert et al., 2022) investigated the seasonal variation in blubber thickness in harbour porpoises in the southern Baltic Sea using GAM and found a strong seasonal effect in adult and juvenile individuals with thickest blubber recorded in winter and spring and the lowest values in late summer and early autumn, see figure 16. Looking at the predicted curves in section 3.2.2 we see a similar pattern with the highest weights predicted in winter and spring and lowest weights in late summer and early autumn indicating that starvation do affect blubber thickness or, at the very least, that weight and blubber thickness co-variate due to other factors.

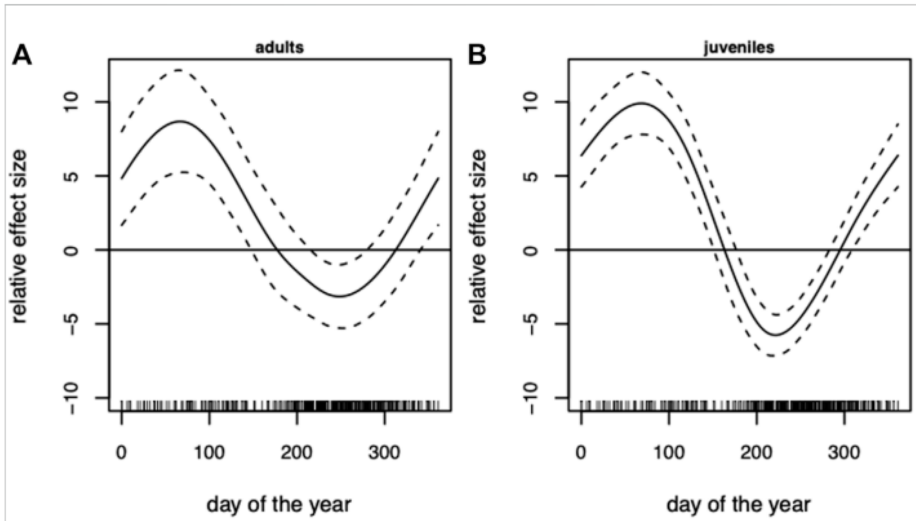


Figure 16: Seasonal variation of blubber thickness (Siebert et al., 2022)

Of the the groups of animals investigated in the log linear regression of blubber thickness against weight in this work, the group “juveniles” stood out with all models being highly significant and in general higher R^2 compared to the other groups. This, coupled with the findings discussed above, makes me believe that nutritional level do have some effect on blubber thickness, maybe up to a point, but most of the correlation with weight can be explained by the fact that animals in different development phases have different blubber thickness (i.e. younger animals have thicker blubber).

Between the groups “Adult males” and “Juveniles and adults” only one model using blubber thickness where significant and only one model produced an $R^2 > 0.1$. The group “Adult females” showed a bit higher R^2 and had four significant models, which I attribute to the fact that pregnant females build up a thicker layer of blubber and that lactating females are leaner. Looking at scatter plots of blubber thickness against weight it’s evident that for these three groups blubber thickness doesn’t vary much with weight, figure 17 shows an example on how weight vary with blubber thickness measured at the ventral side of the neck.

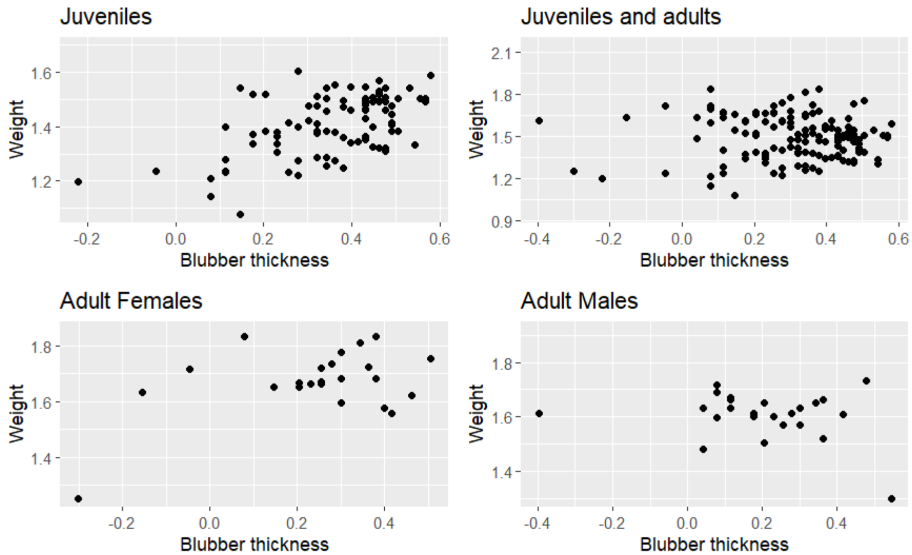


Figure 17: Blubber thickness measured at the ventral side of the neck versus weight (logaritmized).

Overall I find that blubber thickness not was very useful to explain weight in the log linear regression and because of this blubber thickness might not be good metric to infer nutritional status in individual animals. Starving porpoises have been shown, possibly due to the need of insulation, to maintain what may be a minimum level of blubber thickness of 1 cm, while at the same time having consumed a portion of their swimming muscles (W.A. McLellan, pefs. comm., H. N. Koopman, in litt.) (Koopman, 1998). I speculate that in starving porpoises blubber thickness decrease within a fixed interval but as the blubber thickness is down to 1 cm, the loss of mass is due to loss of muscle tissue. This phenomenon may be the reason why the log linear regression wasn't more successful, possibly other methods would be more successful in inferring nutritional status from blubber thickness.

Although blubber thickness might not be useful to infer nutritional status in individual animals, there are still evidence that loss of blubber thickness may cause harmful effects at group levels, such as failures to reproduce and declines in survival rates (Spraker et al., 2020; IJsseldijk et al., 2021). Because of this blubber thickness may still be a useful metric to determine health status at the population level.

4.3.2 Partial correlation

Circumference variables

I found it to a bit surprisingly that in all groups, except the group "adult males", the models using the circumference variables produced a lower R^2 compared to the log linear regression. I would've expected that when we account for the influence of confounding variables that the partial correlation also would have created models yielding higher R^2:s. In our models we have tried to explain the influence of the confounding variables for length, day of the year and, age class. Let's see if can find a explanation for the lower coefficient of determination by investigating the influence on the circumference these variables may have.

In the model we presume that the relation between circumference and length can be written as

$$\text{circumference} = \alpha \cdot \text{total length}^{\beta}$$

$$\Leftrightarrow$$

$$\log_{10}(\text{circumference}) = \log_{10}(\alpha) + \beta \cdot \log_{10}(\text{total length}).$$

I suspected that $\log_{10}$ (circumference) and $\log_{10}$ (total length) may not have a linear relationship and that that would be the cause of the decreased R^2. However, looking at scatter plots of $\log_{10}$ (circumference) and $\log_{10}$ (total length) i haven't been able to find evidence that support this explanation. In these plots it looks like these two variables have a linear relationship and, in fact, the group "adult males" which showed an increase in R^2 also seems to have the least linear relation between $\log_{10}$ (circumference) and $\log_{10}$ (total length).

We tried in our model to account for the seasonal variation in circumference by using GAM. One would expect that if circumference showed a considerable seasonal variation, using partial correlation should also result in higher increase in R^2 and, by contrast, low seasonal variation should lead to a smaller increase in R^2 . We know from the log linear regression that circumference, maybe with the exception of the circumference around the hip, and weight have a strong correlation. Because of this I think it's safe to say that the seasonal variation in weight also reflect the seasonal variation in circumference, perhaps with the exception of the circumference around the hip. In section 3.2.2 we have curves predicting the seasonal variation in weight for porpoises, looking at this curves may give an idea of the seasonal variation in circumference. Since we have used the age class variable *length_sex_class* as a factor variable, the figures of interest would be figure 8 and, figures 12 through 14.

The groups "juveniles and adults" (figure 8) and "juveniles" (figure 12 both showed a somewhat small seasonal variation in weight (and, hence, in circumference) with a maximum difference in variation of around 8-10kg. This week seasonal variation might be an explanation to why we see decreased R^2 in these groups. The group "adult females" (figure 13) showed a higher seasonal variation than the other groups in weight, possibly due to lactation and pregnancy, with this in mind we should have seen an increase in R^2 too. The group "adult males" were the only group where we saw increased R^2:s. This group showed a lower seasonal variation in weight (figure 14) than the other groups, again I found it a bit confusing since lower seasonal variation should have produced lower $\mathbf{R}^2$:s.

All in all I find that the way we have used partial correlation not was very useful when using the variables for circumference. The only pattern I can infer is that it seems that in groups where we saw high R^2 in the log linear regression ("juveniles", "juveniles and adults", "adult females") we also found a decreased R^2 in the partial correlation. By contrast the group "adult males" where we saw a lower R^2 compared to the other groups we found that these models did produced an increased R^2 in the partial correlation.

Variables for blubber thickness

As discussed in the discussion about log linear regression above porpoises show a considerable variation in blubber thickness between different groups of animals of different sexes and in different stages of maturity (Lockyer, 1995. Koopman, 1998). Because of this we would expect an increase in the coefficient of determination when we account for this confounding factors in the partial correlation. In all groups except for adult males we do see increased R^2 for nearly all of our models using blubber thickness as explanatory variables this increase, however, it is quite modest with an maximum ΔR^2 of 0.256.

Since we in our models for the partial correlation have tried to account for the confounding influences of the variables for length, day of the year and, age class, we should look at these variables to see if we can find an explanation to why we don't see a higher increase in coefficient of determination in the partial correlation. As mentioned above there is a considerable variation in blubber thickness between the different groups of animals (Koopman, 1998) and, similarly to what have been discussed in the section about the circumference variables we have presumed in our model that $\log_{10}$(blubber thickness) and $\log_{10}$(total length) have a linear relationship. We know from discussion about log linear regression that Siebert et al. (Siebert et al., 2022) found strong seasonal variation in blubber thickness which should contribute to an increase in R^2 in the partial correlation.

The group "Juveniles and adults" had the overall largest increase in R^2 with four models having a ΔR^2 higher or equal to 0.2. That this group had the highest increase is what we would expect since this group included animals of different sexes and in different stages of maturity and, accounting for these confounding variables should also produce higher R^2 compared to the log linear regression.

The groups "Juveniles" and "Adult females" had about the same amount of increase in R^2 with one and two models with a ΔR^2 higher or equal to 0.2, respectively. As discussed above a juvenile porpoise that grow into an adult gets a thinner layer of blubber (Koopman, 1998), because of this phenomena, I'm a bit surprised that we don't see bigger increase in R^2 in the group juveniles. For the adult females I speculate that they have stronger seasonal variation in blubber thickness due to periods of suckling calves and pregnancy which may contribute to higher coefficient of determination in the partial correlation.

In the group "adult males" most models using blubber thickness as a regressor had a ΔR^2 around zero or lower. Perhaps this reflect that the blubber thickness of adult males may not vary much between animals of different lengths nor between difference times of the year. This is, however, a speculation on my part and I have no concrete evidence to support this hypothesis.

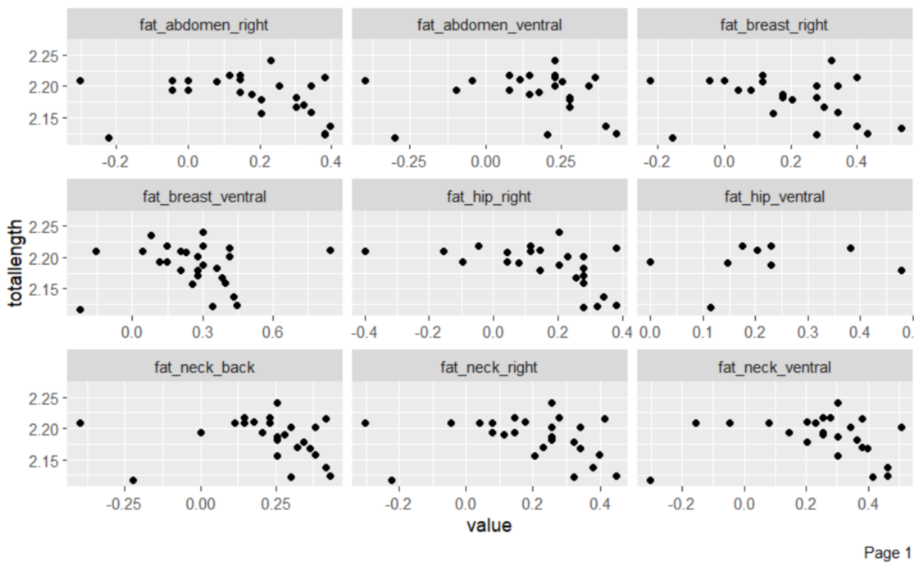


Figure 18: Scatter plots of blubber thickness against total length for adult females.

I have been looking at scatter plots of $\log_{10}$(blubber thickness) against $\log_{10}$(total length) to see if I can determine if they share a linear relationship. Looking at these plots for all groups I haven't been able to see any clear sign of a linear relationship and, this may be the reason why we haven't seen a higher increase in R^2 in the partial correlation. However, to get a definitive proof of the linear relationship between these two variables regression analysis should be performed.

Although we do see an increase in R^2 for most groups when using partial correlation, the R^2 yielded in this models were generally low and, with maximum R^2 of 0.43. In our results we also see the same tendency as in the log linear regression; measurements for the circumference generally give an higher R^2 than that for the measurements of blubber thickness. This may reflect that our methods and models when using the variables for blubber thickness not have been very useful and that with different methods we could have obtained a better result. It may also reflect, which I tend to believe, that blubber thickness not is a very useful variable to predict the animals weight.

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5 Appendix

5.1 Correlation between the continuous variables

5.1.1 All propoises >100 cm and older than 0 years

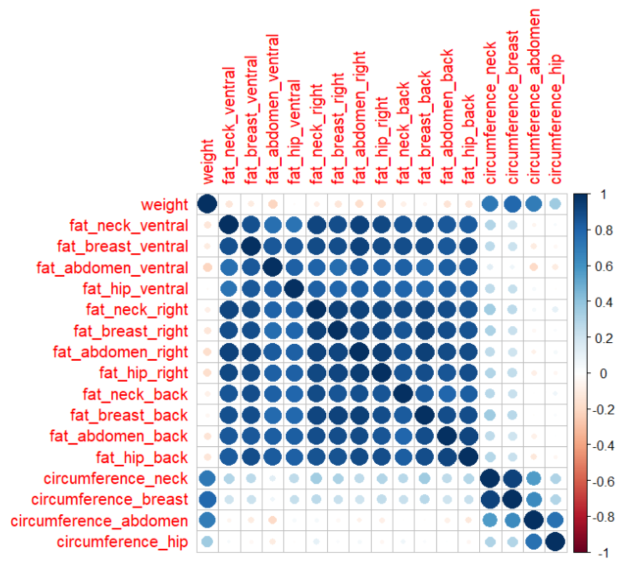

Figure 19: The correlation between weight, blubber thickness and circumference for all porpoises >100 cm

5.1.2 Groups of animals based on length and gender

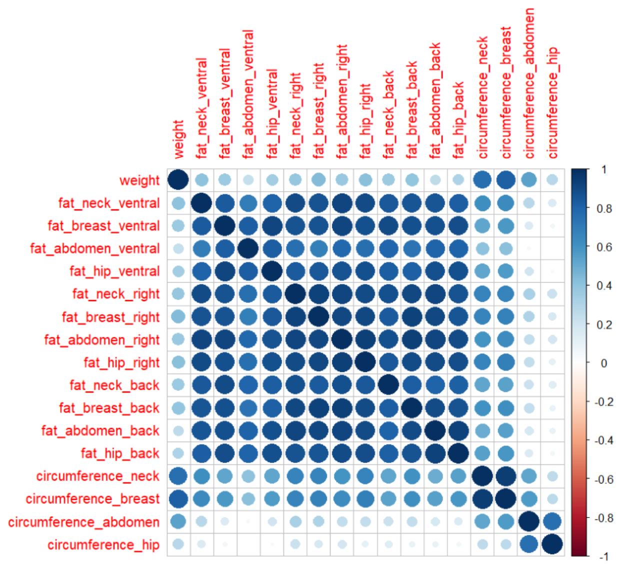

Figure 20: The correlation between weight, blubber thickness and circumference for juveniles, 100-130 cm

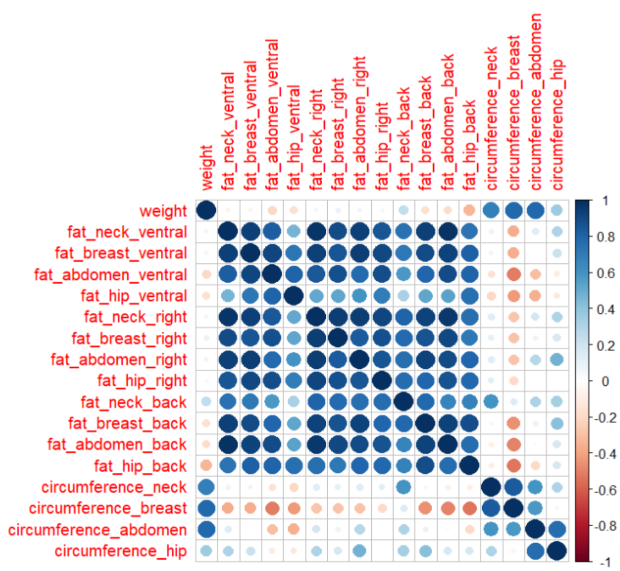

Figure 21: The correlation between weight, blubber thickness and circumference for males longer than 130cm

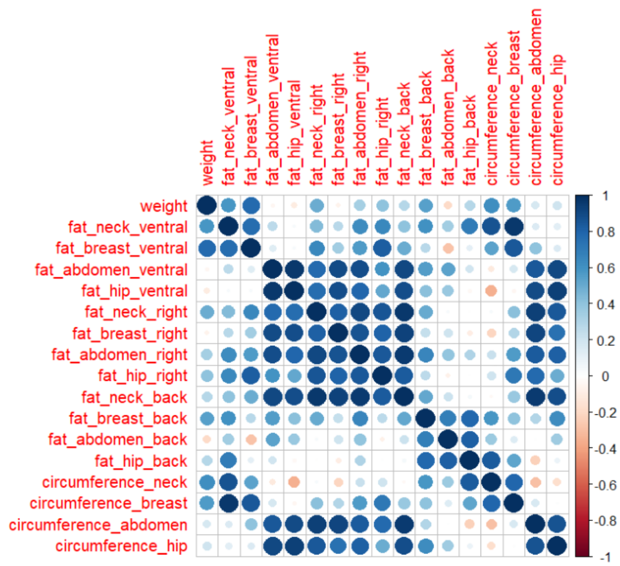

Figure 22: The correlation between weight, blubber thickness and circumference for females longer than 130cm

5.1.3 Groups of animals based on age and gender

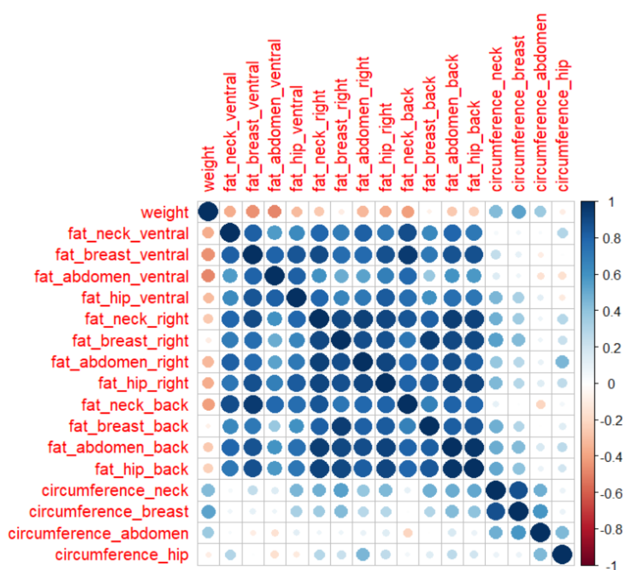

Figure 23: The correlation between weight, blubber thickness and circumference for juveniles, 1-5 years old

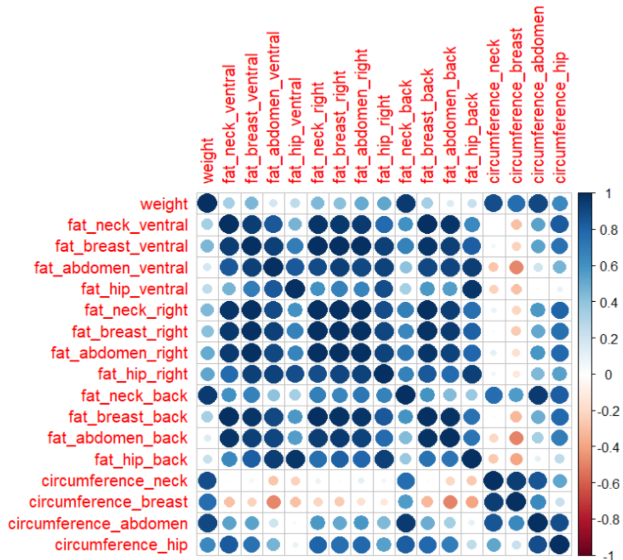

Figure 24: The correlation between weight, blubber thickness and circumference for males older than 5 years

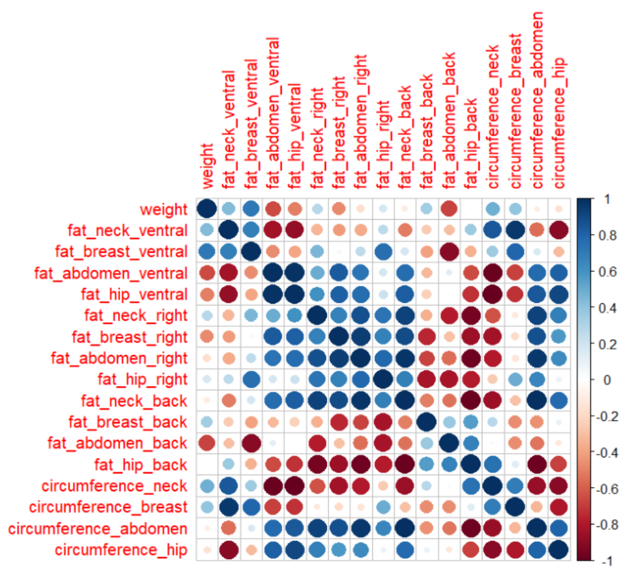

Figure 25: The correlation between weight, blubber thickness and circumference for females older than 5 years