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Edgeworth Expansions for Studying Rate of Convergence towards Normality - with Applications to Variance-Stabilizing Transformations

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Abstract

The aim of this thesis is to present a theoretical framework through which the rate of convergence towards normality can be studied (in the univariate case) - thus providing insight as to what factors to consider when applying the central limit theorem. This is done using Edgeworth expansions of the cumulative distribution function and much focus is placed on cumulants, which they rely upon. We also derive a more general version of the Edgeworth expansion that allows for expectation zero and unit variance to appear only asymptotically, as opposed to the standard expansion that assumes these properties to hold in general. The framework is then applied to variance-stabilizing transformations for two underlying sampling distributions - the Poisson and the exponential. The results tell us that applying the variance-stabilizing transformation in its most simple form does not lead to improved convergence, due to the introduction of a bias and the variance not really being a constant for finite samples. Standardizing the variance-stabilized variable, however, we do see signs of improved convergence for both distributions. In the Poisson case the improvements depend on the product of the rate parameter and the sample size, and in the exponential case the improvements permeate through all tested sample sizes. As such we advise to search for other transformations if the underlying sampling distribution is Poisson, but suggest the standardized variance-stabilized transformation as a viable option if the distribution is exponential.

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