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Tail Estimation: a Comparative Simulation Study of Extreme Value Theory and Importance Sampling

Betty Frankl*

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Abstract

Extreme value theory was developed specifically for analysing extreme events that failed to be analysed by conventional methods. It is difficult to find alternative methods that could be used in practice. However, in a simulation study where the underlying distribution is known, there are alternative methods that could be used. Importance sampling is a Monte Carlo variance reduction technique, that can be used to handle rare events. In this simulation study we will compare these two methods in order to highlight differences and gain deeper knowledge.

We focus on tail estimation and place particular emphasis on the extreme right tails. In these cases there are typically very few, or no, observations in the area of interest. The methodology we use from extreme value theory is the peak over threshold model and the generalized Pareto distribution (GPD) method. For importance sampling we use an extreme order biasing technique.

The comparison of the methods reveals key differences in their approaches and performance. Importance sampling proves both effectiveness, compared to Monte Carlo, and reliability, due to its ability to generate unbiased estimates. However, both importance sampling and Monte Carlo show their limitations for small sample sizes, often resulting in zero estimates. The GPD method exhibits variability in performance across distributions and sample sizes. The deviation from the true values are high in many cases. Importantly, though, the GPD method has the capacity to produce non-zero estimates which is significant for real-world applications where zero probability assumptions may not be valid.

*Postal address: Mathematical Statistics, Stockholm University, SE-106 91, Sweden. E-mail: betty.frankl@gmail.com. Supervisor: Johannes Heiny Ola Hössjer.