

Geometrically Aware markov Chain Monte Carlo

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Abstract

In this thesis we study the generalisation of the Metropolis Adjusted Langevin Algorithm to the Riemannian manifold of symmetric positive definite matrices $P(n)$. Specifically, an application to hierarchical models that involve the Wishart distribution are considered. A concrete example is given for modelling the rates of synonymous and non-synonymous substitution in a phylogeny. It is proven that a large class of uniformly log-concave posterior densities attain ϵ -bounded Wasserstein distance from their invariant measures in $O(1/\epsilon^2)$ iterations of the Riemannian Metropolis-adjusted Langevin Algorithm. It is also shown that common generalisations of the LKJ-distribution never satisfy a set of sufficient conditions for this bound. Lastly, it is conjectured that certain conditions for attaining the iteration complexity bound may be weakened to hold probabilistically only.

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