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Tail Asymptotics: Large Deviation Theory and the Principle of a Single Big Jump

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Abstract

In insurance mathematics and risk modelling, the law of large numbers and the central limit theorem describe only the average case and do not quantify the probability of extreme aggregate losses. Letting $S(t)$ denote the total claim payout up to time t, this concern arises in two forms: a long-run question, the decay rate of $P(S(t) \geq xt)$ as $t \to \infty$, and a fixed-horizon question, the behaviour of $P(S(t) > x)$ as $x \to \infty$ for fixed t. Each question requires a different class of claim size distributions and is addressed by a different framework: the long-run question by large deviation theory, the fixed-horizon question by the Principle of a Single Big Jump.

Large deviation theory requires the claim sizes to be light-tailed. Its central object is the rate function $I(x)$, which characterises the exponential rate at which $P(S(t) \geq xt)$ decays. The Principle of a Single Big Jump instead requires claims to be subexponential, a subclass of heavy-tailed distributions, under which $P(S(t) > x) \sim \lambda t \cdot P(X > x)$ as $x \to \infty$.

These results are verified through simulation, and each framework is shown to break down when applied outside its conditions. Despite combining a random number of claims with random claim sizes, the compound aggregate loss process reduces to a single factor times a single tail under the right distributional conditions.

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