## Stockholms universitet

# Macroeconomic indicators and their effect on Tactical Asset Allocation 

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#### Abstract

In this thesis we develop a strategy for tactical asset allocation that takes into account macroe- conomic variables. The relationship between the macroeconomic variables and financial asset returns is modeled by using an M-GARCH error structure. This relationship will be used as a signaling value and implemented in the asset allocation strategy. In order to evaluate the effect the signaling value has on the performance of the portfolio, three allocation strategies will be compared, two of which will take into account the macroeconomic environment. We illustrate the benefits of the macroeconomic-based strategies by looking at three Swedish financial securities while using inflation and PMI as a macroeconomic factors. The portfolio performance will be measured by its' Sharpe ratio. The results show that the best performing strategy is the mean-variance allocation including both of the macroeconomic factors. In this study the dynamic modeling techniques that contain the influence of the macroeconomic environment outperforms the model that excludes it, thus offering greater risk-return combinations.


[^0]'Optimizing based on historical correlations is like driving by looking through the rear view mirror
' Craig Israelsen 2012'
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#### Abstract

In this thesis we develop a strategy for tactical asset allocation that takes into account macroeconomic variables. The relationship between the macroeconomic variables and financial asset returns is modeled by using an M-GARCH error structure. This relationship will be used as a signaling value and implemented in the asset allocation strategy. In order to evaluate the effect the signaling value has on the performance of the portfolio, three allocation strategies will be compared, two of which will take into account the macroeconomic environment. We illustrate the benefits of the macroeconomic-based strategies by looking at three Swedish financial securities while using inflation and PMI as a macroeconomic factors. The portfolio performance will be measured by its' Sharpe ratio.

The results show that the best performing strategy is the mean-variance allocation including both of the macroeconomic factors. In this study the dynamic modeling techniques that contain the influence of the macroeconomic environment outperforms the model that excludes it, thus offering greater risk-return combinations.


## Sammanfattning

I denna uppsats utvecklar vi en strategi för taktisk tillgångsallokering som tar hänsyn till makroekonomiska variabler. Sambandet mellan de makroekonomiska variablerna modelleras med en M-GARCH struktur för feltermerna. Detta samband används som ett signalvärde och implementeras i allokeringsstrategin. För att utvärdera effekten av signalvärdet på portföljens prestanda jämförst tre optimeringsstrategier, varav två tar hänsyn till det makroekonomiska klimatet. Vi illustrerar fördelarna med den makroekonomiska strategin genom att utvärdera tre index för olika tillgångsklasser, medan inflation och PMI används som makroekonomiska faktorer. Portföljens prestanda mäts sedan med sharpe-kvot.

Resultaten visar att den mest framgångsrika strategin är den som tar hänsyn till de makroekonomiska faktorerna PMI och inflation. I denna studie överträffar modelleringsteknikerna som tar hänsyn till markoekonomiska faktorer de tekniker som inte gör det, ivad gäller portföljprestanda, mätt som riskjusterad avkastning.

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I I would like to thank Margareta Strockenfeldt, quantitative analyst at Mercer Sweden, for supervising me throughout the whole project. Margareta came with great advice on the subject of portfolio analysis and helped me gain a lot knowledge when it comes to financial modeling and markets in general.

II In addition, I would like to thank Taras Bodnar, university professor in financial mathematics at Stockholm University, for being my supervisor. Taras made important suggestions to the thesis structure.

III Lastly, I would like to thank Markus Andersson, former KTH mathematical statistics student, for the opportunity of interviewing him. Markus did a similar study in 2015, and he helped me figure out a good modeling procedure that allowed for testing different investor strategies.

Stockholm, August 6, 2018.

Niclas Englesson

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## 1 Introduction

Financial markets have a complex structure and are typically hard to predict. Financial modeling is defined as building an abstract representation of a real world financial situation. This requires that all key elements in the model are explicitly and consistently forecasted ${ }^{1}$.Investors are always looking for more sophisticated modeling techniques, especially considering the financial crisis of 2007-2008.

This thesis evaluates how macroeconomic factors can be utilized within a tactical asset allocation model for portfolio improvements. We build a tactical asset allocation model that focuses primarily on estimating the joint distribution of volatility of financial assets combined with macroeconomic variables. The model does not try to predict financial asset returns, as previous studies suggest there is greater accuracy in predicting asset return volatility ${ }^{2}$. The main idea of this thesis is to try and find a volatility model that can exploit the volatility contagion between the financial asset returns and macroeconomic factors. We want to test if macroeconomic information can help predict the variance-covariance matrix of asset returns. If this is the case, we can then use the gained information to improve tactical asset allocation. We do this by connecting Harry Markovitz mean-variance portfolio frontier to volatility time series modeling.

The mean-variance portfolio frontier is traditionally based on a constant, time-invariant volatility of asset returns. However, evidence suggests that the covariance matrix of asset returns is timevarying, creating the need for a time-dependent estimation. This means changing the inputs of the model from the unconditional covariance matrix to a time-varying conditional covariance matrix. When employing a time-varying conditional covariance matrix, the frontier also becomes time varying. It therefore requires continuous re-balancing of the portfolio weights, based on the time varying estimations. We model the volatility of returns by using a M-GARCH process that captures the volatility contagion between assets. This model is then extended by adding the macroeconomic factors PMI and inflation. The extended model will then account for the influence that the macroeconomic variables have on the conditional variance estimates of asset returns. The volatility estimates will then be used in the tactical asset allocation strategy where the holdings of each asset class are frequently updated based on the those estimates, which in turn take into account the macroeconomic factors.

We compare the different strategies by looking at measures of risk-adjusted return and plots

[^1]of the aggregated efficient frontiers. We also test each strategy for a one year period for a number of different portfolios. We find that the investment strategy that takes into account the macroeconomic variables seem to outperform the strategy that excludes them, for portfolios with relatively low risk/return targets. We find that portfolios containing macroeconomic variables can offer superior risk-return combinations of asset classes, suggesting investors in the Swedish market to adding these kinds of variables to their financial model.

We set up the thesis as follows; in chapter 2 we describe the basics of asset allocation and why investors should allocate among asset classes in the first place. We also introduce the set of assets that we later use for the financial modeling. Chapter 3 breaks down modern portfolio theory and explains all the necessary theory regarding the efficient frontier. It also introduces the concept of tactical asset allocation. Chapter 4 introduces volatility modeling and the necessary inputs in such models. Here we also discuss how adding the macroeconomic variables to the model will alter the variance/covariance estimates of asset returns. Chapter 5 shows the modeling procedure and the data that was used in the analysis. Chapters 6,7 and 8 shows the results and concludes the finding of modeling procedure and the data. We also discuss developments that could be made to the study.

## 2 Asset Allocation

This chapter explains the concept of asset allocation and the reasons behind why investors should allocate among asset classes. We begin by going through the different asset classes and what role they play in a portfolio. In this thesis we use one index for each asset class when looking at performance.

### 2.1 Asset classes

We will be looking at the following asset classes:

- Swedish Equity
- Swedish Bonds
- Cash

Throughout this thesis we measure the performance of an asset class by the performance of an underlying broad index. We use one index for each asset class. Each index represents the average development of the whole market of that asset class, i.e the Swedish equity index that we use represents the development in the Swedish equity market, the bond index represents the development in the Swedish bond market, and so on. The combination of the asset classes is what forms our asset allocation. For the asset classes we have chosen the following indices:

Swedish Equity is represented by SIX PRX, which reflects the average development of companies on the Stockholm Stock Exchange, adjusted for the investment restrictions that applies to equity funds. It contains around 250 securities, all listed on the Stockholm Exchange.

Swedish Bonds is represented by OMRX Total Bond Index, which contains government bonds issued by the Swedish state and covered bonds issues by mortgage housing agencies.

Cash is represented by Swedish 3-month Treasury Bills.

### 2.1.1 Historical Data

In order to get a better understanding of each asset class and how they perform relative to each other, we look at the historical data of the returns. Table 2.1 shows the annualized returns in percent for each asset class, over a 10 year period.

As indicated by the table, the standard deviation of the Swedish equity is almost twice as large as the average return. This is a common characteristic of equity assets. A risky asset has the characteristic that the mean return is roughly twice the size as its standard

| 10 Year Period | Annualized <br> Return | Std. Of <br> Annual Returns | Growth of <br> 100 SEK | Risk/return <br> ratio |
| :--- | :--- | :--- | :--- | :--- |
|  | $10,15 \%$ | $16,93 \%$ | $270,08 \mathrm{kr}$ | 1,7 |
| Swedish Bonds | $3,80 \%$ | $2,76 \%$ | $145,63 \mathrm{kr}$ | 0,7 |
| Cash | $0,72 \%$ | $0,44 \%$ | $107,74 \mathrm{kr}$ | 0,6 |
| Inflation | $1,42 \%$ | $0,20 \%$ | $115,69 \mathrm{kr}$ | 0.1 |

Table 2.1. Table of Annualized Returns of the Asset classes and Inflation. Data from 2008-01-31 to 2018-01-31.
deviation. However, for bonds and cash, the standard deviation is roughly two thirds of the size of the return. This informs us that bonds and cash are fixed income asset classes. Fixed income assets tend to move slower than equities and are also associated with lower risk.

Another way to investigate how the asset classes behave in terms of risk is to look at the worst one-year return of the last 10 years and compare it to the average annualized return. Figure 2.1 shows the average annualized return on the $y$-axis and worst one-year return on the $x$-axis. Each of the dots represents an asset class. We can see that the equity asset class is placed in the top-right of the plot. In the bottom left part of the plot are the risk return characteristics of fixed income asset classes, which is where we find the dots for cash and Bonds. A portfolio typically contains a mixture of asset classes, which is why we call it asset allocation. So, how do we allocate among the asset classes? The blue dot for instance (Six PRX) has 250 securities in it. However, investing only in Six PRX does not ensure diversification. Six PRX is a diversified set within a single asset class. This is known as intra-diversification, or depth.


Figure 2.1. Risk return Analysis showing worst yearly return in \% against Average annualized Return over the time period 2008-2018.

### 2.1.2 What is true Diversification?

The idea with allocating between asset classes is to create a dot into the top left corner of figure 2.1. The objective is to build a portfolio that has relatively high return with as low risk as possible. Craig Israelsen (2010) states that "meaningful portfolio diversification requires both depth and breadth". Depth means diversifying within an asset class, while breadth means investing in a wide variety of funds in different asset classes.

## 3 Modern Portfolio Theory

If we were to break down portfolio management into one key takeaway, it would be that diversification matters. Harry Markowitz (1952),who later won the nobel price, proved this in the journal of finance, where he effectively demonstrated benefits of diversification through the efficient frontier. A few decades later, William Sharpe (1990) made important extensions the model that was presented by Markowitz by introducing the Sharpe ratio, which he won the Nobel price for in 1990.

### 3.1 Modeling requirements in Modern Portfolio Theory

When modeling in the framework of modern portfolio theory, the following inputs must be considered:

1. The expected returns $E\left[R_{i}\right]$.
2. The variance of the returns $\sigma_{i}^{2}$.
3. The covariance between all assets $\rho \sigma_{i} \sigma_{j}$.

Point one will form a vector of expected returns of each asset and points two and three will form a variance-covariance matrix. These inputs need to be estimated for the modeling. There are various methods to calculate the expected returns and the variance covariance matrix. A common procedure for mean-variance analysis is to use, for example, the CAPM to calculate the expected returns and to assume a constant covariance matrix of the asset returns. However, research suggests that the volatility of assets tend to be time dependent and hence not constant through time ${ }^{1}$. This is illustrated in figure B. 1 in appendix B, which shows historical prices and volatility of the stock index SIX PRX. As indicated by the purple line in the graph, the standard deviation seems to be time dependent, and spike both in times of recession and economic growth. This suggests we should extend to a more sophisticated model that is time variant when modeling the covariance matrix. More on this in chapter 4, where we introduce multivariate time series and volatility models. By extending to a time-varying covariance matrix of returns, the mean variance portfolio frontier also becomes conditionally time varying. This will require continuous re-balancing of the portfolio weights $w_{i}$ 's. The weight $w_{i}$ is the fraction of the total value of the portfolio of which is invested in security $i$. For all portfolios, it is true that

$$
\begin{equation*}
\sum_{i=1}^{n} w_{i}=1 \tag{3.1}
\end{equation*}
$$

However, in this thesis we require all weights $w_{i} \geq 0$, i.e we do not allow short sales. We make this distinction because we are allocating among asset classes and for some

[^2]of them it is not possible to short sell. If one were to perform a similar study with for example only stocks as the underlying assets in the portfolio, then short selling would be acceptable.

### 3.2 Why does diversification matter?

In order to prove why diversification matters, we start by looking at the risk of an asset, i.e. its volatility, or standard deviation. The risk of an asset can be decomposed into two components: its systematic risk and its unsystematic risk, where systematic risk is the variability due to co-movements with aggregate markets, and the unsystematic risk is the variability in the asset price due to specific factors of that asset. Through diversification we can reduce the unsystematic risk drastically. The reason behind why we are able to do so is as follows: when we diversify, i.e. take multiple assets and combine them together in a portfolio, the expected return of that portfolio is given by weighted average of the expected returns of its parts:

$$
\begin{equation*}
E\left[\mathbf{R}_{p}\right]=E\left[\sum_{i=1}^{n} w_{i} R_{i}\right]=\sum_{i=1}^{n} w_{i} E\left[R_{i}\right] \tag{3.2}
\end{equation*}
$$

However, when we measure the risk of a portfolio the standard deviation is not the weighted average of its parts, its less. The portfolio variance can is calculated as follows:

$$
\begin{equation*}
\sigma_{p}^{2}=\operatorname{Var}\left(\mathbf{R}_{p}\right)=\sum_{i=1}^{n} \sum_{j=1}^{n} w_{i} w_{j} \operatorname{cov}\left(r_{i}, r_{j}\right)=\sum_{i=1}^{n} \sum_{j=1}^{n} w_{i} \sigma_{i j} w_{j} . \tag{3.3}
\end{equation*}
$$

If we let $\Sigma$ denote the covariance matrix, and $\mathbf{w}$ denotes the vector of weights, then the variance of the portfolio can be written as

$$
\begin{equation*}
\sigma_{p}^{2}=\mathbf{w}^{\mathbf{T}} \Sigma \mathbf{w} \tag{3.4}
\end{equation*}
$$

Consider a portfolio with three assets (such as in this thesis). The variance of that portfolio is comprised by three variance terms times the squared weight of each of the individual assets plus 6 covariance terms, which are $2 w_{i} w_{j} \sigma_{i, j}$. Since the portfolio has 3 variance terms and 6 covariance terms, the covariance becomes the key driver of the portfolio variance. If we would expand to an $n$ asset portfolio, we see that when $n$ is large, the variance of the portfolio is almost only determined by the covariance terms between the assets and the variance terms of the individual stocks is less relevant. This tells us that the key determinant when deciding which asset to add to a portfolio should be to look at the correlation between that asset and the underlying assets in our portfolio. The assets that have the lowest correlation with the assets in the portfolio will provide the greatest diversification benefits when adding them to the portfolio.

So given the expected return is the weighted average of its parts and the standard deviation is less than the weighted average of its parts, we already understand a huge part of why diversification is beneficial.

### 3.2.1 Sharpe's Ratio

William Sharpe (1990) created an expression that quantitatively measures portfolio performance in terms of risk adjusted return. The Sharpe's Ratio $S_{p}$ of the portfolio is given by

$$
\begin{equation*}
S_{p}=\frac{r_{p}-r_{f}}{\sigma_{p}} \tag{3.5}
\end{equation*}
$$

where $r_{p}$ is the portfolio return, $r_{f}$ is the risk free rate of interest, and $\sigma_{p}$ is the volatility of the portfolio. $r_{p}-r_{f}$ is the excess return. The Sharpe ratio measures excess return per unit of risk.

If we think in terms of Sharpe ratio as a measurement of portfolio performance, when we diversify between assets, we keep the numerator constant but reduce the denominator since the risk is being dampened, hence we increase the Sharpe ratio. Now, the benefits that we achieve from diversification is closely related to the correlation between the assets. When the assets in a portfolio have low correlation ${ }^{2}$ the diversification benefits are greater. That is because when assets have low correlation, there is less co-movement between their returns and hence there is less covariance in prices. As we add assets to our portfolio, the reduction in risk increases for each added asset. However, the marginal benefit of diversification does decrease for each added asset.

### 3.2.2 Efficient Frontier

Harry Markovitz illustrated the benefits of diversification by developing a system of portfolio selection where one can identify the efficient set of portfolios that will optimize the utility for investors. If we assume that an investor adopts a mean-variance utility function, i.e. they maximize the utility by maximizing the expected return and minimizing the variance of returns, the investor will achieve the highest utility by maximizing the Sharpe ratio. When looking at mean-variance plane we can identify that there will be an efficient frontier, which is a set of optimal risky portfolios that comprises all those portfolios that have the highest expected return for each unit of risk. The efficient frontier will start in the minimum variance portfolio, which is the portfolio that has the lowest level of risk. By definition, that portfolio has the minimum variance, i.e. there is no other portfolio with that low level of risk and hence no portfolio has the same level of risk but a higher expected return. Therefore the minimum variance portfolio will always be the start of the the efficient frontier. The frontier will also pass through a very key portfolio called the optimal portfolio, which is the combination of assets, within our feasible set, such that the Sharpe ratio is maximized. When an investor adopts mean-variance utility preferences we know that maximizing the Sharpe ratio is key ${ }^{3}$.

In order to derive the efficient frontier we need estimate all the necessary inputs in the model, that is, the vector of expected returns and the variance covariance matrix of returns. The model assumes an investor forms the portfolio for one time period only, using the information available at the beginning of that time period. Let

$$
\mathbf{R}_{t+1}=\left(R_{1, t+1}, R_{2, t+1}, \ldots, R_{n, t+1}\right)
$$

denote an $n x 1$ vector of asset returns that were realized during time period $t$ and paid out in the beginnning of $t+1$. The model assumes that all funds are invested, thus

$$
\mathbf{w}^{\mathbf{T}}{ }_{t} \mathbf{1}=1
$$

where $\mathbf{1}$ is a vector of ones and $\mathbf{w}_{t}^{t}$ is the vector of weights at time $t$. Now, let us adopt a mean-variance utility function. As presented by Lee (2000), we assume it is sufficient to express the utility function in terms of expected return and volatility. Under some simplified assumptions, such as returns follow a multivariate normal distribution, short selling is accepted, and that investors have a constant relative risk aversion, we can write

[^3]the expected utility of wealth as
\[

$$
\begin{equation*}
E[U(W)]=-\exp \left\{-\gamma\left(E\left[\mathbf{R}_{p, t+1}\right]-\frac{\gamma}{2} \sigma_{p, t+1}^{2}\right)\right\} \tag{3.6}
\end{equation*}
$$

\]

where $\gamma$ is the CRRA, or constant relative risk aversion coefficient. The portfolio has expected return

$$
\begin{equation*}
E\left[\mathbf{R}_{p, t+1}\right]=\mathbf{w}^{\mathbf{T}}{ }_{t} E\left[\mathbf{R}_{t+1}\right] \tag{3.7}
\end{equation*}
$$

and variance

$$
\begin{equation*}
\sigma_{p, t+1}^{2}=\mathbf{w}^{\mathbf{T}}{ }_{t} \Sigma_{t+1} \mathbf{w}_{t} \tag{3.8}
\end{equation*}
$$

where $\Sigma_{t+1}$ is the variance-covariance matrix. Now we want to maximize the utility in equation 3.6. This is equivalent to solving:

$$
\begin{array}{ll}
\underset{\mathbf{w}}{\operatorname{maximize}} & \mathbf{w}_{t}^{T} E\left[\mathbf{R}_{t+1}\right]-\frac{\gamma}{2} \mathbf{w}^{\mathbf{T}}{ }_{t} \Sigma_{t+1} \mathbf{w}_{t}  \tag{3.9}\\
\text { subject to } & \mathbf{w}_{t}^{T} \mathbf{1}=1, \quad i=1, \ldots, m .
\end{array}
$$

The Lagrangian becomes

$$
\begin{equation*}
L=\mathbf{w}^{\mathbf{T}}{ }_{t} E\left[\mathbf{R}_{t+1}\right]-\frac{\gamma}{2} \mathbf{w}^{\mathbf{T}}{ }_{t} \Sigma_{t+1} \mathbf{w}_{t}-\lambda\left(\mathbf{w}^{\mathbf{T}}{ }_{t} \mathbf{1}-1\right) \tag{3.10}
\end{equation*}
$$

We now apply first order conditions and solve, for $\mathbf{w}_{t}$ and $\lambda$ respectively:

$$
\begin{gather*}
\frac{\partial L}{\partial \mathbf{w}_{t}}=E\left[\mathbf{R}_{t+1}\right]-\gamma \Sigma_{t+1} \mathbf{w}_{t}-\lambda \mathbf{1}=0  \tag{3.11}\\
\Rightarrow \quad \mathbf{w}^{*}{ }_{t}=\frac{\Sigma_{t+1}^{-1}}{\gamma}\left(E\left[\mathbf{R}_{t+1}\right]-\lambda \mathbf{1}\right) \tag{3.12}
\end{gather*}
$$

For $\lambda$ we get:

$$
\begin{align*}
\frac{\partial L}{\partial \lambda} & =-\left(\mathbf{w}^{\mathbf{T}}{ }_{t} \mathbf{1}-1\right)=0  \tag{3.13}\\
& \Rightarrow \quad \mathbf{w}^{\mathbf{T}}{ }_{t} \mathbf{1}=1 \tag{3.14}
\end{align*}
$$

Now we substitute equation 3.12 into 3.14 and rearrange in order to obtain

$$
\begin{equation*}
\lambda=\frac{\mathbf{1}^{\mathbf{T}} \Sigma_{t+1}^{-1} E\left[\mathbf{R}_{t+1}\right]}{\mathbf{1}^{\mathbf{T}} \Sigma_{t+1} \mathbf{1}}-\frac{\gamma}{\mathbf{1}^{\mathbf{T}} \Sigma_{t+1} \mathbf{1}} . \tag{3.15}
\end{equation*}
$$

Now we can solve the vector of optimal weights by substituting equation 3.15 into equation 3.12. This gives

$$
\begin{equation*}
\mathbf{w}^{*}{ }_{t}=\left(1-\frac{\mathbf{1}^{\mathbf{T}} \Sigma_{t+1}^{-1} E\left[\mathbf{R}_{t+1}\right]}{\gamma}\right) \frac{\Sigma_{t+1}^{-1} \mathbf{1}}{\mathbf{1}^{\mathbf{T}} \Sigma_{t+1} \mathbf{1}}+\left(\frac{\mathbf{1}^{\mathbf{T}} \Sigma_{t+1}^{-1} E\left[\mathbf{R}_{t+1}\right]}{\gamma}\right) \frac{\Sigma_{t+1}^{-1} E\left[\mathbf{R}_{t+1}\right]}{\mathbf{1}^{\mathbf{T}} \Sigma_{t+1}^{-1} E\left[\mathbf{R}_{t+1}\right]} \tag{3.16}
\end{equation*}
$$

As stated by Lee (2000) ${ }^{4}$, equation 3.16 is the the well known-known Mutual Fund Separation Theorem. This theorem gives an expression for the optimal portfolio in the mean variance framework.

[^4]The classical standard Markowitz (1952) setup looks rather similar ${ }^{5}$ :

$$
\begin{array}{ll}
\underset{\mathbf{w}}{\operatorname{minimize}} & \mathbf{w}^{\mathbf{T}}{ }_{t} \Sigma_{t+1} \mathbf{w}_{t} \\
\text { subject to } & \mathbf{w}^{\mathbf{T}}{ }_{t} E\left[\mathbf{R}_{t+1}\right]=\mu_{t+1}  \tag{3.17}\\
& \mathbf{w}^{\mathbf{T}}{ }_{t} \mathbf{1}=1,
\end{array}
$$

where $\mu_{t+1}$ is the target return of the portfolio. By solving this for our feasible set of expected returns, the optimization will result in the efficient frontier. Every dot on the frontier represents a portfolio that has the highest expected return of all the possible portfolios with that same level of risk.

### 3.2.3 A method for every investor?

Markowitz portfolio selection is a mathematically tractable and a very intuitively appealing method of investing. So why doesn't every investor adopt this approach? Even though it is the most theoretically appealing approach to asset allocation that an investor can take, there are some key issues with the assumptions of the model. The results rely on the accuracy of the inputs that are used in the model, which aren't necessarily perfect measurements. If we use an asset pricing model such as CAPM to estimate the expected returns, the results rely on prediction power of the CAPM. It may turn out that CAPM is an imperfect model for calculating expected returns and that there are factors other than beta that explain returns. In that case, if we can estimate returns better than if we would use the CAPM, we can come up with a better combination of assets within the Markovitz portfolio selection model. There is a lot of empirical evidence that suggests CAPM is certainly not a perfect model in predicting future returns. Fama \& French (2004) states that "the empirical record of the CAPM is poor, poor enough to invalidate the way it is used in applications". So where does this leave us? Markowitz portfolio theory will form the basis of the investment strategy that is used in this thesis. Instead of using the standard inputs of the model, we want to further improve the portfolio selection model by identifying better models of expected returns and volatility, one of which will contain macroeconomic indicators.

### 3.3 Tactical Asset Allocation

Tactical Asset allocation ${ }^{6}$, as opposed to strategic asset allocation, is a dynamic modeling approach that involves market timing. Its goal is to predict future performances across asset classes and dynamically allocating across classes based on those predictions. So if one predicts that certain markets in the economy is going to perform well in the future then one should allocate more of their assets towards those high-growth asses classes, while if one predicts markets are going to perform poorly then one should allocate towards more defensive asset class. This allocation shifts with your prediction. One of the main benefits of this strategy relates to the fact that the vast majority of fund manager performance can be attributed to the asset allocation decision. This was asserted by Gary P. Brinson, CFA, Randolph Hood, and Gilbert L. Brinson et al. (1995). They found that asset allocation is the primary determinant of a portfolio's returns, while security selection within asset classes (active management) plays a minor roll. In their study, they conclude that asset allocation explained $93.6 \%$ of the variation in the quarterly returns of the portfolio. Given this decision is a key component in generating abnormal returns, if we are actually able

[^5]to predict future movements of asset classes and invest accordingly, we should be able to generate significantly positive and abnormal returns.

However, there are some negatives with this investment approach. The first issue lies in fact that predicting future performance of a market is hard. Evidence suggest that professional fund managers are not able to continually predict future movements of asset classes ${ }^{7}$. Another issue is the incurring transaction costs that comes from buying and selling across asset classes. Also, when applying tactical asset allocation, one can get in and out of the market at the wrong times. For example, during the end of the financial crisis of 2008 you might have moved a lot of your assets away from equities. In October that year we saw some huge downward movement in the equities asset class ${ }^{8}$. The asset allocation strategy then suggest you move away from the equities asset class because of the recently high volatility. If you were to stay out of that asset class for too long you would have missed a huge bounce back that occurred in early 2009. The net result may have been that even though you might have missed some of the downturn in 2008, you may also have missed the upward bounce back and that you probably performed as poorly as someone who followed a long term buy and hold strategic asset allocation. This shows that while tactical asset allocation might be beneficial it also has some significant drawbacks.

### 3.3.1 Signaling values

There are three main signals that investors use in order to predict the future movement of asset classes, they are

- Sentiment indicators,
- Economical indicators,
- Technical indicators.

Nowadays, most market participants will agree that sentiment can have some effect on price levels. The issue with sentiment is that it is very hard to measure quantitatively. A study by Baker \& Wurgler (2006) found that "when beginning-of-period proxies for sentiment are low, subsequent returns are relatively high for small stocks, young stocks, high volatility stocks, unprofitable stocks, non-dividend-paying stocks, extreme growth stocks, and distressed stocks. When sentiment is high, on the other hand, these categories of stock earn relatively low subsequent returns." More recently, people have tried measuring sentiment of for example social media. This may include aggregating the views of people's twitter or Facebook accounts.

The second type of signal are economic indicators. The idea behind using economic indicators to predict market prices is based on the fact that any financial asset should be the present value of future cash flows. For example, if we look at the equity asset class and look at an aggregated broad equity index for that asset class, the present value of future cash flows is the present value of future corporate profits. This is where economic indicators come into play. Economic indicators suggest that we can measure where in the business cycle we are and use that information for tactical asset allocation. The idea is to model the economic indicators combined with the asset class indices in order to predict which way the asset class as a whole is moving.

The last signal is a technical indicator, which is basically the idea that you can use historical data in order to predict future prices. This is where we apply time series modeling, more on this in chapter 4.

[^6]
### 3.4 Asset Returns and Macroeconomics

There are quite a few intuitive reasons for adding macroeconomic variables to an investment strategy model. We know that asset prices tend to move in the same direction as aggregate markets, which in turn are largely affected by the macroeconomic environment. If we define asset returns as nominal returns, and since investors usually are concerned with real returns, we can assume that nominal returns in the long run is a function of real returns and inflation. In the short term however, we cannot make any assumptions as to how large this effect is. During the past 30 years there has been growing literature in this field. The research in most of the performed studies suggest that the information obtained from macroeconomic variables can improve asset allocation through better prediction power of expected returns and asset price volatility. Flavin \& Wickens (2003) provide an entire list of previous work done in this field. They also perform a study of their own, where they illustrate how to model a tactical asset allocation strategy that incorporates the effects of macroeconomic variables. They use three risky UK assets and inflation as their macroeconomic factor and model the asset returns by using a VAR with an M-GARCH error structure. They find that "taking account of inflation generates portfolio frontiers that lie closer to the origin, and offers investors superior risk-return combinations." 9

### 3.5 Re-balancing

When building a portfolio of different asset classes and adopting an investment strategy such as tactical asset allocation, an investor has to continually re-balance the portfolio to match the investment strategy. So how often should an investor re-balance the portfolio? The disadvantages of re-balancing often is the incurring transaction costs from buying and selling assets, and it tends to be a bigger workload for the investor. In his presentation on asset allocation, Craig Israelsen (2010) suggests that balancing monthly tends to have the worst performance. He means that re-balancing quarterly or annually are better alternatives, and that an investor could gain margins of 30 to 50 basis points adopting this approach rather than the monthly approach. He suggests annual re-balancing is the best protocol for an investor. Other people argue that there is no optimal time window for re-balancing a portfolio. According to Ping (2015) no re-balancing approach produces significantly superior returns. In this thesis we will use monthly re-balancing when investigating portfolio strategies. This is due to the fact that we only have 10 years of historical data and would require a longer data series in order to perform out modeling procedure on a yearly basis.

[^7]
## 4 Econometric Modeling

This chapter will go through the necessary theory regarding multivariate time series and volatility models. We begin by introducing some basic concepts of time series analysis.

### 4.1 Time series Notation

The theory and notation in the following subsections are taken from Tsay (2005) ${ }^{1}$. Let $\mathbf{z}_{\mathbf{t}}$ be a time series matrix of size $k \times l$, where $k$ is the number of inputs and $l$ is the length of the time series. In the case of asset returns, $k$ is the number of assets and $l$ is the number of days/moths/years in the series (depending on the time frame).

### 4.1.1 Stationarity

$\mathbf{z}_{\mathrm{t}}$ is weakly stationary if

$$
\begin{align*}
& E\left[\mathbf{z}_{\mathbf{t}}\right]=\boldsymbol{\mu} \\
& \operatorname{Cov}\left(\mathbf{z}_{\mathbf{t}}\right)=E\left[\left(\mathbf{z}_{\mathbf{t}}-\boldsymbol{\mu}\right)\left(\mathbf{z}_{\mathbf{t}}-\boldsymbol{\mu}\right)^{T}\right]=\mathbf{\Sigma}_{\boldsymbol{z}} \tag{4.1}
\end{align*}
$$

and for any t and h, $\operatorname{Cov}\left(z_{t+h}, z_{t}^{\top}\right)=\Gamma(h)$,
where $\boldsymbol{\mu}$ is a constant vector of length $k$ and $\boldsymbol{\Sigma}_{\boldsymbol{z}}$ is a constant positive definite $k \times k$ matrix. The last part means that the covariances between two observation vectors depend only on $h$, but not on $t$.

### 4.1.2 Lag-/ Matrices

The lag- $l$ covariance matrix for $\mathbf{z}_{\mathbf{t}}$ is defined as

$$
\begin{equation*}
\boldsymbol{\Gamma}_{l}=\operatorname{Cov}\left(\mathbf{z}_{\mathbf{t}}, \mathbf{z}_{\mathbf{t}-\mathbf{l}}\right)=E\left[\left(\mathbf{z}_{\mathbf{t}}-\boldsymbol{\mu}\right)\left(\mathbf{z}_{\mathbf{t}-\mathbf{l}}-\boldsymbol{\mu}\right)^{T}\right] \tag{4.2}
\end{equation*}
$$

and the lag- $l$ cross-correlation matrix for $\mathbf{z}_{\mathbf{t}}$ is defined as

$$
\begin{equation*}
\rho_{l}=\mathbf{D}^{-1} \boldsymbol{\Gamma}_{l} \mathbf{D}^{-1} \tag{4.3}
\end{equation*}
$$

where $\mathbf{D}$ is a diagonal matrix containing the standard deviations of the components of $\mathbf{z}_{\mathbf{t}}$, the volatility of returns.

### 4.1.3 Vectorization of a matrix

If

$$
\mathbf{A}=\left[\begin{array}{ll}
a & b  \tag{4.4}\\
c & d
\end{array}\right]
$$

[^8]then the vector operator $v e c$ stacks the elements of $\boldsymbol{A}$, such as
\[

\operatorname{vec}(\mathbf{A})=\left[$$
\begin{array}{c}
a  \tag{4.5}\\
b \\
c \\
d
\end{array}
$$\right]
\]

Also, denote vech as the vector-half operator that stacks the lower triangular portion of a symmetric matrix in a vector. vech is similar to vec but would exclude the letter b in the example above.

### 4.2 Modeling Asset Returns

As we discussed in chapter 3, modern portfolio theory requires two key components, that is, the expected returns and the variance-covariance matrix. We are not interested in estimating expected returns, since evidence suggest there is more accuracy in predicting the volatility of an asset rather than the asset return. Therefore we will not use asset price prediction model in this thesis. We will instead adopt a similar approach as in the study by Flavin \& Wickens (2003), namely to use historical means as expected returns. More on this in chapter 6.2.

### 4.3 Multivariate Volatility Models

There are several different approaches to estimating volatility in finance. As a reminder, the definition of volatility is a periodic standard deviation, i.e. it's the standard deviation over some time period of interest. Commonly, investors tend to look at annualized standard deviation, which is the standard deviation for a one-year period. Different time periods may also be of interest, such as monthly volatility, daily volatility, or even something more frequent such as intra-day volatility.

The simplest approach of estimating the volatility is by using the historical unweighted volatility:

$$
\begin{equation*}
\operatorname{vol}_{h i s t}=\sqrt{\frac{1}{m-1} \sum_{t=1}^{m}\left(R_{i, t}-\bar{R}_{i}\right)^{2}} \tag{4.6}
\end{equation*}
$$

where $R_{i, t}$ is the return of asset $i$ in time $t$ and $\bar{R}_{i}$ is the average return of asset $i$. Notice if we are dealing with daily returns, one can simply create for example the annualized volatility by multiplying by $\sqrt{250}$ ( 250 trading days in a year). We look at the historical volatility as a realized moving average, with a time window that is chosen depending on the scenario. This approach is considered to be a simple, but perhaps not the most useful way to measure volatility. There are however more practical approaches to estimating volatility. The methods we are about to introduce are conditionally weighted volatility models, meaning today's estimated volatility is conditional on yesterday's volatility.

### 4.3.1 Multivariate GARCH Models

The first selection of multivariate volatility models we introduce are Multivariate generalized autoregressive conditional heteroskedasticity (MGARCH) -models ${ }^{2}$. This set of parametric models are typically used for modeling time-varying dynamic covariances and dynamic correlations. We know that financial variables usually have time-dependent

[^9]conditional moments and therefore an MGARCH model is a natural choice when modeling. We begin by introducing the univariate model and thereafter we extend to the multivariate model. Let
\[

$$
\begin{equation*}
\mathbf{R}_{\mathbf{t}}=\left(R_{1, t}, \ldots, R_{n, t}\right) \tag{4.7}
\end{equation*}
$$

\]

be an $n x 1$ vector of asset returns and let

$$
\begin{equation*}
\mathbf{R}_{\mathbf{t}}-\mu_{t}=\epsilon_{t}=\Sigma_{t}^{-1 / 2} \mathbf{z}_{t} \tag{4.8}
\end{equation*}
$$

where the $z_{t}$ 's have the following characteristics:

$$
\begin{array}{ll} 
& E\left[\mathbf{z}_{\mathbf{t}}\right]=\mathbf{0}, \\
\text { and } & E\left[\mathbf{z}_{\mathbf{t}} \mathbf{z}_{\mathbf{t}}^{T}\right]=\boldsymbol{I}_{d} . \tag{4.9}
\end{array}
$$

$\boldsymbol{I}_{d}$ is the identity matrix of size $d$. We also know that

$$
\begin{equation*}
E\left[\boldsymbol{\epsilon}_{\boldsymbol{t}} \boldsymbol{\epsilon}_{\boldsymbol{t}}^{T} \mid \mathbf{\Phi}_{t-\mathbf{1}}\right]=\boldsymbol{\Sigma}_{t} \tag{4.10}
\end{equation*}
$$

where $\Phi_{t-1}$ is the $\sigma$-field that is generated from the past observations up to time $t-1$. Multivariate volatility models provide a parametric structure for $\boldsymbol{\Sigma}_{\boldsymbol{t}}$. The models must satisfy the following constraints:

1. The diagonal elements of $\boldsymbol{\Sigma}_{\boldsymbol{t}}$ must be greater than zero, i.e. $\operatorname{diag}\left(\boldsymbol{\Sigma}_{\boldsymbol{t}}\right)>0$. The elements cannot have negative or zero variance.
2. $\boldsymbol{\Sigma}_{\boldsymbol{t}}$ is positive definite.
3. Stationarity: meaning there exist a long-run average of $\boldsymbol{\Sigma}_{t}$ that is finite and constant with respect to $t$.

For a univariate $\operatorname{GARCH}(1,1)$ model, the variance can be described as:

$$
\begin{equation*}
\sigma_{t}^{2}=\omega+\alpha \epsilon_{t-1}^{2}+\beta \sigma_{t-1}^{2} \tag{4.11}
\end{equation*}
$$

In this case the long-run average variance is

$$
\begin{equation*}
\sigma^{2}=E\left[\sigma_{t}^{2}\right]=\omega(1-\alpha-\beta)^{-1} \tag{4.12}
\end{equation*}
$$

under the assumption that $(\alpha+\beta)<1$. Engle \& Kroner (1995) extended the univariate $\operatorname{GARCH}(1,1)$ to a $\operatorname{MGARCH}(1,1)$ model by a general formulation termed $\operatorname{BEKK}(\mathrm{p}, \mathrm{q})$ :

$$
\begin{equation*}
\boldsymbol{\Sigma}_{\boldsymbol{t}}=\boldsymbol{C} \boldsymbol{C}^{\boldsymbol{T}}+\sum_{i=1}^{q} \boldsymbol{A}_{\boldsymbol{i}} \boldsymbol{\epsilon}_{\boldsymbol{t}-\mathbf{1}} \boldsymbol{\epsilon}_{\boldsymbol{t}-\mathbf{1}}^{\boldsymbol{T}} \boldsymbol{A}_{\boldsymbol{i}}^{T}+\sum_{i=1}^{p} \boldsymbol{B}_{\boldsymbol{i}} \boldsymbol{\Sigma}_{\boldsymbol{t}-\mathbf{1}} \boldsymbol{B}_{\boldsymbol{i}}^{\boldsymbol{T}} \tag{4.13}
\end{equation*}
$$

For $n=2$ assets, and $p=q=1$, we get
$\Sigma_{t}=$

$$
\begin{align*}
& {\left[\begin{array}{ll}
\sigma_{11, t} & \sigma_{12, t} \\
\sigma_{21, t} & \sigma_{22, t}
\end{array}\right]=\boldsymbol{C} \boldsymbol{C}^{\boldsymbol{T}}+\left[\begin{array}{ll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right]\left[\begin{array}{cc}
\epsilon_{1, t-1}^{2} & \epsilon_{1, t-1} \epsilon_{2, t-1} \\
\epsilon_{2, t-1} \epsilon_{1, t-1} & \epsilon_{2, t-1}^{2}
\end{array}\right]\left[\begin{array}{ll}
a_{11} & a_{21} \\
a_{12} & a_{22}
\end{array}\right]+} \\
& {\left[\begin{array}{ll}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{array}\right]\left[\begin{array}{ll}
\sigma_{11, t-1} & \sigma_{12, t-1} \\
\sigma_{21, t-1} & \sigma_{22, t-1}
\end{array}\right]\left[\begin{array}{ll}
b_{11} & b_{21} \\
b_{12} & b_{22}
\end{array}\right] } \tag{4.14}
\end{align*}
$$

with number of parameters equal to 11 for $n=2$ assets. The number of parameters to estimate drastically increases as $n$ increases. For $n=3$ we have 24 parameters to estimate. The large number of parameters to estimate is a disadvantage of using the BEKK (1,1)-model. Since we are working with $n=3$ assets classes and a number of macroeconomic variables, the $\operatorname{BEKK}(1,1)$ may be hard to estimate.

### 4.3.2 How does the inclusion of Macroeconomic variables affect the volatility of asset returns?

By adding the return series of a macroeconomic variable to our multivariate GARCH model, we effectively increase the dimension of the variance covariance matrix. By doing so, the volatility of the asset returns will be affected by the conditional volatility of the macroeconomic variable. This is governed by the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$. If we define $\boldsymbol{A}$ and $\boldsymbol{B}$ as lower triangular matrices then

$$
\begin{equation*}
\boldsymbol{\sigma}_{11, t}=c_{11}^{2}+a_{11}^{2} \epsilon_{1, t-1}^{2}+b_{11}^{2} \tag{4.15}
\end{equation*}
$$

This implies that the volatility of asset returns is unaffected by the volatility of the added macroeconomic variable. However, if we instead define $\boldsymbol{A}$ and $\boldsymbol{B}$ as full symmetric matrices, such as in the $\operatorname{BEKK}(1,1)$ model, then

$$
\begin{align*}
\boldsymbol{\sigma}_{\mathbf{1 1}, \boldsymbol{t}}= & c_{11}^{2}+ \\
& \left(a_{11}^{2} \epsilon_{1, t-1}^{2}+2 a_{11} a_{12} \epsilon_{1, t-1} \epsilon_{2, t-1}+a_{12}^{2} \epsilon_{2, t-1}^{2}\right)+  \tag{4.16}\\
& \left(b_{11}^{2} \boldsymbol{\sigma}_{\mathbf{1 1}, \mathbf{t - 1}}+2 b_{11} b_{12} \boldsymbol{\sigma}_{\mathbf{1 2}, \boldsymbol{t} \mathbf{1}}+b_{12}^{2} \boldsymbol{\sigma}_{\mathbf{2 2}, \boldsymbol{t} \mathbf{1}}\right) .
\end{align*}
$$

In this scenario, the volatility of the macroeconomic variable does affect the volatility of the asset returns through a one period lag. The formulation in equation 4.16 is the one we use in the modeling procedure. The matrix $C$ is defined as a lower triangular matrix, and by multiplying $\boldsymbol{C} \boldsymbol{C}^{\boldsymbol{T}}$ we obtain the long-run variance covariance matrix $\boldsymbol{H}$.

### 4.3.3 Alternative models

Volatility is a measure of risk and can be estimated using various methods. In appendix D we introduce another improvement on the simple volatility calculation, namely the Exponentially weighted Moving Average (EWMA).

The EMWA and the $\operatorname{BEKK}(1,1)$ are quite similar models in estimating volatility. The EMWA essentially lags two variables, variance and squared return, while the MGARCH adds one more term to the model. The essential difference between the MGARCH and the EMWA is that the MGRACH is going to incorporate mean reversion by adding the third term, the long rung average. This comes from the idea that if the variance strays from a long run average it will be somewhat persistent to that long run average. In this thesis we will use the $\operatorname{BEKK}(1,1)$ model for the modeling procedure, however, one could have performed the study with a EMWA model ${ }^{3}$.

### 4.4 Testing Conditional Heteroskedasticity

Before using any of the volatility models described above, we need to perform some test on the data to see if the errors are in fact conditionally heteroskedastic. If no heteroskedastic effects are present in the data, then $\boldsymbol{\Sigma}_{\boldsymbol{t}}$ is time invariant. There are various statistical tests for testing conditional heteroskedasticity. In the following tests $\boldsymbol{\epsilon}_{\boldsymbol{t}}$ is a noise process and we employ the second moment of $\epsilon_{t}$ (volatility) when testing for heteroskedasticity. We want to test the null hypothesis of an uncorrelated noise process against the alternative hypothesis of heteroskedastic effects being present.

[^10]
### 4.4.1 The Portmanteau Test

Here we want to test if the first $m$ values in the autororrelation function is 0 . The classical Portmanteau statistic was proposed by Box \& Pierce (1970) and will be used in our time series framework to test

$$
\begin{array}{ll} 
& H_{0}: \rho_{1}^{(a)}=\rho_{2}^{(a)}=\ldots=\rho_{m}^{(a)}=0 \\
\text { against } & H_{0}: \rho_{i}^{(a)} \neq 0 \text { for some } i \in\{1, \ldots, m\} \tag{4.17}
\end{array}
$$

Here $\rho_{i}^{(a)}$ is the $i^{\prime}$ th lagged correlation matrix of $\boldsymbol{\epsilon}_{t}^{2}$. We use the Ljung-Box test statistic:

$$
\begin{equation*}
Q_{k}^{*}(m)=T^{2} \sum_{k=1}^{m} \frac{1}{T-i} \boldsymbol{b}_{\boldsymbol{i}}^{\boldsymbol{T}}\left(\hat{\boldsymbol{\rho}}_{\mathbf{0}}^{(\boldsymbol{a})-\mathbf{1}} \otimes \hat{\boldsymbol{\rho}}_{\mathbf{0}}^{(\boldsymbol{a})-\mathbf{1}}\right) \boldsymbol{b}_{\boldsymbol{i}} . \tag{4.18}
\end{equation*}
$$

Where $k=\operatorname{dim}\left(\boldsymbol{\epsilon}_{\boldsymbol{t}}\right), \mathrm{T}$ is the length of the time series, $\boldsymbol{b}_{\boldsymbol{i}}=\operatorname{vec}\left(\hat{\rho}_{i}\right)$ and $\otimes$ is the Kronecker product.

### 4.4.2 The Rank-Based Test

According to Bradley \& Taqqu (2003), there is a lot of empirical evidence to suggest that the distribution of financial returns is in fact not normal. Asset returns tend to have slightly heavier tails than the normal distribution. Extreme versions of this can have effects on the Portmanteau Test, and it may result in the test presenting a false result. Therefore we consider another test, the Rank-Based test, in combination with the Portmanteau Test. The two tests will both be considered and the results will be compared in order to get a better result. We will see that when taking into consideration the heavy tails of asset returns, the likelihood of rejecting $H_{0}$ becomes slightly larger.

The test statistic for this model is presented by Šiman (2006) and takes the following form:

$$
\begin{equation*}
Q_{R B}=\sum_{i=1}^{m} \frac{\left(\tilde{\rho}_{i}-E\left[\tilde{\rho}_{i}\right]\right)^{2}}{\operatorname{Var}\left(\tilde{\rho}_{i}\right)} \tag{4.19}
\end{equation*}
$$

where $\tilde{\rho}_{l}$ is the lag-l rank autocorrelation of the series. For a complete derivation of the test, see Šiman (2006), pages $5-9$.

### 4.5 Method of Estimation

We estimate the parameters of the models by maximizing the log likelihood function. Let $f$ denote the multivariate normal density function, then the log likelihood is given by

$$
\begin{align*}
l & =\sum_{t=1}^{T} l_{t}  \tag{4.20}\\
\text { where } \quad l_{t} & =-\frac{N}{2} \ln (2 \pi)-\frac{1}{2} \ln \left(\left|\boldsymbol{\Sigma}_{\boldsymbol{t}}\right|\right)-\frac{1}{2} \boldsymbol{\epsilon}_{\boldsymbol{t}}^{\boldsymbol{T}} \boldsymbol{\Sigma}_{\boldsymbol{t}}^{-\mathbf{1}} \boldsymbol{\epsilon}_{\boldsymbol{t}}
\end{align*}
$$

In order to maximize $l$ we use the recursive algorithm introduced by Berndt et al. (1974). This algorithm is implemented when maximizing the log likelihood, and is very useful for estimating $\operatorname{BEKK}(1,1)$ processes.

### 4.6 Asymptotic Theory for the MGARCH process

Under the correct model specification, and a sample size large enough, the univariate GARCH model enables statistical inference with a good amount of confidence. For the multivariate GARCH, however, asymptotic theory is rare. Users of the MGARCH typically assume asymptotic normality as a rule of thumb. The theory of estimation for MGARCH processes is not as comprehensible as that of the univariate case. Some have proposed different methods and theory on the asymptotic properties of the MGARCH process. Tuncer (1994) for example, established weak convergence of the maximum likelihood estimator of the $\operatorname{BEKK}(1,1)$ model. In this section we establish the asymptotic theory of the $\operatorname{BEKK}(1,1)$ process.

### 4.6.1 Notation

Recall the BEKK representation presented in chapter 4.3.1:

$$
\begin{equation*}
\boldsymbol{\Sigma}_{\boldsymbol{t}}=\boldsymbol{C} \boldsymbol{C}^{\boldsymbol{T}}+\sum_{i=1}^{q} \boldsymbol{A}_{\boldsymbol{i}} \boldsymbol{\epsilon}_{\boldsymbol{t}-\mathbf{1}} \boldsymbol{\epsilon}_{\boldsymbol{t}-\mathbf{1}}^{\boldsymbol{T}} \boldsymbol{A}_{\boldsymbol{i}}^{T}+\sum_{i=1}^{p} \boldsymbol{B}_{\boldsymbol{i}} \boldsymbol{\Sigma}_{\boldsymbol{t}-\mathbf{1}} \boldsymbol{B}_{\boldsymbol{i}}^{\boldsymbol{T}} \tag{4.21}
\end{equation*}
$$

where $\boldsymbol{C} \boldsymbol{C}^{\boldsymbol{T}}$ is positive definite, and $\boldsymbol{A}_{\boldsymbol{i}}$ and $\boldsymbol{B}_{\boldsymbol{i}}$ are real $d \times d$ matrices. The main superiority of this model comes from the fact that it ensures positive definiteness of $\boldsymbol{\Sigma}_{\boldsymbol{t}}$. The model does not necessarily need to be Gaussian, but we choose to work with the Gaussian likelihood function. So the quasi maximum likelihood estimator $\hat{\theta}$ is given by minimizing equation 4.20 (here we denote the parameter vector as $\theta$ ). We also use the following notation; $\| .| |$ represents the Euclidean norm, applying to both matrices and vectors. $\|A\|^{2}=\operatorname{Tr}\left(A^{T} A\right)=\sum_{i} A_{i}^{2}$. The spectral radius of $\boldsymbol{A}$ is $\rho(\boldsymbol{A}) . \boldsymbol{N}(A)$ is the square root of $\rho\left(\boldsymbol{A}^{\boldsymbol{T}} \boldsymbol{A}\right)$.

### 4.6.2 Strong consistency

Jeantheau (1998) sets up 6 conditions under which the quasi maximum likelihood estimator has strong consistency. They are:

1. $\Theta$ is compact ( $\Theta$ is the parameter space).
2. $\forall \theta_{0} \in \Theta$ has a unique ergodic, strictly stationary solution.
3. There exist a constant $c$ such that $\forall t, \forall \theta \in \Theta, \operatorname{det}\left(\boldsymbol{\Sigma}_{\boldsymbol{t}, \boldsymbol{\theta}}\right) \geq c$.
4. $\forall \theta_{0} \in \Theta, E_{\theta_{0}}\left[\mid \log \left(\operatorname{det}\left(\boldsymbol{\Sigma}_{\boldsymbol{t}, \boldsymbol{\theta}_{0}}\right) \mid\right]<\infty\right.$.
5. The model is identifiable ${ }^{4}$.
6. $\boldsymbol{\Sigma}_{t, \theta_{0}}$ is a continuous function of $\theta$.

We must now make sure that the conditions hold for the $\operatorname{BEKK}(1,1)$ model that we use in this thesis. To begin with, item 1 is always assumed. Item 2 is shown in the below theorem by Boussama (1998). Note that we may represent equation 4.21 as

$$
\begin{equation*}
\operatorname{vech}\left(\boldsymbol{\Sigma}_{t}\right)=\operatorname{vech}\left(\boldsymbol{C} \boldsymbol{C}^{\boldsymbol{T}}\right)+\tilde{A} \operatorname{vech}\left(\boldsymbol{\epsilon}_{\boldsymbol{t}-\mathbf{1}} \boldsymbol{\epsilon}_{\boldsymbol{t}-\mathbf{1}}^{\boldsymbol{T}}\right)+\tilde{B} \operatorname{vech}\left(\boldsymbol{\Sigma}_{t-1}\right), \tag{4.22}
\end{equation*}
$$

where $\tilde{A}$ and $\tilde{B}$ are functions of $\theta$.

[^11]Theorem 1 (Boussama, 1998) In the model given by equation 4.21, assume that the $\epsilon_{t}$ 's admit a density absolutely continuous with respect to the Lebesgue measure, positive in a neighborhood of the origin. Also, assume that

$$
\rho(\tilde{A}+\tilde{B})<1
$$

and let $\boldsymbol{Y}$ be defined by

$$
\boldsymbol{Y}_{t}=\left(\operatorname{vech}\left(\Sigma_{t+1}\right)^{T}, \operatorname{vech}\left(\Sigma_{t}\right)^{T}, \ldots, \operatorname{vech}\left(\Sigma_{t-1}\right)^{T}, \boldsymbol{\epsilon}_{t}^{T}, \boldsymbol{\epsilon}_{t-1}^{T}\right.
$$

Then the recurrence relations between model 4.21 and 4.9 for $Y$ have an almost surely unique strictly stationary causal solution which constitutes a positive Harris recurrent Markov chain which is geometrically ergodic and $\beta$-mixing ${ }^{5}$.

For full proof of theorem 1, see Comte \& Lieberman (2003) and Boussama (1998). Comte \& Lieberman (2003) also prove that the remaining conditions are fulfilled for the BEKK model. This is summarized in the theorem below.

Theorem 2 (Comte and Lieberman, 2003) For the $\operatorname{BEKK}(1,1)$ process defined in equation 4.21, and for $\hat{\theta}$ defined above, assume that

1. $\Theta$ is compact, $(C), \tilde{A}, \tilde{B}$ are continuous functions of $\theta$, and there exists a $c>0$ such that $\inf _{\theta \in \Theta} \operatorname{det} C(\theta) \geq c \geq 0$.
2. The model is identifiable.
3. The rescaled error admit a density absolutely continuous w.r.t. the Lebesgue measure and positive in a neighborhood of the origin.
4. $\forall \theta \in \Theta, \rho(\tilde{A}(\theta)+\tilde{B}(\theta))<1$.

Then $\hat{\theta}$ is strongly consistent, that is, $\hat{\theta}_{n} \rightarrow \theta_{0}$, as $n \rightarrow \infty$.

### 4.6.3 Asymptotic normality

As employed by Basawa et al. (1976), the conditions for asymptotic normality of the maximum likelihood estimator for a general stochastic process are:

1. $-\frac{1}{T} \sum_{t=1}^{T} \frac{\partial^{2} l_{t}\left(\theta_{0}\right)}{\partial \theta \partial \theta} \xrightarrow{\boldsymbol{P}} C_{1}$ when $T \rightarrow+\infty$ for a positive definite, nonrandom ma$\operatorname{trix} C_{1}$.
2. $\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \frac{\partial l_{t}\left(\theta_{0}\right)}{\partial \theta} \xrightarrow{\mathcal{L}} N(0, C)$ when $T \rightarrow+\infty$ for a nonrandom $C$.
3. For all $i, j, k E\left(\sup _{\left\|\theta-\theta_{0}\right\| \leq \delta}\left|\frac{\partial^{3} l_{t}(\theta)}{\partial \theta_{i} \partial \theta_{j} \partial \theta_{k}}\right|\right)$ is bounded for all $\delta>0$.

Under the assumptions of theorem 1 and 2 we see that condition 1 is satisfied if $C_{1}$ is finite and positive definite. Since

$$
\begin{equation*}
\frac{\partial l_{t}(\theta)}{\partial \theta_{i}}=T R\left(\frac{\partial \Sigma_{t, \theta}}{\partial \theta_{i}} \Sigma_{t, \theta}^{-1}-\epsilon_{t} \epsilon_{t}^{T} \Sigma_{t, \theta}^{-1} \frac{\partial \Sigma_{t, \theta}}{\partial \theta_{i}}\right) \tag{4.23}
\end{equation*}
$$

we get, by using 4.10 , that

$$
\begin{equation*}
E_{\theta_{0}}\left[\left.\frac{\partial l_{t}}{\partial \theta_{i}}\left(\theta_{0}\right) \right\rvert\, \mathbf{\Phi}_{\boldsymbol{t}-\mathbf{1}}\right]=0 \text { a.s. } \tag{4.24}
\end{equation*}
$$

[^12]Further, by using theorem 1 and 2 it follows that

$$
\begin{equation*}
\frac{\partial l_{t}\left(\theta_{0}\right)}{\partial \theta} \tag{4.25}
\end{equation*}
$$

is an ergodic, strictly stationary process. Hence, condition 2 is obtained ${ }^{6}$. Lastly, note that the third condition follows from Basawa et al. (1976)'s condition $B 7$.

Now that we have verified the conditions for asymptotic normality we can present a theorem for convergence of the MLE. Before doing so, we must state that we require the $z_{t}$ 's to be independent.

Theorem 3 (F. Comte, O.Lieberman, 2003) Under the assumptions

1. 1-4 of theorem 2, and $C(\theta), \tilde{A}(\theta), \tilde{B}(\theta)$ admit continuous derivatives up to order 3 on $\Theta$,
2. the components of $z_{t}$ are independent,
3. $\epsilon_{\boldsymbol{t}}$ admits bounded moments of order 8 ,
4. the initial value in $\mathbf{\Sigma}$ is drawn for the stationary ergodic law,

$$
\sqrt{n}(\hat{\theta}-\theta) \xrightarrow{\mathcal{D}} N\left(0, C_{1}^{-1} C C_{1}^{-1}\right) \text { as } n \rightarrow \infty, \text { under } \boldsymbol{P}_{\theta_{0}} .
$$

For complete derivation and proof of theorem 3, see Comte \& Lieberman (2003), pages 61-81. The theorem establishes the asymptotic theory for the MGACH model (and more specifically, the BEKK $(1,1)$ model). By appealing to the conditions presented by Jeantheau (1998) and by satisfying the assumptions of theorem 2, we can achieve asymptotic normality ${ }^{7}$. The result justifies applying statistical inference tools of the $\operatorname{BEKK}(1,1)$ model by trusting the asymptotic normality, whereas without this theorem the tools were used inattentively.

[^13]
## 5 Data issues Modeling Procedure

This chapter begins by discussing the choice of macroeconomic variables and by stating the source of data. Section 5.3 covers the modeling part of the study.

### 5.1 Macroeconomic Variables

There are many potential macroeconomic variables that could be added to our allocation model. However, we choose only to include two variables in our model, Swedish inflation (CPI) and Purchaser Manager Index (PMI), as we find these variables most relevant and since previous studies suggest including them. The motivation behind using inflation in the model is because since investors seek real returns, the nominal return depend on inflation. Also, Schwert (1989) argues that if inflation is uncertain, then volatility of asset returns is reflected by the volatility of inflation. Theoretically there is a positive relation between returns and inflation ${ }^{1}$. However, some research on the relation between asset returns and inflation suggests the opposite of a positive relation ${ }^{2}$. We want to investigate the effects of inflation on the three chosen asset classes.

The Purchasing Managers Index measures the current business cycle of the Swedish economy, partly for the manufacturing industry, and partly for the service sector. An index of PMI over 50 indicates growth while an index below 50 indicates economic decline. The inclusion of PMI could positively effect our asset allocation strategy as we may get a better understanding of where in the business cycle we are.

### 5.2 Data

We include three financial assets, each representing an asset class. These are Swedish Equity, Swedish Bonds and Cash (a short government bond). As stated in section 2.1, the asset classes are represented by one index each:
Swedish Equity is represented by SIX PRX, which reflects the average development of companies on the Stockholm Stock Exchange.

Swedish Bonds is represented by OMRX Total Bond Index, which contains government bonds issued by the Swedish state and secured mortgage bonds issues by mortgage housing agencies.

## Cash is represented by Swedish 3-month Treasury Bills.

The data consists of the monthly returns of the 3 market indexes as well as the monthly

[^14](Swedish) CPI and PMI. The data is from 2008 to 2018 and were sourced from the Mercer Data Bank. ${ }^{3}$

### 5.3 Modeling Procedure

When looking at mutually traded public funds, there are essentially two main forms of portfolio management, passive- and active management. Passive funds track a market index and active funds aim to beat the index and generate alpha. In this thesis we consider a passive strategy within each asset class, but actively managing how much of each asset class to hold. The idea is to model the three financial assets with the goal of obtaining the highest risk-adjusted return (sharpe's ratio). We want to prove that the model containing the influence of macroeconomic variables outperform the other investment strategies.

### 5.3.1 Investment strategies

Consider the following investment strategies of three investors:

Investor A invests under the mean-variance framework and invests using time varying estimates of the $\boldsymbol{\Sigma}_{\boldsymbol{t}}$ from a $\operatorname{BEKK}(1,1)$ model.

Investor $\mathbf{B}$ also invests using time varying estimates for $\boldsymbol{\Sigma}_{\boldsymbol{t}}$, similar to investor A. However, Investor B also incorporates the effects of the macroeconomic factor Swedish inflation into the model.

Investor $\mathbf{C}$ invests with time varying estimates for $\boldsymbol{\Sigma}_{\boldsymbol{t}}$ and the effects of macroeconomic variables, similar to investor B. Investor C incorporates the effects of Swedish inflation and PMI into the model.

The strategies adopted by Investor A, B and C are all self financing, meaning no additional money is invested after time $t=0$. Also, none of the investment strategies allow short selling. Notice that the portfolios of investor A-C consists of three assets, where $w_{1}+w_{2}+w_{3}=1$. This means each investor has to be invested with his entire capital in at least one of the three asset classes at all times. In this thesis we do not account for any transaction costs for buying and selling assets. Since this simplification is adopted by all investors, it should not affect the main result if macroeconomic variables can have a positive influence on the asset allocation. For the same reason we assume all manager fees are zero. In reality, passively managed funds tend to have lower manager fees that funds with active management. However, we will not make any assumptions as to how big price difference there is between the investors, we instead assume they all charge zero in manager fee.

### 5.3.2 Time-varying procedure

Investor $A, B$ and $C$ compute the historical mean and the time-variant historical variancecovariance matrix in each time step. Based on those estimates the weights are computed. The investors then compute a new vector of means and a new variance covariance matrix in each time point based on data from the past 9 years. Preferably we would have chosen a longer time period, however, the data of the chosen indexes does not stretch back further in time. In the study by Flavin \& Wickens (2003) they use 30 years of historical data.

[^15]Only using 9 years of historical data may be a slight drawback, however, since 2008 the Swedish financial market has experienced both extreme bullish- and bearish conditions, which indicates the data should be sufficient to draw some conclusions.

The investment strategies are compared by the following procedure: ${ }^{4}$

1. Begin by creating log returns of the data.
2. Test for Cross-correlation.
3. Test for Conditional heteroscedasticity.
4. Estimate the expected returns by using a 9 -year rolling average.
5. Estimate the $\boldsymbol{\Sigma}_{\boldsymbol{t}}$ with the $\operatorname{BEKK}(1,1)$ model.
6. Compute portfolio weights $\boldsymbol{w}$ at time $t$.
7. Calculate the return for the investor by multiplying the weights $\boldsymbol{w}$ by the actual returns at time $t+1$.
8. Repeat step 1-7 for the remaining time periods.

For Investor $\mathrm{A}, \boldsymbol{\mu}$ is a $3 \times 1$ vector and $\boldsymbol{\Sigma}_{\boldsymbol{t}}$ is a $3 \times 3$ matrix. Investor B incorporates the macroeconomic variable inflation in the model, and thus $\boldsymbol{\mu}$ becomes a $4 \times 1$ vector and $\boldsymbol{\Sigma}_{\boldsymbol{t}}$ becomes a $4 \times 4$ matrix. In that case, a sub-vector and sub-matrix of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}_{\boldsymbol{t}}$ are used to calculating the portfolio weights $\boldsymbol{w}$. Hence, for investor $\mathrm{C}, \boldsymbol{\Sigma}_{\boldsymbol{t}}$ is a $5 \times 5$ matrix. This means the macroeconomic variables inflation and PMI both effect both $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}_{\boldsymbol{t}}$, but neither are part of the weights, i.e. investor B and C do not invest in inflation nor PMI, since those are not asset classes.
All of the investment strategies are conducted from the perspective of a Swedish investor. Each of the investors is assumed to choose an optimal portfolio, a minimum variance portfolio, and a few other portfolios with the lowest possible level of volatility for a given return on investment.

As mentioned above, the variance covariance matrix is modeled by using the $\operatorname{BEKK}(1,1)$ model, and the portfolio weights are rebalanced each month based on the estimates of the BEKK model. The returns of the portfolios are then computed by

$$
\begin{equation*}
R_{p}=C_{o} \prod_{t=1}^{12}\left(w_{1, t} R_{1, t}+w_{2, j} R_{2, t}+w_{3, j} R_{3, t}\right) \tag{5.1}
\end{equation*}
$$

where $C_{0}$ is the invested capital at $t=0$, which is 2017-01-31. The re-balancing is performed for next 12 months for a number of different portfolios. The goal is to compare the three investment strategies in terms of risk adjusted returns and ultimately comment if the addition of macroeconomic variables can be beneficial for tactical asset allocation.

[^16]
### 5.4 Testing

After transforming the price data to log returns, we start of with testing for cross correlation by using the Portmanteau test. See section 4.4.1 test details. In R, the function $m q$ from the MTS package can be utilized for obtaining the Portmanteau test statistic. We test

Listing 5.1. R output Portmanteau test
> test <- mq (rtnn3, lag = 10)
Ljung-Box Statistics:

|  | $m$ | Q(m) | df | p-value |
| :---: | ---: | ---: | ---: | ---: |
| $[1]$, | 1 | 121 | 25 | 0 |
| $[2]$, | 2 | 243 | 50 | 0 |
| $[3]$, | 3 | 343 | 75 | 0 |
| $[4]$, | 4 | 436 | 100 | 0 |
| $[5]$, | 5 | 532 | 125 | 0 |
| $[6]$, | 6 | 619 | 150 | 0 |
| $[7]$, | 7 | 702 | 175 | 0 |
| $[8]$, | 8 | 774 | 200 | 0 |
| $[9]$, | 9 | 827 | 225 | 0 |
| $[10]$, | 10 | 871 | 250 | 0 |

for 10 lags and find that the null hypothesis of zero cross correlation can be rejected on any level of significance. We also test for zero conditional heteroscedasticity. By using the Marchtest in R, we get:

Listing 5.2. R output Testing for conditional heteroscedasticitytest

```
> MarchTest(rtnn3, lag = 10)
Q(m) of squared series(LM test):
Test statistic: 137.8069 p-value: 0
Rank-based Test:
Test statistic: 66.34859 p-value: 0.0000000002236091
Q_k(m) of squared series:
Test statistic: 776.4933 p-value: 0
Robust Test(5%) : 504.3756 p-value: 0
```

The results show that the null hypothesis of zero conditional heteroscedasticity can be rejected on any level of significance. This indicates we can start modeling with a time series model such as the $\operatorname{BEKK}(1,1)$.
The next step is to calculate the historical means of each of the assets, by creating a 9year rolling average. After this, the BEKK $(1,1)$ model is formed for each of the investors. Portfolio weight are hereby computed based on the historical means and the time dependent variance covariance matrices from each time point

## 6 Results

### 6.1 What does the data say?

Figure 6.1 shows the development of the market indexes SIX PRX, OMRX Bond, T-bill and the macroeconomic variables the most recent 10 -year period. SIX PRX, OMRX Bond, T-bill and CPI (inflation) are all indexed to 100 in the beginning of 2008, while PMi is not transformed, taking on values from around 50 , where values over 50 indicates bullish market conditions and values below 50 indicate bearish market conditions. The


Figure 6.1. Figure showing historical data of the 10-year period 2008-2018 of the the securities SIX PRX, OMRX Bond, T-bill combined with the macroeconomic variables Swedish CPI (inflation) and PMI
graph shows that the development on the Swedish equity market has been very strong over the past 10 years, while the bond market has progressed slower. An interesting observation is the seemingly large correlation between PMI and SIX PRX, especially during the first 2-3 years of the 10-year period. After the end of 2013, the correlation seems to vanish, and one can even suspect a negative relation between the two time series. Based on the figure, it seems that PMI is strongly correlated with the Stock market during times of extreme market stress, while not as strongly connected during calmer market conditions.

### 6.2 Conditional Mean

The 9 -year rolling averages from the data can be found in table A. 1 in appendix B. One could have adopted a time series model in order to predict the expected means, however, Flavin \& Wickens (2003) argues that there is little evidence in good predictability of asset returns. In their study, the estimates for their asset returns based on a $\operatorname{BEKK}(1,1)$ model show that the financial asset returns cannot be easily predictable, based on their
data. They found that the only significant coefficient in their BEKK model was for UK equities. They argue that obtaining significance in the coefficient for equity is consistent with having a substantial equity premium.

Because of the lack of significance we choose to adopt another approach for calculating expected returns. Given the poor evidence of good predictability in the time series models, we choose to use a vector of historical means as our expected return data. Jobson \& Korkie (1981) show that this approach can improve portfolio performance when adopting the Markovitz portfolio selection. In their original paper from 1998, Flawin and Wickens also argue that this approach can be beneficial over the estimated parameter means from the BEKK model. Re-balancing based on changes in the predicted returns is counter productive because the lack of evidence of good predictability of asset returns. This means taking on transactions costs that ultimately may not lead greater portfolio performance.

### 6.3 Conditional Variance-Covariance Matrix

Due to greater persistence, the same argument is not true for re-balancing due to changes in the conditional variance. The estimated time varying variance-covariances matrices show that both inflation and PMI play an important role, both short term and long term. The long run variance-covariance matrix is obtained by multiplying $\boldsymbol{C} \boldsymbol{C}^{\boldsymbol{T}}=\boldsymbol{H}$. For investor C, it's given by (in \%):

$$
\boldsymbol{H}=\left[\begin{array}{ccccc}
\text { EQ } & \text { Bonds } & \text { Cash } & \text { Inf } & \text { PMI }  \tag{6.1}\\
0,1679 & -0,0559 & 0,0007 & 0,0000 & -0,0046 \\
-0,0559 & 0,0198 & -0,0002 & -0,0013 & -0,0019 \\
0,0007 & -0,0002 & 0,0002 & 0,0063 & -0,0019 \\
0,0000 & -0,0013 & 0,0063 & 0,0024 & -0,0110 \\
-0,0046 & -0,0019 & -0,0019 & -0,0110 & 0,3393
\end{array}\right]
$$

It seems that the strongest correlation between a macroeconomic variable and the securities is the one between inflation and T-bill, which is $\boldsymbol{H}_{43}=0.0063 \%$. This makes intuitive sense because inflation and inflation expectations are key factors in determining the interest rates of Treasury bills. Historically, periods of high inflation are usually associated with with relatively high interest rates on Treasury bills. ${ }^{1}$. We also make the observation that there is a negative sign on the covariance between Swedish equities and PMI $\left(\boldsymbol{H}_{51}\right)$. Based on figure 6.1 it seems that in times of market stress PMI and Swedish equity have a strong positive correlation, whilst in the long run the correlation between the two time series is not as strong, and has a negative coefficient. Of course, some of the relationships inflation has with the securities may in large be explained by the strong negative correlation between inflation and PMI. Therefore it could be of interest to look at the long run correlation matrix of investor $B$, where inflation is the only present

[^17]macroeconomic variable. $\boldsymbol{H}$ for investor B is given by (in \%):
\[

\boldsymbol{H}=\left[$$
\begin{array}{cccc}
\mathbf{E Q} & \text { Bonds } & \text { Cash } & \text { Inf }  \tag{6.2}\\
0,203 & -0,003 & 0,028 & -0,018 \\
-0,003 & 0,005 & 0,012 & 0,008 \\
-0,028 & 0,012 & 0,115 & 0,073 \\
-0,018 & 0,008 & 0,073 & 0,046
\end{array}
$$\right]
\]

The matrix shows there is a positive relationship between inflation and both the Swedish Bond market and the Swedish Treasury bill, however, the relation with the T-bill is of much greater significance. This can be explained, in part, by the actions taken by the Swedish central bank based on inflation and inflation expectations. Rising inflation may cause the central bank to raise the interest rate on the (short term) Treasury bill in order to reduce the demand for credit and thus prevent the economy from overheating. When the central bank imposes such a raise to the short term interest rate, then long term interest rates tend to go up as well. This can be seen in the positive sign of both the correlation between inflation and the bond market $\boldsymbol{H}_{42}$, and the correlation between the short term treasury bill and the bond market $\boldsymbol{H}_{32}$. The negative sign on the covariance between Swedish equity and inflation $\boldsymbol{H}_{41}$ indicates a long term inverse relationship between equity returns and inflation. This is consistent with the findings by Groenewold \& Fraser (1997) in their study on share prices and macroeconomic factors. A negative long-run covariance between the Swedish stock market and inflation suggests that high inflation volatility is associated with lower volatility on Swedish equity returns.

There is quite a large number of significant estimates in the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, to be found in appendix A. 2 with their corresponding $t$-statistics. This suggests that the in the short run, the conditional variance covariance matrices differ from the long run covariance $\boldsymbol{H}$. In models such as the $\operatorname{BEKK}(1,1)$, there are quite a large number of coefficients to estimate. The most relevant ones are placed along the main diagonals of the matrices $\boldsymbol{C}, \boldsymbol{A}$ and $\boldsymbol{B}$. Those parameters should be statistically significant, otherwise there might be some doubt in using the MGARCH error structure in the first place. Many of the coefficients in the off diagonal are not significant, which suggests they may be zero. However, one can still choose to keep them in the model. In rough terms, the greater the diagonal elements of $\boldsymbol{A}$ and $\boldsymbol{B}$, the larger deviation in the short run covariances from the long run covariances.

Figures B.2, B. 3 and B. 4 in appendix B, show the conditional variances, together with the long run variance for each of the three securities, based on investor B's volatility model. We see that the short term deviances from the long run are quite substantial during certain times (especially during 2008-2009). The conditional variances are usually above the long-run values, especially for the Swedish equity. This indicates that Investor B could, for some time periods, hold less less Swedish equity compared to what the long-run variance implies. The inverse is true for other time periods. Similar plots can be found for investor C (B.5,B.6,B.7), where a similar pattern can be spotted. The impact of the macroeconomic variables on the short term volatility of asset returns is an important factor. When looking at the t -values for investor C , presented in appendix A.2, one can see that both $\boldsymbol{B}_{51}, \boldsymbol{A}_{51}$ and $\boldsymbol{A}_{15}$ are highly significant coefficients. This suggests PMI does have a strong impact on Swedish equity variance in the short-run. PMI does seem to impact OMRX Bond and the treasury bill as well, but not to the same extent as the impact on SIX PRX. The coefficients $\boldsymbol{A}_{52}$ and $\boldsymbol{A}_{53}$ are marginally significant, while $\boldsymbol{B}_{52}$ and $\boldsymbol{B}_{53}$ are not. Inflation also seems to play a vital role in determining short term volatility. SIX PRX seems to be least affected of the impact of inflation, suggested by the low $t$-values of the coefficient affecting inflation in matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, both for investor

B and C. Inflation seems to have greater impact on T-bill and OMRX Bond. Taking all the $t$-values of the estimated coefficients into account, the results as a whole suggest that even though asset returns are seemingly hard to predict, asset return volatility is both time varying and more predictable, especially given the effects of the macroeconomic environment. This preliminary result suggests that the inclusion of macroeconomic variables will cause some different re-balancing compared to a model without macroeconomic variables, especially for portfolios with expected return between the minimum variance portfolio and the optimal portfolio.

## 7 Portfolio Selection

### 7.1 The Efficient Frontier

In chapter 3 we described all the necessary tools that are needed to calculate portfolios along the efficient frontier. To do so, one needs estimates of the expected returns and the variance covariance matrix. With the theory on modern portfolio theory combined with the theory on volatility modeling (presented in chapter 4), we are now able to calculate a set of efficient portfolios that lie on the efficient frontier. Based on the historical data, we calculate monthly 9 -year rolling averages as a proxy for the expected returns. Next we use the BEKK model to generate time varying variance covariance matrices, one for each of the 12 time periods.For each investor, the distribution of frontiers generated from the 12-month re-balancing period will be plotted in the mean-variance field. Figure 7.1 shows the aggregated set of portfolios for the three investors. We see that the inclusion of the macroeconomic factor offers superior risk return combinations for portfolios with relatively low expected return. For portfolios with higher expected return, the effect seems to vanish, and none of the compared investment strategies seems to be superior. Another


Figure 7.1. Figure showing the distributions of efficient frontiers aggregated in the mean-variance plane.
important observation is that the optimal portfolio for each of the investors is drastically reduced when including the macroeconomic variables. Investor $B$ and $C$ achieve their optimal portfolio at a monthly return of $0.13 \%$ and $0.11 \%$ respectively, while investor A's optimal portfolio has a expected return of $0.31 \%$. Figure 7.1 really shows that taking into account the macroeconomic factors results in part of the distribution of efficient frontiers shifts to the left. It seems that the macroeconomic factors help to predict the asset return volatilities and thus making investors shift away from more volatile asset classes when needed, based on the macroeconomic environment. Tables $7.1,7.2$, and 7.3 show each of the the portfolios with their corresponding mean and standard deviation, for each of the investors.

### 7.2 Portfolio Selection

When comparing the different investors and their investment strategies, we look at 5 different portfolios along the efficient frontier. They include the minimum variance portfolio (MVP), the optimal portfolio (OP), the portfolios with $0.2 \%, 0.3 \%$ - and $0.4 \%$ monthly target return. We also calculate some other dots along the portfolio line to match the expected return of investor A optimal portfolio with investor B- and C's optimal portfolio, and vice versa.

Table 7.1. Average mean, standard deviation, sharpe ratio and weightings for investor A over the 12 rebalancing periods.

| Averages | Minimum | $\mathbf{0 . 1 3 \%}$ | Optimum |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Target Return: | $0.05 \%$ | $0.13 \%$ | $0.31 \%$ | $0.20 \%$ | $0.30 \%$ | $0.40 \%$ |
| SD | $0.29 \%$ | $0.38 \%$ | $0.74 \%$ | $0.51 \%$ | $0.78 \%$ | $0.99 \%$ |
| Sharpe | 0.23 | 0.37 | 0.44 | 0.40 | 0.39 | 0.40 |
| Average Weights |  |  |  |  |  |  |
| SIX PRX | $0.19 \%$ | $2.51 \%$ | $7.55 \%$ | $4.30 \%$ | $6.97 \%$ | $12.54 \%$ |
| OMRX Bond | $0.00 \%$ | $20.14 \%$ | $69.66 \%$ | $39.86 \%$ | $67.54 \%$ | $87.41 \%$ |
| T-bill | $99.81 \%$ | $77.35 \%$ | $22.79 \%$ | $55.84 \%$ | $25.49 \%$ | $0.05 \%$ |

Tables 7.1, 7.2 and 7.3 show the aggregate set of portfolios for different levels of expected return for each of the investors. By comparing the average Sharpe ratios over for the different investors, we see that for the lower values of expected return, investor B and C clearly outperform investor A in terms of Sharpe's ratio.

Table 7.2. Average mean, standard deviation, Sharpe ratio and weightings for investor B over the 12 rebalancing periods.

| Averages | Minimum | $\mathbf{0 . 3 0 \%}$ | Optimum |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Target Return: | $0.06 \%$ | $0.30 \%$ | $0.13 \%$ | $0.20 \%$ | $0.40 \%$ | $0.50 \%$ |
| SD | $0.16 \%$ | $0.71 \%$ | $0.27 \%$ | $0.45 \%$ | $1.02 \%$ | $1.79 \%$ |
| Sharpe | 0.34 | 0.44 | 0.49 | 0.47 | 0.41 | 0.31 |
| Average Weights |  |  |  |  |  |  |
| SIX PRX | $0.25 \%$ | $5.49 \%$ | $2.25 \%$ | $3.40 \%$ | $13.17 \%$ | $30.31 \%$ |
| OMRX Bond | $0.39 \%$ | $71.47 \%$ | $21.11 \%$ | $42.18 \%$ | $84.59 \%$ | $69.69 \%$ |
| T-bill | $99.36 \%$ | $23.04 \%$ | $76.64 \%$ | $54.42 \%$ | $2.25 \%$ | $0.00 \%$ |

We also notice some differences in terms of the weights of each of the strategies, especially for portfolios that lie in-or around the optimum. As expected return increases, the weightings of the investors converge and the difference between the strategies fades.

Table 7.3. Average mean, standard deviation, Sharpe ratio and weightings for investor C over the 12 rebalancing periods.

| Averages | Minimum | $\mathbf{0 . 3 0 \%}$ | Optimum |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Target Return: | $0.06 \%$ | $0.30 \%$ | $0.11 \%$ | $0.20 \%$ | $0.40 \%$ | $0.50 \%$ |
| SD | $0.16 \%$ | $0.76 \%$ | $0.23 \%$ | $0.48 \%$ | $1.07 \%$ | $2.25 \%$ |
| Sharpe | 0.39 | 0.39 | 0.48 | 0.42 | 0.38 | 0.24 |
| Average Weights |  |  |  |  |  |  |
| SIX PRX | $0.00 \%$ | $9.39 \%$ | $1.64 \%$ | $5.39 \%$ | $13.76 \%$ | $29.30 \%$ |
| OMRX Bond | $3.56 \%$ | $60.53 \%$ | $14.27 \%$ | $36.72 \%$ | $83.16 \%$ | $72.80 \%$ |
| T-bill | $96.44 \%$ | $30.08 \%$ | $84.10 \%$ | $57.90 \%$ | $3.08 \%$ | $0.15 \%$ |

The average equity shares for the optimal portfolios differs quite substantially between investor A and B,C. This is due to the large difference in expected return in the optimal portfolio of each investor. When looking at other portfolios, we see that investor C has increased equity shares compared to the other investors. This can be explained by the negative correlation of Swedish equity returns and PMI. Another difference is in the MVP weightings, where we can see that investor $C$ seems to prioritize OMRX Bond over Swedish equity, compared to the other investors. This seems to yield an overall greater Sharpe's ratio for the MVP, especially compared to investor A.

### 7.3 Performance

In order to further determine whether the macroeconomic factors improve portfolio performance, we now compare the returns of each of the investment strategies for the given 12 -month testing period. Ultimately, a longer testing period would be preferable, but longer data for some of the indexes was not available by the data provider ${ }^{1}$. For a number of different portfolios, we compare the portfolio returns of each investor and compare the weights in each time period. Table 7.4 shows the weights for each of the investors

Table 7.4. Returns and calculated weights in each time period for each investor in the Minimum Variance Portfolio.

| MVP | 2017-03-31 | 2017-04-30 | 2017-05-31 | 2017-06-30 | 2017-07-31 | 2017-08-31 | 2017-09-30 | 2017-10-31 | 2017-11-30 | 2017-12-31 | 2018-01-31 | 2018-02-28 | Total 12 month |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weight Manager A SIX PRX | 0.19\% | 0.19\% | 0.19\% | 0.19\% | 0.19\% | 0.19\% | 0.19\% | 0.19\% | 0.18\% | 0.19\% | 0.19\% | 0.19\% |  |
| Weight Manager B SIX PRX | 0.48\% | 0.00\% | 0.53\% | 0.00\% | 0.03\% | 0.22\% | 0.00\% | 0.36\% | 0.55\% | 0.18\% | 0.00\% | 0.65\% |  |
| Weight Manager C SIX PRX | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% |  |
| Return SIX PRX | 2.12\% | 4.27\% | 1.71\% | -1.98\% | -3.07\% | -0.86\% | 5.45\% | 2.10\% | -3.56\% | -1.27\% | 1.56\% | -0.73\% | 5.426\% |
| Weight Manager A OMRX Bond | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% |  |
| Weight Manager B OMRX Bond | 1.11\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 1.48\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 1.27\% | 0.80\% |  |
| Weight Manager C OMRX Bond | 2.32\% | 1.09\% | 2.39\% | 5.25\% | 4.52\% | 3.78\% | 2.18\% | 3.94\% | 2.49\% | 4.93\% | 6.00\% | 3.85\% |  |
| Return OMRX Bond | -0.21\% | 0.23\% | 0.40\% | -0.58\% | -0.05\% | 0.33\% | -0.26\% | 0.42\% | 0.30\% | -0.36\% | -0.30\% | 0.40\% | 0.312\% |
| Weight Manager A T-bill | 99.81\% | 99.81\% | 99.81\% | 99.81\% | 99.81\% | 99.81\% | 99.81\% | 99.81\% | 99.82\% | 99.81\% | 99.81\% | 99.81\% |  |
| Weight Manager B T-bill | 98.41\% | 100.00\% | 99.47\% | 100.00\% | 99.97\% | 98.30\% | 100.00\% | 99.64\% | 99.45\% | 99.82\% | 98.73\% | 98.55\% |  |
| Weight Manager C T-bill | 97.68\% | 98.91\% | 97.61\% | 94.75\% | 95.48\% | 96.22\% | 97.82\% | 96.06\% | 97.51\% | 95.07\% | 94.00\% | 96.15\% |  |
| Return T-bill | -0.06\% | -0.06\% | -0.05\% | -0.04\% | -0.07\% | -0.09\% | -0.07\% | -0.05\% | -0.05\% | -0.08\% | -0.12\% | -0.02\% | -0.742\% |
| Total return A | -0.06\% | -0.05\% | -0.05\% | -0.04\% | -0.07\% | -0.09\% | -0.06\% | -0.04\% | -0.05\% | -0.08\% | -0.11\% | -0.02\% | -0.7300\% |
| Total return B | -0.05\% | -0.06\% | -0.04\% | -0.04\% | -0.07\% | -0.08\% | -0.07\% | -0.04\% | -0.06\% | -0.08\% | -0.12\% | -0.02\% | -0.7369\% |
| Total return C | -0.06\% | -0.06\% | -0.04\% | -0.07\% | -0.06\% | -0.07\% | -0.07\% | -0.03\% | -0.04\% | -0.10\% | -0.13\% | 0.00\% | -0.7292\% |

allocation in each time period during the twelve months. The actual returns of each asset class is also shown. As indicated by the table, all investors perform poorly for the MVP. This is due to the clear overweight in T-bill for each investor, and T-bill has had negative returns throughout the entire time period. We see a slight superiority in in the return of investor C, but the difference is minimal. When we look at portfolios with higher expected return than for the MVP, investor C outperforms investor A in most cases. It seems that investor C produces better risk adjusted returns for lower values of expected returns, but the effect seems to vanish as expected return goes up. Table 7.5 shows that investor C is the only investor that generates a positive return over the 12 -month period for portfolios with expected return $0.2 \%$. In fact, for all the investigated portfolios, the only times investor A outperforms investor C in the $12-$ month period is for the optimal portfolio and for the portfolio with expected return of $0.4 \%$. This result in the OP can be explained by the fact that investor A has much higher expected return in the OP $(0.31 \%$ for investor A compared to $0.11 \%$ ). This means investor A will likely generate a higher return than investor C in the OP , but compensated with a much larger volatility. As suggested by figure 7.1, the superiority of adding macroeconomic factors seems to vanish when increasing expected return, and thus when we reach the point of $0.4 \%$ it seems that the effect has faded. Based on our data, the fast majority of excess gains that can be made

[^18]from taking into account the macroeconomic environment lies in the portfolios ranging between the MVP and the OP of investor A. Similar tables for the optimal portfolio, and portfolios with expected return of $0.3 \%$ and $0.4 \%$ can be found in appendix B. (B.1, B.2, B.3).

Table 7.5. Returns and calculated weights in each time period for each investor, with expected return $0.2 \%$.

| 0.20\% | 2017-03-31 | 2017-04-30 | 2017-05-31 | 2017-06-30 | 2017-07-31 | 2017-08-31 | 2017-09-30 | 2017-10-31 | 2017-11-30 | 2017-12-31 | 2018-01-31 | 2018-02-28 | Total 12-month |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weight Manager A SIX PRX | 3.35\% | 3.55\% | 3.47\% | 3.36\% | 3.56\% | 3.81\% | 3.89\% | 4.29\% | 5.03\% | 5.35\% | 6.06\% | 5.93\% |  |
| Weight Manager B SIX PRX | 1.45\% | 0.00\% | 4.03\% | 1.11\% | 3.36\% | 2.95\% | 0.88\% | 4.18\% | 8.08\% | 6.05\% | 5.09\% | 3.62\% |  |
| Weight Manager C SIX PRX | 5.46\% | 4.83\% | 5.69\% | 4.08\% | 4.17\% | 4.35\% | 6.83\% | 4.99\% | 5.79\% | 4.96\% | 6.27\% | 7.25\% |  |
| Return SIX PRX | 2.12\% | 4.27\% | 1.71\% | -1.98\% | -3.07\% | -0.86\% | 5.45\% | 2.10\% | -3.56\% | -1.27\% | 1.56\% | -0.73\% | 5.426\% |
| Weight Manager A OMRX Bond | 38.40\% | 39.45\% | 39.29\% | 39.43\% | 37.37\% | 40.19\% | 41.67\% | 39.37\% | 38.08\% | 39.48\% | 43.75\% | 41.82\% |  |
| Weight Manager B OMRX Bond | 42.83\% | 48.24\% | 37.94\% | 44.50\% | 37.91\% | 42.52\% | 49.71\% | 39.74\% | 24.38\% | 36.19\% | 48.56\% | 53.65\% |  |
| Weight Manager C OMRX Bond | 33.47\% | 36.27\% | 33.96\% | 37.79\% | 35.72\% | 38.72\% | 33.82\% | 37.01\% | 34.69\% | 41.33\% | 42.70\% | 35.11\% |  |
| Return OMRX Bond | -0.21\% | 0.23\% | 0.40\% | -0.58\% | -0.05\% | 0.33\% | -0.26\% | 0.42\% | 0.30\% | -0.36\% | -0.30\% | 0.40\% | 0.312\% |
| Weight Manager A T-bill | 58.25\% | 57.00\% | 57.24\% | 57.22\% | 59.07\% | 56.00\% | 54.44\% | 56.34\% | 56.88\% | 55.17\% | 50.19\% | 52.24\% |  |
| Weight Manager B T-bill | 55.72\% | 51.76\% | 58.03\% | 54.39\% | 58.73\% | 54.53\% | 49.41\% | 56.07\% | 67.54\% | 57.76\% | 46.34\% | 42.73\% |  |
| Weight Manager C T-bill | 61.08\% | 58.90\% | 60.35\% | 58.13\% | 60.11\% | 56.93\% | 59.35\% | 58.00\% | 59.52\% | 53.71\% | 51.02\% | 57.65\% |  |
| Return T-bill | -0.06\% | -0.06\% | -0.05\% | -0.04\% | -0.07\% | -0.09\% | -0.07\% | -0.05\% | -0.05\% | -0.08\% | -0.12\% | -0.02\% | -0.742\% |
| Total return A | -0.05\% | 0.21\% | 0.19\% | -0.32\% | -0.17\% | 0.05\% | 0.06\% | 0.23\% | -0.09\% | -0.26\% | -0.10\% | 0.11\% | -0.119\% |
| Total return B | -0.09\% | 0.08\% | 0.19\% | -0.30\% | -0.16\% | 0.07\% | -0.12\% | 0.23\% | -0.25\% | -0.26\% | -0.12\% | 0.18\% | -0.545\% |
| Total return C | 0.01\% | 0.25\% | 0.20\% | -0.32\% | -0.18\% | 0.04\% | 0.24\% | 0.23\% | -0.13\% | -0.26\% | -0.09\% | 0.08\% | 0.071\% |

The OP's of investor B and C has a lot lower expected return than the OP of investor A, however the main difference lies in the risk of the OP's. In every time period the risk associated with the OP is much lower for investor B and C. This is evidenced by the increased average Sharpe ratio of 0.48 for investor C, compared to 0.44 for investor A, which is an increase of $9 \%$.

In order to get a better view of how the different investors re-allocate between asset classes we study figure 7.2 below, which shows the portfolio weight of equities for each of the investors combined with the development of SIX PRX during the given time period, with a monthly target return of $0.20 \%$. The figure shows that Investor C is slightly


Figure 7.2. Figure showing the portfolio weights of Equities for each of the investor for the whole testing year. The weights are based on a target portfolio return of $0.20 \%$.
superior at predicting asset returns of SIX PRX, compared to the other two investors. This can be recognized by looking at the first three months of the observed time period. Here the stock market is performing well, and hence Investor $C$ has an overweight in equities compared to the other investors. However, in the poorly performing months of the year, such as in November, Investor $C$ has a relative underweight in equities compared to his competitors. A similar plot for the fixed income portion of the portfolio can be found below in figure 7.3. The figure shows the weight of OMRX Bond for each of the investors, with a monthly target return of $0.20 \%$. As indicated by figure 7.3, investor C


Figure 7.3. Figure showing the portfolio weights of Bonds for each of the investor for the whole testing year. The weights are based on a target portfolio return of $0.20 \%$.
seems to time the market slightly better than his opponents. Let us look at the month of November again. We saw in figure 7.2 that equities were performing poorly during this month, and thus investor C had an underweight in equities. We can also see that the Bond market had two consecutive good months in October and November, and investor C seems to catch that upward trend slightly better than investor A and B. This is indicated by the relative overweight of OMRX bond of investor C relative to other months of the year. Other months, when the Bond market wasn't performing as well, we can see that investor $C$ shifts away from that asset class relative to the other investors. Look at June and December for instance. Those are both months of negative monthly return and the two lowest returns of the entire year. During those months investor C had the least amount of share in the bond market of the three investors. This indicates that the investment strategy adopted by investor $C$ seems to move away from certain asset classes during times of market stress and decline, while shifting capital towards more attractive asset classes during those time periods. Similar patterns can be spotted for investors A and B, but the pace which which those patterns occur is somewhat decreased when comparing to investor C . This can be explained by the inclusion of the macroeconomic variables in investor C's investment strategy. Similar plots with a different monthly return target of $0.30 \%$ can be found in appendix B. Analogous conclusions can be drawn from those plots as well.

Table 7.6 below shows a summary of the performance data from each of the portfolios along the efficient frontier. The returns presented in the table are the realized yearly returns for each of investors during the one year time period. The performance figures marked bold show the winning investor for each given level of target return. We see that

Table 7.6. Table summarizing the results of each portfolio for the three investor strategies over the 12-month testing period.

| Portfolio | Minimum | $\mathbf{0 , 1 3 \%}$ | $\mathbf{0 , 2 0 \%}$ | Optimum | $\mathbf{0 , 3 0 \%}$ | $\mathbf{0 , 4 0 \%}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Total return A | $-0,7300 \%$ | $-0,422 \%$ | $-0,119 \%$ | $\mathbf{0 , 3 4 5 \%}$ | $0,282 \%$ | $\mathbf{1 , 2 6 0 \%}$ |
| Total return B | $-0,7379 \%$ | $-0,683 \%$ | $-0,545 \%$ | $-0,618 \%$ | $-0,346 \%$ | $0,857 \%$ |
| Total return C | $\mathbf{- 0 , 7 2 9 5 \%}$ | $\mathbf{- 0 , 3 4 5 \%}$ | $\mathbf{0 , 0 7 1 \%}$ | $-0,489 \%$ | $\mathbf{0 , 6 6 6 \%}$ | $1,156 \%$ |

investor C is superior in most cases in regards to portfolio returns, followed by Investor A, and thereafter investor B. Investor C outperforms investor A for every level of expected return, except for the OP and the portfolio with $0.4 \%$ target return. The former can be
explained in the increased target return in the OP for investor A makes creates an overweight equity compared to investor $C$. However, investor $C$ compensates for the lower return by a lower portfolio standard deviation and thus an increase in the Sharpe's ratio in the OP.

Based on this data, combined with the aggregated efficient frontier figure, we can conclude that the greatest benefit of adding inflation and PMI to to the asset allocation model lies in the potential of lowering the portfolio risk. This suggests that investors who seek monthly returns in line with investor A's optimal portfolio (or lower) should consider adding macroeconomic factors to their investment model in order to reduce risk and thus potentially generating greater risk adjusted returns.

## 8 Conclusion

In this thesis we describe an asset allocation method that involves taking into account the effects of macroeconomic variables. The model is based on the standard Markowitz (1952) minimum-variance portfolio selection, but extended to such that we allow the variance-covariance matrix to vary over time, and to reflect changes in the macroeconomic environment. We build a tactical asset allocation strategy that involves continuous re-balancing based on the predictions of short term changes in the variance covariance matrix. We show that investors who extend this time varying model by adding macroeconomic factors can offer superior risk adjusted combinations of assets. We also show, based on our data, that this methodology allows for significant increase in the Sharpe's ratio by gains in risk reduction.

The analysis involves three types of financial assets, and two macroeconomic variables. The financial assets are Swedish Equity, Swedish Bonds, and a 90-day Swedish Treasury Bill. The macroeconomic variables are Swedish inflation and PMI. We model the joint distribution of asset returns and the macroeconomic variables in such a way that permits a time-varying variance-covariance structure of the joint distribution. This allows the macroeconomic variables to influence the estimated variance-covariance matrices in each time step, and thus influencing the portfolio weights and the efficient frontier. We use a variant of a $\operatorname{M-GARCH}(1,1)$, called $\operatorname{BEKK}(1,1)$, to model the covariance of the joint distribution. Other methods such as the EMWA could also be used.

We show that the impact of macroeconomic factors seem to reduce the risk compared to the portfolio model with only asset returns. This is reflected in the Sharpe's ratios significant increase when adding CPI and PMI to the model. This means that risk-adjusted returns are significantly higher when modeling with the influence of macroeconomic factors, compared to the model excluding them. As we saw in figure 7.1, the efficient fronter shifts to the left in the lower part of the graph, allowing for greater risk adjusted returns. The effect however, seems to fade as we increase expected return. Inflation and PMI have significant impact on the conditional covariance of the financial asset returns. PMI is negatively correlated with equity in the long run, but short run deviances from the long run estimates are very obvious. Inflation also plays a vital role in estimating the short term covariance matrices.

For future studies it could be of interest to investigate how this model would behave when adding different types of asset classes. One could for instance categorize the equity asset class by adding indexes for small cap, mid cap and large cap. One could also add foreign stocks and bonds, in addition to foreign CPI. Then comes the addition of alternative investments, such as real estate and hedge funds. It would be interesting to see how these asset classes would behave in relation to macroeconomic factors. Allowing a greater number of asset classes might result in further diversification benefits and ultimately greater risk-return combinations, especially considering macroeconomic factors. Another interesting addition to this fairly simple model would be adding a more complex
model for predicting asset returns. By using historical means we are able to prove that macroeconomic factors may in fact improve asset allocation, but we could perhaps have generated better returns by using a more complex model for calculating expected returns.

Another interesting extension to the study would be to test this tactical allocation with annual re-balancing instead of monthly. As argued by Craig Israelsen (2010), annual rebalancing seems to be the best protocol for a financial investor.

Conclusively we see that the gained information we get by taking into account the macroeconomic environment clearly refines the allocation procedure under the MPT framework. From a risk-return perspective, investors adding macroeconomic variables to their tactical asset allocation model can offer greater risk-return combinations, notably for low return/low risk targets.

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## A Appendix Estimations

## A. 1 Historical Means

Table A.1. Table presenting the 9 year rolling averages of monthly mean log returns

| Dates | 9 year monthly return |  |  |  |
| ---: | :--- | ---: | ---: | ---: |
| Start date | End date | SIX PRX | OMRX Bond | T bill |
| $2008-02-29$ | $2017-01-31$ | $0,71 \%$ | $0,33 \%$ | $0,08 \%$ |
| $2008-03-31$ | $2017-02-28$ | $0,71 \%$ | $0,35 \%$ | $0,07 \%$ |
| $2008-04-30$ | $2017-03-31$ | $0,74 \%$ | $0,34 \%$ | $0,07 \%$ |
| $2008-05-31$ | $2017-04-30$ | $0,74 \%$ | $0,35 \%$ | $0,07 \%$ |
| $2008-06-30$ | $2017-05-31$ | $0,72 \%$ | $0,36 \%$ | $0,06 \%$ |
| $2008-07-31$ | $2017-06-30$ | $0,87 \%$ | $0,36 \%$ | $0,06 \%$ |
| $2008-08-31$ | $2017-07-31$ | $0,84 \%$ | $0,34 \%$ | $0,05 \%$ |
| $2008-09-30$ | $2017-08-31$ | $0,82 \%$ | $0,34 \%$ | $0,05 \%$ |
| $2008-10-31$ | $2017-09-30$ | $1,01 \%$ | $0,33 \%$ | $0,05 \%$ |
| $2008-11-30$ | $2017-10-31$ | $1,22 \%$ | $0,30 \%$ | $0,04 \%$ |
| $2008-12-31$ | $2017-11-30$ | $1,22 \%$ | $0,29 \%$ | $0,04 \%$ |
| $2009-01-31$ | $2017-12-31$ | $1,16 \%$ | $0,26 \%$ | $0,03 \%$ |
| $2009-02-28$ | $2018-01-31$ | $1,24 \%$ | $0,27 \%$ | $0,03 \%$ |

## A. 2 Bekk model estimates

## A.2.1 Investor A

Investor $A$, parameter estimates of matrix $C, A$ and $B$

$$
\begin{gather*}
\boldsymbol{C}=\left[\begin{array}{ccc}
0,0546 & & \\
-0,0015 & 0,0085 & \\
-0,0001 & 0,0006 & 0,0009
\end{array}\right]  \tag{A.1}\\
\boldsymbol{A}
\end{gather*}=\left[\begin{array}{lll}
0,0999 & 0,0996 & 0,0990  \tag{A.2}\\
0,1000 & 0,1000 & 0,1000  \tag{A.3}\\
0,1000 & 0,1000 & 0,1000
\end{array}\right]-\left[\begin{array}{lll}
-0,0051 & -0,0053 & -0,0054 \\
-0,0051 & -0,0053 & -0,0053 \\
-0,0051 & -0,0053 & -0,0053
\end{array}\right] .
$$

Investor $A, t$ statistic of matrix $C, A$ and $B$

$$
\begin{align*}
& \text { t-values for the parameters in } \boldsymbol{C}=\left[\begin{array}{ccc}
30,412 & & \\
-1,986 & 13,969 \\
-0,135 & 3,197 & 18,421
\end{array}\right]  \tag{A.4}\\
& \text { t-values for the parameters in } \boldsymbol{A}=\left[\begin{array}{lll}
3,216 & 0,871 & 4,495 \\
0,147 & 2,472 & 5,582 \\
0,009 & 0,134 & 1,796
\end{array}\right]  \tag{A.5}\\
& \text { t-values for the parameters in } \boldsymbol{B}=\left[\begin{array}{lll}
-2,030 & -0,394 & -1,874 \\
-0,036 & -3,265 & -0,304 \\
-1,033 & -1,401 & -1,954
\end{array}\right] \tag{A.6}
\end{align*}
$$

## A.2.2 Investor B

Investor $B$, parameter estimates of matrix $C, A$ and $B$

$$
\begin{align*}
\boldsymbol{C} & =\left[\begin{array}{cccc}
0,044 & & & \\
0,000 & 0,006 & & \\
0,000 & 0,000 & 0,000 & \\
0,008 & -0,004 & -0,034 & -0,021
\end{array}\right]  \tag{A.7}\\
\boldsymbol{A} & =\left[\begin{array}{cccc}
-0,070 & 0,042 & -0,001 & -0,487 \\
1,531 & -0,271 & 0,016 & 2,016 \\
2,676 & -2,118 & -1,050 & 1,570 \\
-0,097 & -0,006 & 0,002 & 0,603
\end{array}\right]  \tag{A.8}\\
\boldsymbol{B} & =\left[\begin{array}{cccc}
-0,006 & 0,028 & 0,008 & -0,050 \\
-2,211 & -0,265 & -0,023 & 0,166 \\
2,629 & 0,208 & 0,013 & -2,016 \\
0,074 & 0,000 & -0,001 & -0,227
\end{array}\right] \tag{A.9}
\end{align*}
$$

Investor $B$, $t$ statistic of matrix $C, A$ and $B$
t-values for the parameters in $\boldsymbol{C}=\left[\begin{array}{ccrrr}5,619 & & & \\ -0,203 & 5,596 & & \\ -0,371 & -12,224 & 0,186 & \\ 0,460 & -0,483 & -12,231 & -6,216\end{array}\right]$
t-values for the parameters in $\boldsymbol{A}=\left[\begin{array}{cccc}-3,589 & 1,679 & -0,610 & -3,735 \\ 1,734 & -5,872 & 1,054 & 2,209 \\ 0,205 & -2,543 & -6,190 & 0,199 \\ -1,409 & -0,173 & 0,891 & 3,662\end{array}\right]$
t -values for the parameters in $\boldsymbol{B}=\left[\begin{array}{cccc}-4,024 & 1,810 & 3,245 & -0,473 \\ -2,177 & -3,261 & -0,614 & 0,221 \\ 0,518 & 0,514 & 3,119 & -0,756 \\ 0,288 & -0,063 & -0,845 & -2,011\end{array}\right]$

## A.2.3 Investor C

Investor $C$, parameter estimates of matrix $C, A$ and $B$

$$
\begin{array}{cc}
\boldsymbol{C}=\left[\begin{array}{ccccc}
0,0410 & & & \\
-0,0136 & -0,0035 & & & \\
0,0002 & -0,0001 & 0,0013 & & 0,0033 \\
0,0000 & 0,0036 & 0,0001 & \\
-0,0011 & 0,0098 & -0,0143 & -0,0441 & 0,0339
\end{array}\right] \\
\boldsymbol{A}=\left[\begin{array}{ccccc}
0,212188 & 0,069795 & 0,017215 & 0,04577 & -0,2061 \\
1,716601 & 0,436405 & -0,04125 & 0,248986 & 0,670923 \\
0,461565 & -0,91643 & -0,0836 & -1,14787 & 0,319919 \\
-0,32118 & -0,13101 & 0,052992 & -1,85551 & 0,844907 \\
-1,28298 & 0,190471 & -0,01326 & 0,023403 & -0,37688
\end{array}\right]  \tag{A.14}\\
\boldsymbol{B}=\left[\begin{array}{ccccc}
-0,29025 & 0,007705 & -0,00393 & 0,051305 & 0,194701 \\
0,47728 & 0,079119 & -0,03775 & 0,114304 & 1,158644 \\
0,354745 & -0,10254 & -0,19427 & -0,38245 & 0,405342 \\
0,497329 & -0,24222 & -0,02131 & -0,21961 & 0,002724 \\
0,669242 & 0,017128 & 0,004693 & -0,04322 & 0,239951
\end{array}\right]
\end{array}
$$

(A.13)
(A.15)

Investor $C$, $t$ statistic of matrix $C, A$ and $B$
t-values for the parameters in $\boldsymbol{C}=\left[\begin{array}{ccccc}9,471 & & & & \\ -39,467 & -2,261 & & & \\ 0,326 & -0,449 & 11,076 & & 2,919 \\ 0,002 & 3,006 & 0,098 & \\ -0,166 & 1,077 & -1,463 & -3,551 & 4,551\end{array}\right]$
t-values for the parameters in $\boldsymbol{A}=\left[\begin{array}{ccccc}6,146 & 2,945 & 1,736 & 1,266 & -3,199 \\ 1,262 & 5,879 & -1,009 & 2,972 & 2,589 \\ 0,055 & -2,405 & -4,178 & -1,652 & 0,060 \\ -1,073 & -0,272 & 0,689 & -5,899 & 0,385 \\ -14,751 & 3,363 & -2,049 & 1,074 & -3,526\end{array}\right]$
(A.17)
t-values for the parameters in $\boldsymbol{B}=\left[\begin{array}{ccccc}-2,976 & 0,203 & -0,600 & 2,941 & 1,995 \\ 1,204 & 6,220 & -3,237 & 4,278 & 1,928 \\ 0,076 & -0,111 & -4,536 & -1,461 & 0,180 \\ 1,775 & -2,872 & -0,970 & -4,059 & 0,003 \\ 6,538 & 0,561 & 1,735 & -1,712 & 2,093\end{array}\right]$
(A.18)

## B Figures and Tables



Figure B.1. Figure showing historical data of the 10-year period 2008-2018 of the equity index Six PRX.

## B.0.1 Investor B Conditional Variance plots



Figure B.2. Figure showing conditional volatility of the Swedish equity index Six PRX.


Figure B.3. Figure showing conditional volatility of the Swedish Bond index OMRX Bond.


Figure B.4. Figure showing conditional volatility of the Swedish 90 day Treasury bill.

## B.0.2 Investor C Conditional Variance plots



Figure B.5. Figure showing conditional volatility of the Swedish equity index Six PRX.


Figure B.6. Figure showing conditional volatility of the Swedish Bond index OMRX Bond.

## B. 1 Performance tables with weights



Figure B.7. Figure showing conditional volatility of the Swedish 90 day Treasury bill.


Figure B.8. Figure showing the portfolio weights of Equities for each of the investor for the whole testing year. The weights are based on a target portfolio return of $0.30 \%$.


Figure B.9. Figure showing the portfolio weights of Bonds for each of the investor for the whole testing year. The weights are based on a target portfolio return of $0.30 \%$.

Table B.1. Returns and calculated weights in each time period for each investors optimal portfolio.

| Optimal | 2017-03-31 | 2017-04-30 | 2017-05-31 | 2017-06-30 | 2017-07-31 | 2017-08-31 | 2017-09-30 | 2017-10-31 | 2017-11-30 | 2017-12-31 | 2018-01-31 | 2018-02-28 | Totalt 12 mån |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weight Manager A SIX PRX | 0.76\% | 7.70\% | 3.22\% | 7.60\% | 5.34\% | 8.03\% | 8.12\% | 2.13\% | 12.56\% | 11.29\% | 12.15\% | 11.70\% |  |
| Weight Manager B SIX PRX | 0.85\% | 0.00\% | 2.29\% | 0.01\% | 1.84\% | 1.62\% | 0.29\% | 3.10\% | 7.21\% | 4.30\% | 3.23\% | 0.65\% |  |
| Weight Manager C SIX PRX | 0.91\% | 0.23\% | 1.23\% | 0.75\% | 0.70\% | 0.66\% | 1.77\% | 1.75\% | 2.34\% | 2.15\% | 3.29\% | 0.00\% |  |
| Return SIX PRX | 2.12\% | 4.27\% | 1.71\% | -1.98\% | -3.07\% | -0.86\% | 5.45\% | 2.10\% | -3.56\% | -1.27\% | 1.56\% | -0.73\% | 5.426\% |
| Weight Manager A OMRX Bond | 3.51\% | 92.30\% | 36.01\% | 0.00\% | 59.40\% | 91.97\% | 91.88\% | 16.17\% | 87.44\% | 88.71\% | 87.85\% | 88.30\% |  |
| Weight Manager B OMRX Bond | 16.87\% | 2.85\% | 17.92\% | 0.00\% | 19.20\% | 22.61\% | 30.58\% | 28.21\% | 20.83\% | 24.94\% | 31.76\% | 29.53\% |  |
| Weight Manager C OMRX Bond | 7.97\% | 6.82\% | 9.40\% | 5.25\% | 11.44\% | 10.75\% | 10.81\% | 16.00\% | 15.52\% | 21.26\% | 25.82\% | 20.73\% |  |
| Return OMRX Bond | -0.21\% | 0.23\% | 0.40\% | -0.58\% | -0.05\% | 0.33\% | -0.26\% | 0.42\% | 0.30\% | -0.36\% | -0.30\% | 0.40\% | 0.312\% |
| Weight Manager A T-bill | 95.74\% | 0.00\% | 60.77\% | 0.00\% | 35.26\% | 0.00\% | 0.00\% | 81.70\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% |  |
| Weight Manager B T-bill | 82.28\% | 97.15\% | 79.79\% | 91.94\% | 78.97\% | 75.77\% | 69.14\% | 68.70\% | 71.96\% | 70.76\% | 65.00\% | 68.21\% |  |
| Weight Manager C T-bill | 91.12\% | 92.95\% | 89.37\% | 84.56\% | 87.86\% | 88.59\% | 87.42\% | 82.26\% | 82.14\% | 76.59\% | 70.89\% | 75.43\% |  |
| Return T-bill | -0.06\% | -0.06\% | -0.05\% | -0.04\% | -0.07\% | -0.09\% | -0.07\% | -0.05\% | -0.05\% | -0.08\% | -0.12\% | -0.02\% | -0.742\% |
| Total return A | -0.05\% | 0.54\% | 0.17\% | -0.15\% | -0.22\% | 0.24\% | 0.20\% | 0.07\% | -0.19\% | -0.47\% | -0.07\% | 0.27\% | 0.345\% |
| Total return B | -0.07\% | -0.05\% | 0.07\% | -0.04\% | -0.12\% | -0.01\% | -0.11\% | 0.15\% | -0.23\% | -0.20\% | -0.12\% | 0.10\% | -0.618\% |
| Total return C | -0.05\% | -0.03\% | 0.01\% | -0.08\% | -0.08\% | -0.05\% | 0.01\% | 0.06\% | -0.07\% | -0.17\% | -0.11\% | 0.07\% | -0.489\% |

Table B.2. Returns and calculated weights in each time period for each investor, with an expected monthly return of $0.3 \%$.

| 0.30\% | 2017-03-31 | 2017-04-30 | 2017-05-31 | 2017-06-30 | 2017-07-31 | 2017-08-31 | 2017-09-30 | 2017-10-31 | 2017-11-30 | 2017-12-31 | 2018-01-31 | 2018-02-28 | Totalt 12 mån |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weight Manager A SIX PRX | 5.67\% | 5.94\% | 5.74\% | 5.49\% | 5.78\% | 6.12\% | 6.20\% | 6.79\% | 7.88\% | 8.34\% | 9.37\% | 10.27\% |  |
| Weight Manager B SIX PRX | 2.27\% | 0.00\% | 6.59\% | 2.07\% | 5.59\% | 4.91\% | 1.88\% | 6.70\% | 12.54\% | 9.62\% | 8.24\% | 5.47\% |  |
| Weight Manager C SIX PRX | 10.08\% | 9.00\% | 10.18\% | 7.79\% | 7.62\% | 7.72\% | 11.66\% | 8.56\% | 9.58\% | 8.33\% | 10.41\% | 11.78\% |  |
| Return SIX PRX | 2.12\% | 4.27\% | 1.71\% | -1.98\% | -3.07\% | -0.86\% | 5.45\% | 2.10\% | -3.56\% | -1.27\% | 1.56\% | -0.73\% | 5.426\% |
| Weight Manager A OMRX Bond | 69.70\% | 70.54\% | 69.42\% | 68.70\% | 64.79\% | 68.68\% | 70.41\% | 66.19\% | 63.41\% | 65.29\% | 71.30\% | 62.06\% |  |
| Weight Manager B OMRX Bond | 77.67\% | 85.25\% | 67.37\% | 76.41\% | 65.31\% | 71.97\% | 81.95\% | 66.51\% | 42.43\% | 59.28\% | 76.94\% | 86.57\% |  |
| Weight Manager C OMRX Bond | 59.39\% | 62.96\% | 58.75\% | 63.49\% | 59.84\% | 64.33\% | 55.82\% | 60.19\% | 55.75\% | 65.37\% | 66.12\% | 54.29\% |  |
| Return OMRX Bond | -0.21\% | 0.23\% | 0.40\% | -0.58\% | -0.05\% | 0.33\% | -0.26\% | 0.42\% | 0.30\% | -0.36\% | -0.30\% | 0.40\% | 0.312\% |
| Weight Manager A T-bill | 24.63\% | 23.52\% | 24.84\% | 25.81\% | 29.43\% | 25.20\% | 23.39\% | 27.02\% | 28.72\% | 26.37\% | 19.33\% | 27.68\% |  |
| Weight Manager B T-bill | 20.07\% | 14.75\% | 26.03\% | 21.52\% | 29.11\% | 23.12\% | 16.17\% | 26.80\% | 45.02\% | 31.11\% | 14.82\% | 7.96\% |  |
| Weight Manager C T-bill | 30.53\% | 28.05\% | 31.07\% | 28.72\% | 32.55\% | 27.95\% | 32.52\% | 31.25\% | 34.67\% | 26.31\% | 23.47\% | 33.93\% |  |
| Return T-bill | -0.06\% | -0.06\% | -0.05\% | -0.04\% | -0.07\% | -0.09\% | -0.07\% | -0.05\% | -0.05\% | -0.08\% | -0.12\% | -0.02\% | -0.742\% |
| Total return A | -0.04\% | 0.40\% | 0.37\% | -0.52\% | -0.23\% | 0.15\% | 0.14\% | 0.41\% | -0.11\% | -0.37\% | -0.09\% | 0.17\% | 0.282\% |
| Total return B | -0.13\% | 0.19\% | 0.37\% | -0.49\% | -0.22\% | 0.18\% | -0.12\% | 0.41\% | -0.34\% | -0.36\% | -0.12\% | 0.31\% | -0.346\% |
| Total return C | 0.07\% | 0.51\% | 0.40\% | -0.53\% | -0.28\% | 0.12\% | 0.47\% | 0.42\% | -0.19\% | -0.37\% | -0.06\% | 0.12\% | 0.666\% |

Table B.3. Returns and calculated weights in each time period for each investor, with an expected monthly return of $0.3 \%$.

| 0.40\% | 2017-03-31 | 2017-04-30 | 2017-05-31 | 2017-06-30 | 2017-07-31 | 2017-08-31 | 2017-09-30 | 2017-10-31 | 2017-11-30 | 2017-12-31 | 2018-01-31 | 2018-02-28 | Totalt 12 mån |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weight Manager A SIX PRX | 14.72\% | 15.06\% | 13.40\% | 12.06\% | 8.11\% | 11.70\% | 13.09\% | 10.25\% | 10.72\% | 5.35\% | 15.58\% | 13.84\% |  |
| Weight Manager B SIX PRX | 14.72\% | 15.06\% | 13.40\% | 12.06\% | 8.11\% | 11.70\% | 13.09\% | 10.25\% | 17.00\% | 13.19\% | 15.58\% | 13.84\% |  |
| Weight Manager C SIX PRX | 14.72\% | 15.06\% | 14.67\% | 12.06\% | 11.06\% | 11.70\% | 16.49\% | 8.56\% | 13.37\% | 11.99\% | 15.58\% | 16.32\% |  |
| Return SIX PRX | 2.12\% | 4.27\% | 1.71\% | -1.98\% | -3.07\% | -0.86\% | 5.45\% | 2.10\% | -3.56\% | -1.27\% | 1.56\% | -0.73\% | 5.426\% |
| Weight Manager A OMRX Bond | 85.28\% | 84.94\% | 86.60\% | 87.94\% | 91.89\% | 88.30\% | 86.91\% | 39.37\% | 88.73\% | 88.01\% | 84.42\% | 86.16\% |  |
| Weight Manager B OMRX Bond | 85.28\% | 84.94\% | 86.60\% | 87.94\% | 91.89\% | 88.30\% | 86.91\% | 39.74\% | 60.49\% | 82.36\% | 84.42\% | 86.16\% |  |
| Weight Manager C OMRX Bond | 85.28\% | 84.94\% | 83.54\% | 87.94\% | 83.95\% | 88.30\% | 77.82\% | 60.19\% | 76.81\% | 88.01\% | 84.42\% | 73.48\% |  |
| Return OMRX Bond | -0.21\% | 0.23\% | 0.40\% | -0.58\% | -0.05\% | 0.33\% | -0.26\% | 0.42\% | 0.30\% | -0.36\% | -0.30\% | 0.40\% | 0.312\% |
| Weight Manager A T-bill | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 56.34\% | 0.55\% | 0.00\% | 0.00\% | 0.00\% |  |
| Weight Manager B T-bill | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 56.07\% | 22.51\% | 4.45\% | 0.00\% | 0.00\% |  |
| Weight Manager C T-bill | 0.00\% | 0.00\% | 1.79\% | 0.00\% | 4.99\% | 0.00\% | 5.68\% | 31.25\% | 9.82\% | 0.00\% | 0.00\% | 10.20\% |  |
| Return T-bill | -0.06\% | -0.06\% | -0.05\% | -0.04\% | -0.07\% | -0.09\% | -0.07\% | -0.05\% | -0.05\% | -0.08\% | -0.12\% | -0.02\% | -0.742\% |
| Total return A | 0.13\% | 0.84\% | 0.58\% | -0.75\% | -0.29\% | 0.19\% | 0.49\% | 0.35\% | -0.12\% | -0.39\% | -0.01\% | 0.24\% | 1.260\% |
| Total return B | 0.13\% | 0.84\% | 0.58\% | -0.75\% | -0.29\% | 0.19\% | 0.49\% | 0.36\% | -0.44\% | -0.47\% | -0.01\% | 0.24\% | 0.857\% |
| Total return C | 0.13\% | 0.84\% | 0.59\% | -0.75\% | -0.38\% | 0.19\% | 0.69\% | 0.42\% | -0.25\% | -0.47\% | -0.01\% | 0.17\% | 1.156\% |

## C Efficient frontier calculations and plots

## C. 1 Investor A

Table C.1. Table of the first months weight for investor A
Start date

| Sta |
| :--- |
| 2008-02-29 |

End date
2017-01-31
Weights:

Table C.2. Weights, mean, SD and Sharpe's ratio for the first month of investor A

|  | Minimum |  |  | Optimum |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean | $0.08 \%$ | $0.13 \%$ | $0.23 \%$ | $0.20 \%$ | $0.40 \%$ | $0.50 \%$ | $0.70 \%$ |  |
| SD | $0.28 \%$ | $0.34 \%$ | $0.55 \%$ | $0.47 \%$ | $1.12 \%$ | $2.39 \%$ | $5.34 \%$ |  |
| Sharpe | 0.28 | 0.385 | 0.423 | 0.4219 | 0.3571 | 0.2087 | 0.1309 |  |
| SIX PRX | 0.001886 | 0.01759 | 0.04284 | 0.03505 | 0.1749 | 0.44244 | 0.97737 |  |
| OMRX Bond | 0 | 0.16379 | 0.49456 | 0.39242 | 0.8250 | 0.55755 | 0.02262 |  |
| T bill | 0.998113 | 0.81860 | 0.46258 | 0.57252 | 0 | 0 | 0 |  |

This re-balancing procedure would then continue in a similar manner for the entire 12-month testing period, for each investor.

## C. 2 Investor B

Table C.3. Table of the first months weight for investor B

| Start date 2008-02-29 weights | End date2017-01-31 |  |  |  |  | Mean 0.30\% |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0.0600 | 0.7188 | 0.2211 | 0 |  |  |
|  |  | SIX PRX | OMRX Bond | T bill | Inflation |  |  |
| 0.0600 | SIX PRX | 0.2149\% | 0.0152\% | 0.0020\% | 0.0005\% |  |  |
| 0.7188 | OMRX Bond | 0.0152\% | 0.0131\% | 0.0013\% | 0.0010\% | SD | 0.97\% |
| 0.2211 | T bill | 0.0020\% | 0.0013\% | 0.0004\% | 0.0007\% | Sharpe | 0.31 |
| 0 | Inflation | 0.0005\% | 0.0010\% | 0.0007\% | 0.0033\% |  |  |
| 1.000000007 |  | 0.0015\% | 0.0076\% | 0.0003\% | 0.0000\% |  |  |

Table C.4. Weights, mean, SD and Sharpe's ratio for the first month of investor B

|  | Minimum |  |  | Optimum |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mean | $0.08 \%$ | $0.30 \%$ | $0.09 \%$ | $0.20 \%$ | $0.40 \%$ | $0.50 \%$ | $0.70 \%$ |  |  |  |  |  |  |  |  |
| SD | $0.19 \%$ | $0.97 \%$ | $0.22 \%$ | $0.59 \%$ | $1.41 \%$ | $2.32 \%$ | $4.54 \%$ |  |  |  |  |  |  |  |  |
| Sharpe | 0.40 | 0.31 | 0.41 | 0.34 | 0.28 | 0.22 | 0.15 |  |  |  |  |  |  |  |  |
| SIX PRX | 0.000 | 0.060 | 0.006 | 0.034 | 0.175 | 0.442 | 0.977 |  |  |  |  |  |  |  |  |
| OMRX Bond | 0.000 | 0.719 | 0.038 | 0.394 | 0.825 | 0.558 | 0.023 |  |  |  |  |  |  |  |  |
| T bill | 1.000 | 0.221 | 0.956 | 0.572 | 0.000 | 0.000 | 0.000 |  |  |  |  |  |  |  |  |
| Inflation | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |  |  |  |  |  |  |  |  |

This re-balancing procedure would then continue in a similar manner for the entire 12 -month testing period, for each investor.

## C. 3 Investor C

Table C.5. Table of the first months weight for investor B

| Start date 2008-02-29 | End date2017-01-31 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weights |  | 0.0133 | 0.0742 | 0.9125 | 0.0000 | 0.0000 |
|  |  | SIX PRX | OMRX Bond | T bill | Inflation | PMI |
| 0.0133 | SIX PRX | 0.77\% | -0.12\% | 0.00\% | 0.03\% | 0.20\% |
| 0.0742 | OMRX Bond | -0.12\% | 0.03\% | 0.00\% | -0.01\% | -0.02\% |
| 0.9125 | T bill | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% |
|  | Inflation | 0.03\% | -0.01\% | 0.00\% | 0.01\% | 0.00\% |
| 0.0000 | PMI | 0.20\% | -0.02\% | 0.00\% | 0.00\% | 0.44\% |
| 1 |  | 0.00\% | 0.00\% | 0.00\% | 0.00\% | 0.00\% |


| Mean | $0.10 \%$ |
| :--- | :--- |
| SD | $0.17 \%$ |
| Sharpe | 0.63 |

The Re-balancing occurs every time period and new weights and Sharpe ratios are calculated for each portfolio. In addition, a plot of the efficient frontiers is generated for every time period. As an example, see plot C. 1 below, which shows the plot from 2017-04-30.

Table C.6. Weights, mean, SD and Sharpe's ratio for the first month of investor C

|  | Minimum |  |  | Optimum |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean | $0.08 \%$ | $0.20 \%$ | $0.10 \%$ | $0.30 \%$ | $0.40 \%$ | $0.50 \%$ | $0.70 \%$ | $0.13 \%$ |
| SD | $0.14 \%$ | $0.42 \%$ | $0.17 \%$ | $0.73 \%$ | $1.03 \%$ | $2.77 \%$ | $7.49 \%$ | $0.22 \%$ |
| Sharpe | 0.57 | 0.47 | 0.63 | 0.41 | 0.39 | 0.18 | 0.09 | 0.59 |
| SIX PRX | 0.000 | 0.055 | 0.009 | 0.101 | 0.147 | 0.422 | 0.970 | 0.022 |
| OMRX Bond | 0.023 | 0.335 | 0.080 | 0.594 | 0.853 | 0.578 | 0.030 | 0.153 |
| T bill | 0.977 | 0.611 | 0.911 | 0.305 | 0.000 | 0.000 | 0.000 | 0.825 |
| Inflation |  |  |  |  |  |  |  |  |
| PMI | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |



Figure C.1. Figure showing Efficient frontier generated 2018-04-30

## D Additional Models and Measurements

## D. 1 Tracking Error

In practice, a portfolio that adopts tactical asset allocation is usually compared to a passive benchmark portfolio. If the tactical asset allocation strategy outperforms the benchmark portfolio, one talks about the manager has successfully generated alpha. This is defined as

$$
\alpha_{t}=R_{p, t}-R_{B M, t}
$$

where $R_{p, t}$ is the return of the portfolio that is adopting tactical asset allocation at time $t$, and $R_{B M, t}$ is the return of the benchmark at time $t$. The volatility of alpha is called tracking error and is defined as

$$
\begin{equation*}
T E_{t}=\sqrt{\frac{1}{T-1} \sum_{i=1}^{T}\left(\alpha_{t}-\frac{1}{T} \sum_{i=1}^{T} \alpha_{t}\right)^{2}} \tag{D.1}
\end{equation*}
$$

The performance of an investment strategy can be measured by the information ratio, which is the ratio between alpha and tracking error. Investors seek to obtain as high information ratio as possible ${ }^{1}$.

## D.1.1 Exponentially weighted Moving Average Model

Volatility is a measure of risk and can be estimated using various methods. We will now introduce another improvement on the simple volatility calculation, namely the Exponentially weighted Moving Average (EWMA). If we look at the simple unweighted approach of estimating volatility we see that this approach gives all return the same weight. This works under the assumption that the covariance between assets is constant through time. We know, however, that covariances tend to increase during periods of market stress and decrease during periods of normality. This problem can be addressed by using the EWMA, in which the more recent returns obtain a greater weight than past returns. It introduces a parameter $\lambda$, called the smoothing parameter, to the model. It results in the following expression for $\Sigma_{t}$ :

$$
\begin{equation*}
\boldsymbol{\Sigma}_{\boldsymbol{t}}=\lambda \boldsymbol{\Sigma}_{\boldsymbol{t}-\mathbf{1}}+(1-\lambda) \boldsymbol{\epsilon}_{\boldsymbol{t}-\mathbf{1}} \boldsymbol{\epsilon}_{\boldsymbol{t}-\mathbf{1}}^{\boldsymbol{T}} \tag{D.2}
\end{equation*}
$$

where $0<\lambda<1$. This formula states that the variance at time $t$ is a function of $\lambda$ multiplied by the variance on the day before plus $(1-\lambda)$ multiplied by the squared return on the day before. This recursion incorporates the entire infinite series that proceeded time $t$. Notice that the sum of the weights always equals one. The parameter $\lambda$ is either

[^19]fixed or can be estimated by using QMLE. Hull \& White (1998) use $\lambda=0.94$ in their study on Value at Risk when daily changes in the market variables are not Normally distributed. In many financial applications the estimates $\lambda=0.94$ or $\lambda=0.96$ tend to be used.


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[^1]:    ${ }^{1}$ Brealey \& Myers (2003), pp. 223.
    ${ }^{2}$ Flavin \& Wickens (2003), pp. 3

[^2]:    ${ }^{1}$ Schwert (1989) showed that asset return volatility can change drastically over time, and that aggregate leverage is significantly correlated with volatility.

[^3]:    ${ }^{2}$ A low correlation corrensponds to a correlation close to 0 .
    ${ }^{3}$ Maximizing the Sharp ratio for a given level of expected return is obtained by minimizing the volatility for that lever of expected return. This creates a portfolio located on the efficient frontier.

[^4]:    ${ }^{4}$ Note that Lee (2000) does allow for short selling, while in our analysis we exclude that option for the investment strategies later to be tested.

[^5]:    ${ }^{5}$ For complete derivation and solution of the Markowits problem, see Luenberger et al. (1997)
    ${ }^{6}$ Lecture notes by UON (2016) in Applied Portfolio Management, lecture on tactical asset allocation.

[^6]:    ${ }^{7}$ see Stockton \& Shtekhman (2010)
    ${ }^{8}$ see Kosakowski (2017)

[^7]:    ${ }^{9}$ Flavin \& Wickens (2003). Note that the original study was performed in 1998 , but later published in 2003 in the Review of Financial Economics. The quote is from the 2001 update of the study.

[^8]:    ${ }^{1}$ see Tsay (2005), page 389-391.

[^9]:    ${ }^{2}$ see Rossi (2010) for complete lecture on Multivariate Volatility Models

[^10]:    ${ }^{3}$ Markus Andersson (2018) used a EMWA model to test if macroeconomic variables could improve Tactical asset allocation in the Swedish stock market.

[^11]:    ${ }^{4}$ A statistical model $\boldsymbol{P}$ is identifiable if the mapping $\theta \mapsto \boldsymbol{P}$ is one-to-one. i.e. distinct values of $\theta$ correspond to distinct probability distributions.

[^12]:    ${ }^{5}$ see Comte \& Lieberman (2003), page 61-84.

[^13]:    ${ }^{6}$ see Comte \& Lieberman (2003), pages 61-84.
    ${ }^{7}$ proven with the help of Basawa et al. (1976).

[^14]:    ${ }^{1}$ Irving Fisher's theory on the nominal interest rate suggests that the real interest rate equals the nominal interest rate minus the expected inflation rate. The theory of rational expectations also suggests a positive relation between inflation and interest rates of returns.
    ${ }^{2}$ see Fama \& Schwert (1977)

[^15]:    ${ }^{3}$ Mercer Data Bank is a large collection of financial data, provided by Mercer Sweden

[^16]:    ${ }^{4}$ Procedure suggested by Markus Andersson (2018) in an interview on tactical asset allocation modeling in R based on macroeconomic variables. Markus performed a similar study in 2015, where he investigated if there was a connection between the components in the macroeconomic environment and portfolios consisting of equities from OMX Stockholm 30.

[^17]:    ${ }^{1}$ This is consistent with an article published by the Federal Reserve of San Fransisco in the Dr. ECON (December 2000) educational section, titled "What makes Treasury bill rates rise and fall? What effect does the economy have on T-Bill rates?"

[^18]:    ${ }^{1}$ Mercer data bank.

[^19]:    ${ }^{1}$ Lee (2000)

