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Optimal premium strategies using reinforcement learning in push-pull competition between two insurance companies

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Abstract

How to set the premiums in an appropriate way have been studied ex- tensively by both practitioners and academics during history. The stan- dard approach in literature is to set the premium level according to a model of the expected loss and then adding a safety loading which is related to distributional properties of the risk. In this study, we take another approach and examine the competition between two insurance companies and analyze different versions of the push and pull game between two insurance companies I_1 and I_2 that have initial capital reserves $R_1(0) > R_2(0)$ (one larger than the other). In the game, the larger company aims to maximize the reserve difference $R_1(t) - R_2(t)$ (push the smaller company away) and the smaller company aims to minimize the same (pull closer to the larger company) by using premiums as controls. Using the results in Asmussen et. al (2019) we analytically derive the Nash-equilibrium premiums in the considered games, when taking market frictions $H \sim beta(a, b)$ into consideration. Later on, we implement dynamic programming and reinforcement learning methods to solve the control problem when assuming a simplified state space (reserve difference) and action space (premium levels). The results in this game show that the optimal premium derived with DP and RL is in agreement with the analytical Nash equilibrium solution. The game is later on extended to a case where the market frictions depend on the number of customers currently insured at respective company. Again, a form of analytical result is computed and lastly, the game is analyzed using DP and RL. The results in this game show that if the reserve difference is large, the larger company benefits from offering a lower premium to gain a market advantage. However, when the reserve difference is low, the company can not afford to lower the premium because of the risk of losing the game. The results in the game also indicate that when the company have gained market advantage it benefits from charging higher premiums. This is also in accordance with the analytical results. To summarize, the results show that it is indeed possible to solve and gain insights of the considered optimal control problems using DP and RL. However, in order to use these methods in a real-insurance context, the specific and simplified settings considered in this thesis would need to be evolved and extended in several ways.

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