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Large Dimensional Analysis of Covariance Matrix Estimators in the Framework of General Minimum Variance Portfolio (GMVP) based on Random Matrix Theory (RMT)

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Abstract

The General Minimum Variance Portfolio (GMVP) has the smallest variance of all the portfolios. The weight of this portfolio depends on the inverse of the population covariance matrix, which is an unknown object in practice and must be replaced by an estimator. Several estimators of the GMV portfolio weights exist in the literature. The preliminary aim of the thesis is their comparison with respect to the out-of-sample performance and their asymptotic behaviors based on Random Matrix Theory (RMT). For numerical simulations synthetic and real data are used. Asymptotic behaviors are analyzed when the number of assets p and the sample size n are going together to infinity at the same convergence rate p/n, which is called in the literature as double-limit regime or high-dimensional asymptotic. The different estimators we are interested in are based on the Sample Covariance Matrix (SCM) and Tyler's robust M-estimator in non-regularized and regularized (shrinkage) forms. The following four approaches are considered: 1-Frahm and Memmel (2010) [2] They treat the case of the linear shrinkage estimator based on the sample covariance estimator and a non-random target under the assumption of serially independent and identically normally distributed asset returns. 2-Bodnar et al. (2018) [1] They improve the estimator of Frahm and Memmel and suggest shrinking the sample estimator for the portfolio weights directly and not the whole sample covariance matrix, which is dominant but not necessarily optimal. A new estimator for the GMV portfolio based on Random Matrix Theory is derived, which is optimal and distribution-free. 3-Rubio et al. (2012) [3] They regularize the SCM estimator, where the shrinkage target is a nonrandom positive-definite matrix. They invert the sum of the SCM estimator and the target matrix and in GMVP implementation find the portfolio weights. To find the minimum realized variance the shrinkage intensity is optimized. 4-Yang et al. (2015) [4] They are in principle following the same procedure as in Rubio et al. using instead for the SCM estimator the Tyler´s M-estimator.

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