STOCKHOLMS UNIVERSITET
MATEMATISKA INSTITUTIONEN
Avd. Matematisk statistik

MT5011 – Part TEOR
EXAM
May 27, 2020

Exam in Basic Insurance Mathematics, 7.5 credits

May 27, 2020 – time: 9–17

Examiner: Mathias Lindholm, `lindholm@math.su.se`

Additional tools and material: Anything you like as long as you do not discuss the exam with anyone!

Return of the exam: Online.

Each correctly solved problem is worth 10 points. All arguments must be clear and easy to follow.

The grades A–E are set according to the following minimum point levels:

Grade	A	B	C	D	E
Points	43	38	33	28	23

Additional information

- If something is unclear or if you experience problems during the exam, please notify me as soon as possible by sending an e-mail to `lindholm@math.su.se`
- To ask questions during the exam you send an e-mail to `lindholm@math.su.se` with the subject "Exam MT5011" together with a Zoom meeting ID.
- I will at least check my e-mail at 10.00, 12.00, 14.00, and 16.00, and will get back to you as soon as I can.
- The exam is supposed to be as close as possible to an ordinary campus exam and you are **not** asked to write thesis type answers.

- **Important:** If I need to get in touch with you during the exam I will use the news forum on the course home page, so please check this regularly.

Problem 0

The following text **must** be written on a separate sheet and handed in together with the solutions:

"I, the author of this document, hereby guarantee that I have produced these solutions to this home exam without the assistance of any other person (except the examiner). This means that I have for example not discussed the solutions or the home exam with any other person (except the examiner)."

Problem 1

Consider the following claims triangle with *incremental claim amounts*:

	1	2	3
1	1523	2320	187
2	785	921	
3	343		

a) Estimate the total expected outstanding claim costs for the relevant accident years using the Chain Ladder model.

b) Estimate the expected outstanding incremental claim amounts for relevant combinations of accident years and development years.

Problem 2

Consider the pure endowment insurance for a today 65 year old individual, which pays 1 unit of money if the individual is alive at its 67th birthday.

a) Calculate the discounted fair price of this contract using the following information (x = age)

x	$1\,000q_x$
64	4.235
65	6.145
66	6.523
67	6.856
68	7.012

and the continuously compounded spot rates $r_1 = 0.012, r_2 = 0.015, r_3 = 0.018, r_4 = 0.020$, and $r_5 = 0.021$, where r_i corresponds to the spot rate i years from today.

b) Calculate the variance of the discounted total payment.

Problem 3

Assume that you are an insuring a line of business which has two types of claims, where one type is rare and expensive ("large"), and the other type is common, but less expensive ("small"). Let N denote the total number of claims during next year and assume that $N \sim \mathrm{Po}(\lambda), \lambda > 0$, and let p denote the probability that a particular claim is a large claim, and that small and large claims occur independently. Further, let $X_j^{(i)}$ denote the size of the jth claim of type $i = 0, 1$, where $i = 0$ corresponds to a small claim, and $i = 1$ corresponds to a large claim. Assume that all $X_j^{(i)}$ are i.i.d. for fixed i and assume that all $X_j^{(0)}$s and $X_j^{(1)}$s are independent, and independent of N.

a) Calculate the mean and variance of the total claim cost for claims of both types that occur during next year.

b) Calculate the mean and variance of the total claim cost for large claims that occur during next year.

Problem 4

Let

$$A(x) = \frac{a}{b}(\mathrm{e}^{bx} - 1), \; x > 0,$$

and calculate the corresponding

a) density,

b) survival function.

Argue for why the distribution is an Exponential distribution or not. Clearly state the conditions on a and b for the implied mortality rate function to be well defined.

Problem 5

Consider an insurance company with two lines of business, with n_1 i.i.d. contracts of type 1, and n_2 i.i.d. contracts of type 2. Calculate

a) Value-at-Risk at the level p for each line of business separately when assuming no discounting and that the one-year losses are normally distributed (both lines of business),

b) use the result from **a)** to calculate the total solvency capital requirement for both lines of business, when the correlation between the two lines of business is ρ. What happens if you double the insured amount? What happens if you double the number of contracts?

Good luck!