| STOCKHOLMS UNIVERSITET |
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| MATEMATISKA INSTITUTIONEN |
| Avd. Matematisk statistik |

MT5011 – Part TEOR EXAM August 21, 2020

Exam in Basic Insurance Mathematics, 7.5 credits

August 21, 2020 – time: 9–17

Examiner: Mathias Lindholm, lindholm@math.su.se

Additional tools and material: Anything you like as long as you do not discuss the exam with anyone!

Return of the exam: Online.

Each correctly solved problem is worth 10 points. All arguments must be clear and easy to follow.

The grades A–E are set according to the following minimum point levels:

| Grade | А | В | С | D | Е |
|--------|----|----|----|----|----|
| Points | 43 | 38 | 33 | 28 | 23 |

Additional information

- IT IS ONLY POSSIBLE TO REPORT YOUR RESULTS IF YOU ARE PROPERLY REGISTERED FOR THE EXAM!!
- If something is unclear or if you experience problems during the exam, please notify me as soon as possible by sending an e-mail to lindholm@math.su.se
- To ask questions during the exam you send an e-mail to lindholm@math.su.se with the subject "Exam MT5011" together with a Zoom meeting ID.
- I will at least check my e-mail at 10.00, 12.00, 14.00, and 16.00, and will get back to you as soon as I can.

- The exam is supposed to be as close as possible to an ordinary campus exam and you are **not** asked to write thesis type answers.
- Important: If I need to get in touch with you during the exam I will use the news forum on the course home page, so please check this regularly.

Problem 0

The following text **must** be written on a separate sheet and handed in together with the solutions:

"I, the author of this document, hereby guarantee that I have produced these solutions to this home exam without the assistance of any other person (except the examiner). This means that I have for example not discussed the solutions or the home exam with any other person (except the examiner)."

Problem 1

Assume that you have n independent contracts that can have at most one claim during next year, where all individual claim costs are i.i.d. and independent of the total number of claims.

a) Calculate the mean total cost for all contracts for next year.

b) Calculate the variance of the total cost for all contracts for next year.

c) Calculate the standard deviation of the total cost for all contracts for next year if you instead assume that there are two types of claims that occur independently, when assuming that there can be at most one claim of each type per contract (max. two claims per contract during next year).

Problem 2

Consider the following claims triangle with *incremental claim amounts*:

| | 1 | 2 | 3 |
|---|------|------|----|
| 1 | 1430 | 2020 | 50 |
| 2 | 681 | 1005 | |
| 3 | 259 | | |

a) Calculate relevant reserves using the chain ladder method.

b) Calculate the discounted cash flow using the continuously compounded spot rates $r_1 = 0.012, r_2 = 0.014$, and $r_3 = 0.017$.

c) Starting from the incremental claim amounts during the first development year, calculate all remaining expected incremental claim amounts using the chain ladder method. Note that this includes parts of the claims triangle that has been observed!

Problem 3

Use the following life table

| x | $1 000 q_x$ |
|----|--------------|
| 64 | 4.235 |
| 65 | 6.145 |
| 66 | 6.523 |
| 67 | 6.856 |
| 68 | 7.012 |

to calculate the single fair premium for a life insurance contract for a today 65 year old individual which pays 0.5 units of money if the individual dies before turning 67, and which pays 1 unit if the individual survives until its 67th birthday. No discounting.

Problem 4

Assume that you have received 10 000 units of money, i.e. premium income, to cover all of next years costs for n i.i.d. contracts, where the mean cost

per contract is 0.1 units of money, and where the standard deviation of the cost per contract is 0.03. Determine the approximate size of n so that the probability that the total claim cost for next year will not exceed your premium income is 0.05.

Problem 5

Re-use the setup from Problem 4 and calculate

a) the total fair premium for all contracts (if you have not solved Problem 4, use n = 5 000),

b) the total premium income for all contracts if the standard deviation principle is used, and determine the additional safety loading needed to match the premium income from Problem 4 (if you have not solved Problem 4, use $n = 5\ 000$ and the alternative premium income 500).

Good luck!