STOCKHOLMS UNIVERSITET
MATEMATISKA INSTITUTIONEN
Avd. Matematisk statistik

MT5011 – Part TEOR
EXAM
August 20, 2021

Exam in Basic Insurance Mathematics, 7.5 credits

August 20, 2021 – time: 9–17

Examiner: Mathias Lindholm, `lindholm@math.su.se`
Additional tools and material: Anything you like as long as you do not discuss the exam with anyone!
Return of the exam: Online, see info on the course homepage.

Each correctly solved problem is worth 10 points. All arguments must be clear and easy to follow.

The grades A–E are set according to the following minimum point levels:

Grade	A	B	C	D	E
Points	43	38	33	28	23

Additional information

- IT IS ONLY POSSIBLE TO REPORT YOUR RESULTS IF YOU ARE PROPERLY REGISTERED FOR THE EXAM!!
- THE NAME OF YOUR FILE WITH SOLUTIONS SHALL BE THE SAME AS YOUR ANNONYMISATION CODE FOR THE EXAM
- If something is unclear or if you experience problems during the exam, please notify me as soon as possible by sending an e-mail to `lindholm@math.su.se`
- To ask questions during the exam you send an e-mail to `lindholm@math.su.se` with the subject "Exam MT5011" together with a Zoom meeting ID.

- I will at least check my e-mail at 10.00, 12.00, 14.00, and 16.00, and will get back to you as soon as I can.
- The exam is supposed to be as close as possible to an ordinary campus exam and you are **not** asked to write thesis type answers.
- **Important:** If I need to get in touch with you during the exam I will use the news forum on the course home page, so please check this regularly.

Problem 0

The following text **must** be written on a separate sheet and handed in together with the solutions:

"I, the author of this document, hereby guarantee that I have produced these solutions to this home exam without the assistance of any other person (except the examiner). This means that I have for example not discussed the solutions or the home exam with any other person (except the examiner)."

Problem 1

Consider the following observed cumulative claim amounts corresponding to the diagonal in a claims triangle: $c_{1,3} = 132, c_{2,2} = 224$, and $c_{3,1} = 206$.

a) Use Mack's chain-ladder model to calculate the ultimo claim cost for relevant accident years given the estimates $\widehat{f}_1 = 1.29$ and $\widehat{f}_2 = 1.04$.

b) By using Mack's chain-ladder model together with the above development factor estimates, estimate the historical incremental payments.

Problem 2

Let $D \sim \text{Po}(\lambda n)$ denote the number of deaths during next year in a population consisting of n i.i.d. individuals.

a) Calculate the probability of survival for a single individual and discuss potential problems with the definition of D.

b) Determine the hazard rate.

Problem 3

Consider a portfolio of insurance contracts where there are 1 000 i.i.d. contracts of type 1, and 1 000 i.i.d. contracts of type 2, and let $X_i^{(j)}$ denote the cost of contract $i, i = 1, \ldots, 1\ 000$, of type $j, j = 1, 2$. Further, assume that

$$\mu^{(1)} := \mathbb{E}[X_i^{(1)}] = 1, \quad \text{and } \sigma^{(1)} = \sqrt{\mathrm{Var}(X_i^{(1)})} = 0.2,$$

and

$$\mu^{(2)} := \mathbb{E}[X_i^{(2)}] = 1.1, \quad \text{and } \sigma^{(2)} = \sqrt{\mathrm{Var}(X_i^{(2)})} = 0.1.$$

a) Calculate the premium for a single type $j, j = 1, 2$, contract using the standard deviation principle when assuming that all contracts are independent.

b) With how much will the total standard deviation premium amount differ from the total fair premium amount?

Problem 4

Consider an insurance company with n insured that have bought insurance of two types. Assume that the insured only can have at most one claim of each type per year, where all claims are assumed to be independent across insured as well as within an insured. Further assume that all claims are i.i.d. within the type of insurance.

a) If each type 1 claim has a Poisson number of claim payments with mean 2 whose payments are i.i.d. with mean 0.5 and standard deviation 0.1, and each type 2 claim has a deterministic payment of size 1, calculate the expected cost for all insured during next year.

a) Calculate the standard deviation of the cost for all insured during next year.

Problem 5

Assume that you have the following estimates of the cumulative hazard rate for ages $x = 64, \ldots, 68$.

x	$\widehat{A}(x)$
64	3.912
65	3.916
66	3.923
67	3.929
68	3.936

Based on the estimates of the cumulative hazard rate, calculate

a) an estimate of the probability of a today 64 year old individual dying within the next year,

b) the expected (undiscounted) cost for a single term insurance contract for a today 64 year old individual that has its last possible payment if dying before its 66th birthday.

Good luck!