STOCKHOLMS UNIVERSITET
MATEMATISKA INSTITUTIONEN
Avd. Matematisk statistik

MT5011 – Part TEOR
EXAM
May 31, 2022

Exam in Basic Insurance Mathematics, 7.5 credits

May 31, 2022 – time: 14–19

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Additional tools and material: Pocket calculator supplied by the department.
Return of the exam: Within three weeks from the exam. Specific date will be notified via the course forum.

Each correctly solved problem is worth 10 points. All arguments must be clear and easy to follow.

The grades A–E are set according to the following minimum point levels:

Grade	A	B	C	D	E
Points	43	38	33	28	23

Problem 1

Consider the following cumulative claims triangle amounts:

	1	2	3
1	–	–	132
2	–	224	**232.96**
3	206	**265.74**	**276.37**

where values in bold face are predictions corresponding to expected values, where the diagonal values are the latest observed cumulative amounts.

a) Calculate the expected cash flow implied by the lower triangle with bold face values.

b) If you assume that the bold face values have been calculated using Mack's chain-ladder model, determine the development factors.

c) Using part **b)**, estimate the incremental historical amounts corresponding to "−" in the table above.

Problem 2

Let $N_i \mid Z_i \sim \text{Pois}(Z_i)$ denote the number of accidents of type $i, i = 0, 1$, during a year for a typical contract, where Z_i is such that $\mathbb{E}[Z_i] = \lambda_i$ and $\text{Var}(Z_i) = \sigma_i^2$. The Z_is are assumed to be independent and the N_is are assumed to be independent.

a) Calculate the mean and variance of $N := N_0 + N_1$.

b) If you introduce $W \sim \text{Be}(p)$, W being independent of all other random variables, calculate the mean and variance of

$$N := WN_0 + (1 - W)N_1.$$

(In this situation you can think of W as an indicator for e.g. a "good year" or a "bad year", and N is a so-called finite mixture distribution.)

Hint: Use that for arbitrary random variables Y and Z with finite second moments it holds that $\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y \mid Z]]$ and $\text{Var}(Y) = \mathbb{E}[\text{Var}(Y \mid Z)] + \text{Var}(\mathbb{E}[Y \mid Z])$.

Problem 3

Let $T \geq 0$ denote a survival time defined by the survival function

$$S(t) = \exp\{-\beta t\}$$

and calculate the

a) density function of T,

b) probability $\mathbb{P}(T \leq t \mid T > s), s \leq t$.

c) Given that $f(t) = \alpha(t)S(t)$, where $\alpha(t)$ is the intensity function, show that $S(t) = \exp\{-\int_0^t \alpha(u)\mathrm{d}u\}$.

Problem 4

Consider the following life table

x	$1\ 000 q_x$
64	4.235
65	6.145
66	6.523
67	6.856
68	7.012

and consider an endowment insurance paying at most 1 SEK if the insured individual survives it's 67th birthday and calculate the fair

a) single premium paid on the 64th birthday,

b) annual premia paid up until the individuals 66th birthday, given that the individual is alive.

All calculations ignore discounting.

Problem 5

The perhaps most commonly used risk measure is value-at-risk: Let $X := A - L$, where A corresponds to a random asset value, and L corresponds to random liability value one year from today, and let $d(0,1)$ be a deterministic discount factor, value-at-risk at the $100p\%$-level is defined according to

$$\mathrm{VaR}_p(X) := F^{-1}_{-d(0,1)X}(1-p),$$

if X is a continuous random variable.

If we assume that $X \sim \mathrm{N}(\mu, \sigma^2)$ it follows that $X \overset{d}{=} \mu + \sigma Z$, where $Z \sim \mathrm{N}(0,1)$, express $\mathrm{VaR}_p(X)$ in terms of a suitable percentile of Z.

Good luck!