STOCKHOLMS UNIVERSITET
MATEMATISKA INSTITUTIONEN
Avd. Matematisk statistik

MT5011 – Part TEOR
EXAM
May 24, 2023

Exam in Basic Insurance Mathematics, 7.5 credits

May 24, 2023 – time: 14–19

Examiner: Mathias Lindholm, `lindholm@math.su.se`
Additional tools and material: Pocket calculator supplied by the department.
Return of the exam: Within three weeks from the exam. Specific date will be notified via the course forum.

Each correctly solved problem is worth 10 points. All arguments must be clear and easy to follow.

The grades A–E are set according to the following minimum point levels:

Grade	A	B	C	D	E
Points	43	38	33	28	23

Problem 1

Consider the following incremental yearly payments:

	1	2	3
1	37	12	2
2	48	5	–
3	72	–	–

Calculate

a) the total reserve,

b) the expected discounted cash flow using the continuously compounded spot rates given by

$r_1 = 0.04, r_2 = 0.045, r_3 = 0.052, r_4 = 0.06, r_5 = 0.061, r_6 = 0.062, r_7 = 0.0623$

N.B. The above interest rates will also be used in problems below.

Problem 2

Derive expressions in terms of T_0, the remaining life of a today 0 year old individual, for the following quantities:

a) $\mathbb{P}(T_{48} \in [2,3] \mid T_{48} > 1)$,

b) $\mathbb{P}(T_{48} > 5 \mid T_{48} > 2)$,

and

c) give interpretations in words of **a)** and **b)**.

Problem 3

Consider the following life table

x	$1\ 000 q_x$
64	4.235
65	6.145
66	6.523
67	6.856
68	7.012

and consider an insurance that pays 100 000 SEK to the insured's family if the insured dies no later than on its 67th birthday, and where the insured receives 40 000 SEK if the insured is alive at its 67th birthday.

If the insured's age today is 64, calculate
a) the expected discounted cash flow using the interest rates from Problem 1 and the life table above,

b) the fair single premium paid at the time point when the insured turns 64 years old.

Problem 4

We consider a non-life insurance company with two lines of business (LoBs), where the total number of claims from LoB $i, i = 1, 2$, during a year follows a Poisson-distribution with intensity $w^{(i)}\lambda^{(i)}$, where $w^{(i)} > 0$ corresponds to the number of contracts in LoB i, and where

$$\lambda^{(1)} = 0.05, \text{ and } \lambda^{(2)} = 0.10,$$

and where all claim sizes for each individual LoB are i.i.d. and all claim sizes and number of claims are independent. Further, let $\mu^{(i)}$ denote the expected claim size for LoB i and let $\theta^{(i)}$ denote the corresponding variance.

a) If $\mu^{(1)} = 2000, \theta^{(1)} = 40\ 000$, and $\mu^{(2)} = 1000, \theta^{(2)} = 90\ 000$. Calculate the coefficient of variation for each claim type and compare – what conclusions can you draw? Argue clearly.

(Recall that the coefficient of variation is given by the ratio of the standard deviation and the expected value.)

b) In addition to the information from part **a)**, assume that $w^{(1)} = 10\ 000$ and $w^{(2)} = 5\ 000$, and calculate the total expected un-discounted claim cost for the insurance company, together with the corresponding variance.

Hint: Use that for arbitrary random variables Y and Z with finite second moments it holds that $\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y \mid Z]]$ and $\mathrm{Var}(Y) = \mathbb{E}[\mathrm{Var}(Y \mid Z)] + \mathrm{Var}(\mathbb{E}[Y \mid Z])$.

c) Can the total un-discounted claim cost be expressed as a compound Poisson-distribution? If yes, argue why. If no, describe what you need to change in the dynamics in order for the total claim cost to be compound Poisson.

Problem 5

Assume that your assets one year from today, A, are $\mathrm{N}(\mu_A, \sigma_A^2)$-distributed, and that your liabilities one year from today, L, are $\mathrm{N}(\mu_L, \sigma_L^2)$-distributed, and let $X = A - L$.

a) Express the one-year value-at-risk of X at the 95%-level, i.e.

$$\mathrm{VaR}_p(X) := F^{-1}_{-d(0,1)X}(1 - p),$$

for where $1 - p$ is the percentile level, in terms of a function of a suitable percentile of $Z \sim \mathrm{N}(0, 1)$.

b) Assume that the current liability value, i.e. at time 0, is given by $l_0 =$ 1 000, and that the corresponding asset value is given by $a_0 = 1\ 500$. If we further assume that $\mathrm{VaR}_p(X - (a_0 - l_0)/d(0, 1)) = 100$, where $d(0, 1)$ is the one-year discount factor based on the interest rates from Problem 1, is it then true that

$$\mathrm{VaR}_p(X) \leq 0?$$

Argue clearly.

c) What does the result from **b)** tell you about your initial position $a_0 - l_0$, when you measure the one-year risk in terms of VaR? Argue clearly.

Good luck!