MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examiner: Oliver Petersen

Written exam
MM5026 VT25
Ordinary differential equations
May 28, 2025

No calculator, book or notes are allowed. Give complete justifications for your answers!
At least 14 points (including bonus) are needed in order to proceed to the ***voluntary oral exam****.*

1. (4 points) Consider the ODE

$$\begin{aligned} x'(t) &= x(t)^2, \\ x(t_0) &= c, \end{aligned}$$

where $c \in \mathbb{R}$ is any constant.

(a) Solve the ODE.

(b) What is the largest interval containing t_0 on which there is a continuously differentiable solution $x(t)$?

2. (4 points) Solve the system of ODE

$$\begin{aligned} \mathbf{x}'(t) &= \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \\ \mathbf{x}(0) &= \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \end{aligned}$$

3. (4 points) Consider the ODE

$$xy''(x) + \frac{1}{2}y'(x) + y(x) = 0.$$

(a) Find the general solution to this ODE using the generalized power series method. It is enough to find the correct recursion formula for the coefficients in the generalized power series expansions.

(b) Are all solutions to this equation continuously differentiable at $x = 0$?

4. (4 points) Solve the following initial value problem using the Laplace transform:

$$\begin{aligned} y''(t) - y(t) &= 3e^{3t}, \\ y(0) &= 2, \\ y'(0) &= 1. \end{aligned}$$

5. (4 points) Let $c_1, c_2 \in \mathbb{R}$ and consider the boundary value problem

$$\begin{aligned} y''(x) + 2y'(x) + 2y(x) &= 0, \\ y(0) &= c_1, \\ y(L) &= c_2. \end{aligned}$$

For what $L > 0$ does there exist a unique solution?

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6. (4 points) Let E denote the function

$$E(x_1, x_2) := x_1^2 + \sin^2(x_2).$$

(a) Find an autonomous system with an equilibrium point $\hat{x}$, for which E is a strict Liapunov function in some open set $\Omega \subseteq \mathbb{R}^2$ containing $\hat{x}$.

(b) Sketch the orbits of your autonomous system.

(c) Find all equilibrium points of your autonomous system.

(d) Determine if the equilibrium points are unstable, stable and/or asymptotically stable.

Good luck!