MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examiner: Oliver Petersen

Written exam
MM5026 VT26
Ordinary differential equations
May 26, 2026

No calculator, book or notes are allowed. Give complete justifications for your answers!
At least 14 points (including bonus) are needed in order to proceed to the ***voluntary oral exam****.*

1. (4 points) Consider the initial value problem

$$\begin{aligned} x'(t) &= te^{x(t)}, \\ x(0) &= 1. \end{aligned}$$

   (a) What is the solution?
   (b) For what $t \in \mathbb{R}$ is the solution defined?

2. (4 points)

   (a) Show that the Laplace transform of the function $f(t) = t$ is $\mathcal{L}[f](s) = \frac{1}{s^2}$ for all $s > 0$.
   (b) Solve the following initial value problem using the Laplace transform:

$$\begin{aligned} y''(t) - y(t) &= t, \\ y(0) &= 0, \\ y'(0) &= -1. \end{aligned}$$

3. (4 points) Find one solution to the system of ODE

$$\mathbf{x}'(t) = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

4. (4 points) Consider the autonomous system

$$\begin{aligned} x'(t) &= ay(t) + bx(t)y(t), \\ y'(t) &= cx(t)y(t) + dx(t), \end{aligned}$$

   for some real numbers a, b, c and d.

   (a) Choose real numbers a, b, c, d such that this system has precisely 2 equilibrium points, and compute the location of the equilibrium points.
   (b) Is the origin an unstable, stable and/or asymptotically stable equilibrium point for your system?

5. (4 points) Consider the ODE

$$x^2 y''(x) - \frac{x}{2} y'(x) + \frac{1}{2} y(x) = 0,$$

   for $x > 0$.

   (a) Find the general solution to this ODE near $x = 0$ using the generalized power series method.
   (b) Do all solutions to this ODE extend to continuously differentiable functions at $x = 0$?

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6. (4 points) Let $c_1, c_2 \in \mathbb{R}$ and consider the boundary value problem

$$\begin{aligned} y''(x) + ay'(x) + y(x) &= \sin(x), \\ y(0) &= c_1, \\ y(\pi) &= c_2, \end{aligned}$$

for some real number a. Show that if $|a| > 2$, then there is a unique solution to this problem, defined for all $x \in [0, \pi]$.

Good luck!