MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examiner: Annemarie Luger

Written exam
MM5026 VT26
Ordinary differential equations
August 28, 2026

No calculator, book or notes are allowed. Give complete justifications for your answers!
At least 14 points (including bonus) are needed in order to proceed to the ***voluntary oral exam****.*

1. (4 points) Consider the initial value problem

$$\begin{aligned} x'(t) + x(t)^2 &= 1, \\ x(0) &= \frac{1}{2}. \end{aligned}$$

   (a) How many solutions does the above initial value problem have?
   (b) Determine all solutions!
   (c) Specify for each solution the maximal interval of existence!

2. (4 points) Consider the ODE

$$(x-1)y''(x) + 2xy'(x) - 3y(x) = 0.$$

   (a) Do all solutions to this ODE sufficiently near $x = 0$ extend to continuously differentiable functions at $x = 0$?
   (b) Find the general solution to this ODE near $x = 0$ using the power series method. It is enough to find the recursion relation.

3. (4 points) Solve the following initial value problem using the Laplace transform:

$$\begin{aligned} y'''(t) + 4y''(t) + 5y'(t) + 2y(t) &= 2e^t, \\ y(0) &= 0, \\ y'(0) &= 0, \\ y''(0) &= 1. \end{aligned}$$

   Hint: You may use the algebraic identites

   - $s^3 + 4s^2 + 5s + 2 = (s+1)^2(s+2)$, for any $s \in \mathbb{R}$,
   - $\frac{1}{(s+1)(s+2)(s-1)} = \frac{1}{6(s-1)} - \frac{1}{2(s+1)} + \frac{1}{3(s+2)}$, for any $s \neq -2, -1, 1$.

4. (4 points) Find the general solution to the system of ODEs

$$\mathbf{x}'(t) = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} 2 \\ 2 \end{pmatrix}.$$

5. (4 points) Consider the autonomous system

$$\begin{aligned} x'(t) &= -x(t)y(t)^2 + x(t)^2, \\ y'(t) &= -2y(t) - 1. \end{aligned}$$

   For what equilibrium points does there exist a Lyaponov function?

Please turn the page!

6. (4 points) Consider the boundary value problem

$$\begin{aligned} y''(x) + 2y'(x) + ay(x) &= 0, \\ y(0) &= 1, \\ y(\pi) &= 2, \end{aligned}$$

for some real number a.

(a) For what $a \in \mathbb{R}$ does there exist a unique solution to this problem, defined for all $x \in [0, \pi]$?

(b) Find all solutions to the boundary value problem for those $a \in (-3, 3)$ for which there does **not** exist a unique solution.