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MT5020: The mathematics and statistics of infectious disease outbreaks

Written Exam, May 28, 2025

Writing time: 14.00-17.00.

Allowed to bring to exam: Pocket calculator (not mobile). Pocket calculators can also be borrowed at exam (but of course you will not be familiar with them). Summary or formula sheet is **not allowed**.

Exam points: Each exercise gives a prescribed maximum points, with an overall maximum sum of 35 points. How these affect final grade is explained on course page.

Your solutions: (that you turn in) must be clearly written and answers need to be motivated (except Exercise 1).

Final size equation: If the basic reproduction number equals R_0 and a fraction r are initially immune, then the final size equation (for a homogeneously mixing community) determining the fraction τ^* among the initially susceptible, is the largest solution to the equation $1-\tau = e^{-R_0(1-r)\tau}$.

The renewal equation: If $\{g_s\}$ is the generation interval distribution, the current reproduction number equals R_t, and $\{I_k\}$ is the observed daily incidence of new cases, then the renewal equation (for a homogeneous community) states that $E(I_t) = R_t \sum_{s\geq 1} I_{t-s} g_s$.

Problem 1 [5P]

Multiple choice exercise. Just answer i), ii), iii) or nothing on each of the 5 subexercises. No motivation or computation! Each correct answer gives 1 point and each wrong answer gives -1 point (so better not to answer if you don't know). A negative total sum gives 0 points on the exercise.

(a) What is the goal of nowcasting incidence?

i) To estimate the current number that infectious people infect on average.

ii) To estimate the current number of new infections (or fatalities), taking reporting delays into account.

iii) To evaluate the efficiency of a forecast method in retrospect.

(b) What is the goal of scenario modelling?

i) To study different scenarios in epidemic modelling in retrospect.

ii) To estimate seasonal scenarios from historical data and include them in the epidemic model.

iii) To model how many that will get infected in the future under different model scenarios.

(c) What is counterfactual analysis?

i) To retrospectively analyse a hypothetical epidemic outcome under a different scenario than actually took place.

ii) To predict the outcome of a different scenario than what is planned for.

iii) To compare different epidemic models for the same situation.

(d) Which statement below describes the meaning of R_t best?

i) The expected number of people getting infected during the epidemic if it stops at time t

ii) The current (around time t) average number of infections by an infected individual.

iii) The average number of infections by an infected individual early in the epidemic.

(e) What is the main effect of an early massive lockdown if all preventions are later dropped?

i) It reduces the peak incidence.

ii) It reduces the final fraction getting infected.

iii) It delays the epidemic.

Problem 2 [10P]

Consider the first wave of the Covid-19 pandemic and assume we can approximate the spread of Covid-19 in the Göteborg region by a simple SIR epidemic model for a homogeneously mixing community. A reasonable estimate for R_0 in the Göteborg region during the first wave is $R_0 = 2.2$.

(a) Assuming a negligible fraction are initially infected from outside and that no preventive measures are put in place, what fraction will get infected in case of a major outbreak? Give an expression for it, but compute it also to numerically using calculator to within 10% from true value. **[3P]**

(b) Suppose that the community manages to insert preventive measures very early in the epidemic thus reducing R_0 by 40%. What fraction would get infected in case of a major outbreak in this scenario? Give an expression for it, but compute it also to numerically using calculator to within 10% from true value. **[2P]**

(c) Assume that 80% of the population remain susceptible in the early autumn of 2020 (so 20% got infected during the first wave and are then completely immune). Assume further that the preventive measures in the autumn are such that the number of infectious contacts as compared with no prevention are reduced by 30% (instead of 40% reduction during the spring). The effective reproduction number is hence reduced both by prevention and by immunity. What value does it take? **[2P]**

(d) Assuming that a second wave takes off and assuming no additional preventive measures (so reduced contacts by 30% althrough), what fraction of those who were susceptible at the start of the second wave will get infected? What fraction of the whole population would get infected? Give an expression for both questions, but compute them also to numerically using calculator to within 10% from true value. **[3P]**

Problem 3 [10P]

Assume a new influenza outbreak has just started in a small town and the following daily incidence has been reported the first week: $I_1 = 3,\ I_2 = 5,\ I_3 = 5,\ I_4 = 7,\ I_5 = 9, I_6 = 12,\ I_7 = 14$. Assume further (for simplicity) that the generation interval is such that all infections happen 3 or 4 days after getting infected, and these two days have equal probability.

(a) State what the generation interval distribution is and compute $E(I_8)$ in terms of R_0 (also depending on $\{g_s\}$ and the reported incidence the first week) using the renewal equation. Since it is the beginning of an outbreak we replace R_t by R_0 in the renewal equation. **[3P]**

(b) Suppose $I_8 = 16$ is observed. Use this to estimate R_0 from the renewal equation. **[3P]**

(c) Suppose that the 8 reported cases from the first two days ($I_1 = 3,\ I_2 = 5$) were contact traced and it was found out that they each infected the following number of individuals: 1, 3, 2, 4, 1, 2, 0, 1. Estimate R_0 using this information. **[4P]**

Problem 4 [10P]

Consider the Stochastic Markovian SIR epidemic model in which infectious individuals have infectious contacts at rate $\beta = 6$ (each contact with a uniformly selected individual) and recover at rate $\gamma = 2$.

(a) What is the mean infectious period and what is R_0? **[3P]**

(b) Suppose a community fraction v is vaccinated prior to the outbreak with a vaccine having 100% efficacy. What is then the reproduction number R_v? And what is the critical vaccination coverage v_c required to surely prevent an outbreak (i.e. obtaining herd immunity)? **[4P]**

(c) Suppose $v = 50\%$. What is R_v? How much does the contact rate have to be reduced (using various preventive measures) to reduce the effective reproduction number R_E (taking both vaccine immunity and preventions into account) below 1? In other words, if the contact rate is reduced by a fraction p, how large must p be for $R_E \leq 1$? **[3P]**