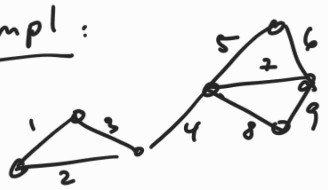
Some examples for matroids on graphs that are not defined in terms of TREES

EXMPL1

Given graph $G = (V, E)$

$\mathcal{F} = \{ E' \mid E' \subseteq E$ & E' is a subset of edges of unions of simple cycles$\}$

Exmpl:



edges of cycles: $C_1 = \{1, 2, 3\}$

$C_2 = \{5, 6, 7\}$

$C_3 = \{7, 8, 9\}$

$C_4 = \{5, 6, 8, 9\}$

• $E' = \{1, 2\} \in \mathcal{F}$ since $E' \subseteq C_1$

• $E' = \{1, 2, 5, 7\} \in \mathcal{F}$ since $E' \subseteq C_1 \cup C_2 \cup C_3$

[Basis here: $C_1 \cup C_2 \cup C_3$]

(M1) $\emptyset \in \mathcal{F}$ ✓

(M2) if $E' \in \mathcal{F}$ & $E'' \subseteq E'$ $\longrightarrow$ E' subsets of union of edges of cycles. & since $E'' \subseteq E'$

$\Rightarrow$ E'' subsets of union of edges of cycles.

$\Rightarrow$ $E'' \in \mathcal{F}$.

(M3) let $E', E'' \in \mathcal{F}$ & $|E'| > |E''|$

let $e \in E' \setminus E''$

$\Rightarrow$ since $e \in E'$ $\Rightarrow$ $\{e\}$ subset of union of edges of cycles.

This together with E'' subset of unions of edges of cycles

$\Rightarrow$ $E'' \cup \{e\}$ ——— " ———

$\Rightarrow$ $E'' \cup \{e\} \in \mathcal{F}$

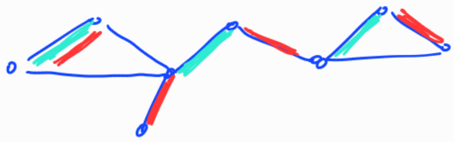
$\Rightarrow$ $(E, \mathcal{F})$ is matroid.

$\square$

Example „MATCHINGS"

For $G=(V,E)$ a matching $M \subseteq E$ is a subset of the edges of G s.t. $\forall\, e,f \in M,\ e \neq f$ holds that $e \cap f = \emptyset$.

["no two edges share a common vertex"]



M_1 = set of green edges

M_2 = set of red edges

Observation: both M_1 & M_2 are inclusion-maximal but have different size.

$\Rightarrow$ $(E, \mathcal{F})$ with $\mathcal{F} = \{E' \subseteq E \mid E' \text{ is subset of matching}\}$

does not form matroid (why? Ex!)

But we may look at vertices that are covered by matchings: $v \in V$ is covered by M if $\exists\, e \in M$ s.t. $v \in e$.

Consider $(V, \mathcal{F})$ with $\mathcal{F} = \{X \subseteq V :$ all vertices in X are covered by some matching $M\}$.

[that is $X \subseteq \bigcup_{e \in M} e$]

$(V, \mathcal{F})$ is matroid

(M1) $\emptyset \in \mathcal{F}$ ✓

(M2) $X \in \mathcal{F}$, $Y \subseteq X$ $\Rightarrow$ $Y \subseteq X$ all vertices in X & thus in Y are covered by some matching M.

$\Rightarrow Y \in \mathcal{F}$.

(M3) difficult to show (beyond this course) via Edmond-Matching alg & augmenting paths.